Schallwellen einer Energie u_α

$$\rho \frac{\partial^2 u_\alpha}{\partial t^2} = \sum_{\beta\gamma\delta} C_{\alpha\beta\gamma\delta} \frac{\partial^2 u_\delta}{\partial r_\beta \partial r_\gamma}$$

Lösung $u_\alpha = u_{0\alpha} \cdot e^{i\omega t - i\underline{q}\underline{r}}$ Wellenvektor $\underline{q}$

kubische Kristalle

$$\begin{cases} \rho \frac{\partial u_x^2}{\partial t^2} = C_{11} \frac{\partial^2 u_x}{\partial x_2} + C_{44}\left(\frac{\partial^2 u_x}{\partial^2 y^2} + \frac{\partial^2 u_x}{\partial z^2}\right) + (C_{12} + C_{44})\left(\frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial^2 u_z}{\partial x \partial z}\right) \\ \rho \frac{\partial^2 u_y}{\partial t^2} = \text{zykl. Vertauschung von } x, y, z \\ \rho \frac{\partial^2 u_z}{\partial t^2} = \text{---} \;\; '' \;\; \text{---} \end{cases}$$

Dispersionsrelation $\boxed{\omega_\alpha = v_\alpha \, q}$ linear $\hat{=}$ dispersionslos

- 3 Moden:

1 longitudinal $\underline{u} \parallel \underline{q}$ L

2 transversal $\underline{u} \perp \underline{q}$ T

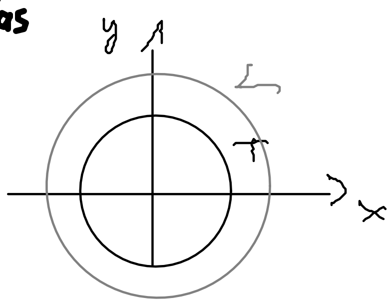
Isotropes Medium

$\vec{u}_1 \parallel \underline{q}$: $\frac{\omega_{\parallel}}{q} = \sqrt{\frac{C_{11}}{\rho}} = v_\parallel$ L

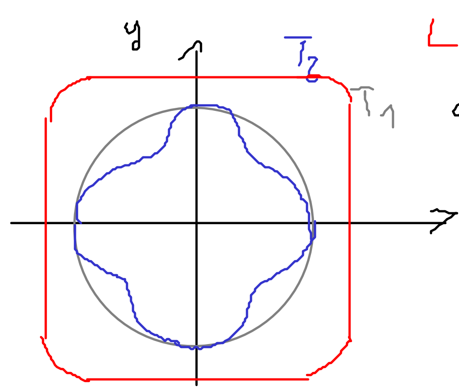
$\vec{u}_2, \vec{u}_3 \perp \underline{q}$: $\frac{\omega_\perp}{q} = \sqrt{\frac{C_{44}}{\rho}} = v_\perp$ T

$v_\alpha \approx 1$ bis $10 \frac{km}{s}$

Glas



GaAs



durch Entartung

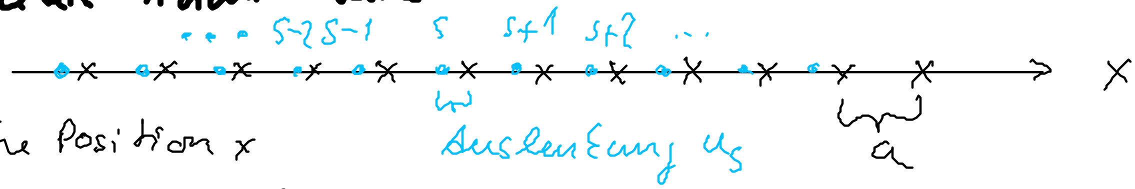
- Messung von : C_{11}, C_{44} $\xrightarrow{\text{Richtg}}$ [100]

[110] $\longleftarrow$ C_{12}

$$\boxed{C_{11} - C_{12} = 2 C_{44}}$$

Gitterschwingungen

linear 1dim Kette



ruhe Position x

Massen alle gleich M

BGL: $M \frac{d^2 u_s}{dt^2} = \sum_{n=-\infty}^{\infty} c_n (u_{s+n} - u_s)$

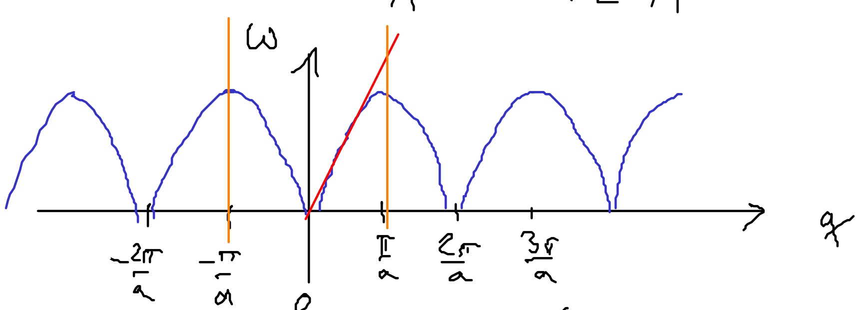
Ansatz: $u_{s+n} = V e^{-i(\omega t - qna)}$

$$\omega^2 M = \sum_{n=-\infty}^{\infty} c_n (1 - e^{iqna}) \quad ; \quad c_{-n} = c_n$$

$$\omega^2 = \frac{1}{M} \sum_{n=1}^{\infty} c_n \left(2 - e^{iqna} - e^{-iqna} \right) = \frac{2}{M} \sum_{n=1}^{\infty} c_n (1 - \cos(qna))$$

$c_1 \gg c_n$ $(n > 2)$ $\quad \omega^2 = \frac{2c_1}{M} (1 - \cos(qa)) = \frac{4c_1}{M} \sin^2\left(\frac{qa}{2}\right)$

$$\omega = 2 \sqrt{\frac{c_1}{M}} \left| \sin\left(\frac{qa}{2}\right) \right|$$



$q \ll a$: $\lambda \gg$ Gitterkonst

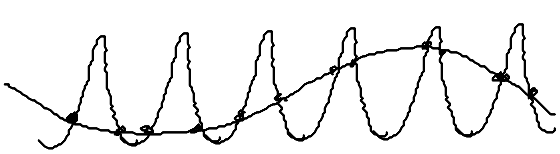
$\sin qa \approx qa$

Einschränkung auf 1. BZ

Bei $c_2 \neq 0$ $\quad \omega^2 = \frac{4c_1}{M} \left(\sin^2\left(\frac{qa}{2}\right) + \frac{c_2}{c_1} \sin(qa) \right)$

Phasenunterschied $\quad \frac{u_{s+1}}{u_s} = e^{iqa} \qquad -\pi < qa < \pi$

$$q' \longrightarrow q + \frac{2\pi N}{a}$$



Phasengeschwindigkeit $\quad v = \frac{\omega}{q}$

Gruppengeschwindigkeit $\quad v_g = \frac{\partial \omega}{\partial q}$

$q \longrightarrow 0, \lambda \longrightarrow \infty \qquad \cos\alpha = 1 - \frac{1}{2}\alpha^2 + \dots$

$$\omega^2 \approx \frac{q^2 a^2}{M} \sum_{n=1}^{\infty} n^2 c_n$$

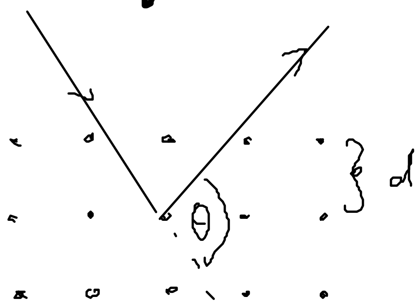
$$\nearrow c_n = \sum_{n=1}^{\infty} \frac{n^2}{a} c_n^L$$

$$\searrow c_n = \sum_{n=1}^{\infty} \frac{n^2}{a} c_n^T$$

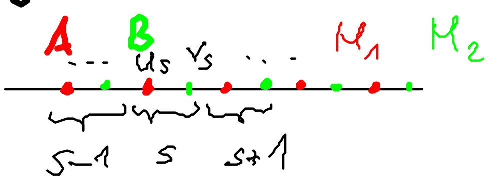
$|q| \longrightarrow \frac{\pi}{a}, \quad \lambda = 2a \qquad \frac{u_{s+1}}{u_s} = e^{\pm i\pi} = 1$

Bragg-Reflexion $\qquad 2d \sin\theta = \lambda$

$\theta = 90°$: $\quad 2d = \lambda$ (am Rand der BZ)



Gitter mit zweiatomiger Basis



$$M_1 \frac{d^2 u_s}{dt^2} = C'(v_s - u_s) - C''(u_s - v_{s-1})$$

$$M_2 \frac{d^2 v_s}{dt^2} = C''(u_{s+1} - v_s) - C'(v_s - u_s)$$

Ansatz $\qquad u_s = u\, e^{-i(\omega t - qsa)} \qquad v_s = v\, e^{-i(\omega t - qsa)}$

- det (Los Matrix) = 0 : $\quad \omega_\pm = \frac{\omega_0^2}{2}\left(1 \pm \sqrt{1 - \gamma^2 \sin^2 \frac{qa}{2}}\right)$

  $\gamma = 4 \frac{\sqrt{C'C''}}{C'+C''} \cdot \frac{\sqrt{M_1 M_2}}{M_1 + M_2} \qquad \omega_0 = (C' + C'')\left(\frac{1}{M_1} + \frac{1}{M_2}\right)$

- für $C' = C''$ und $M_1 = M_2$ $\Rightarrow$ einatomige Basis mit $a' = 2a$