Nanophotonics

Martin Wegener

1. **Basics**
    - 1.1. Analogy between photonics and electronics
    - 1.2. Paradigms of nanophotonics
        - 1.2.1. Evanescent waves & the limit of electrostatics
        - 1.2.2. Mie-scattering of nanoparticles
        - 1.2.3. Nanoscale metal particles: Particle plasmons
        - 1.2.4. A nano-aperture in an ideal metal film: Bethe-Bouwkamp theory

2. **Photonic band gap (PBG) materials**
    - 2.1. One-dimensional Photonic Crystals (i.e., dielectric mirrors)
    - 2.2. Two-dimensional Photonic Crystals
        - 2.2.1. Negative refraction and the superprism effect
        - 2.2.2. “Semiconductors for light“: Waveguides and defect cavities
    - 2.3. Three-dimensional Photonic Crystals
        - 2.3.1. Quantum optics: Modified Planck‘s law
        - 2.3.2. Fabrication: Overview and state-of-the-art
        - 2.3.3. Fabrication: Current efforts within the CFN

3. **Metamaterials**
    - 3.1. Left-handed or Veselago materials (LHM)
    - 3.2. “Perfect lenses“ made from LHM
    - 3.3. Towards metamaterials @ optical frequencies (CFN activities)

4. **Plasmonics**
    - 4.1. Field-enhancement & surface-enhanced Raman scattering (SERS)
    - 4.2. Surface-plasmon amplification by stimulated emission of radiation (SPASER)
    - 4.3. Transmission through sub-wavelength hole arrays

Textbooks

“Principles of Nano-Optics”, L. Novotny and B. Hecht,
Cambride University Press (2006)
(best textbook available, in-depth mathematical discussion)

“Nanophotonics“, P.N. Prasad,
John Wiley & Sons (2004)
(recent comprehensive overview, nothing in depth,
good for finding further references and original work)

“Photonic Crystals“, J.D. Joannopoulos, R.D. Meade, J.N. Winn,
Princeton University Press
(nice textbook introduction into the theory, mostly 2D)

“Periodic Nanostructures for Photonics”, K. Busch … M. Wegener,
Physics Reports 444, 101-202 (2007)
(recent review on photonics crystals and metamaterials)

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The solutions of problems from nanophotonics ...

... are “simply” solutions of the Maxwell equations

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

In macroscopic media:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

see lectures within “Physik II”

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$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

For isotropic linear materials ...

$$\vec{D} = \varepsilon_0 \varepsilon \, \vec{E}$$

$$\vec{B} = \mu_0 \mu \, \vec{H}$$

see lectures within "Physik II"

Remember: The free Maxwell equations are scalable.

This means that if one replaces

$$\vec{r} \longrightarrow s\vec{r}$$

and at the same time

$$t \longrightarrow st$$

the solution remains the same.

see lectures within "Physik II"

Both, electrons and light are waves.

But … the (single-particle, 1D) Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = \left(-\frac{\hbar^2}{2m_\mathrm{e}}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi(x,t)$$

and the (scalar, 1D) wave equation for the electric field

$$\frac{\partial^2}{\partial x^2}E(x,t) - \frac{1}{c^2(x)}\frac{\partial^2}{\partial t^2}E(x,t) = 0$$

seem to be quite different mathematically at first sight.

see “Physik II“ and “THEORIE D“

Just a few simple manipulations ...

With the usual ansatz for the Schrödinger equation

$$\psi(x,t) = \psi(x)\exp(-\mathrm{i}\,\mathrm{E}/\hbar\, t)$$

and the ansatz for the wave equation

$$E(x,t) = E(x)\exp(-\mathrm{i}\omega t) + \mathrm{c.c.}$$

... and we get ...

… strictly the identical mathematical form

for the stationary Schrödinger equation

$$\frac{\partial^2}{\partial x^2}\psi(x)+\left(\frac{2m_\mathrm{e}}{\hbar^2}\left(\mathrm{E}-V(x)\right)\right)\psi(x)=0$$

$$=: a(x)$$

and for the light field

$$\frac{\partial^2}{\partial x^2}E(x)+\left(\frac{\omega^2\, n^2(x)}{c_0{}^2}\right)E(x)=0$$

with the refractive index $n(x)=\dfrac{c_0}{c(x)}$.

Beyond this "piecewise-constant", one-dimensional & scalar case, the analogy between electron waves and light waves is no longer complete because …

… optical materials can be inhomogeneous

$$\vec{H}(\vec{r},t) = \vec{H}(\vec{r})\,\exp(-\mathrm{i}\omega t) + \mathrm{c.c.}$$

$$\boxed{\frac{1}{\mu(\vec{r})}\vec{\nabla}\times\left(\enclose{circle}{\frac{1}{\varepsilon(\vec{r})}}\vec{\nabla}\times\vec{H}(\vec{r})\right) = \frac{\omega^2}{c_0^2}\vec{H}(\vec{r})}$$

Beyond this "piecewise-constant", one-dimensional & scalar case, the analogy between electron waves and light waves is no longer complete because ...

... optical materials can be inhomogeneous

... light waves are vector waves (polarization)

... and have both an electric and a magnetic component

... optical materials can be anisotropic or birefringent

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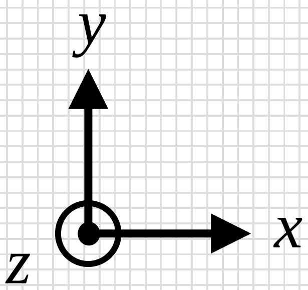
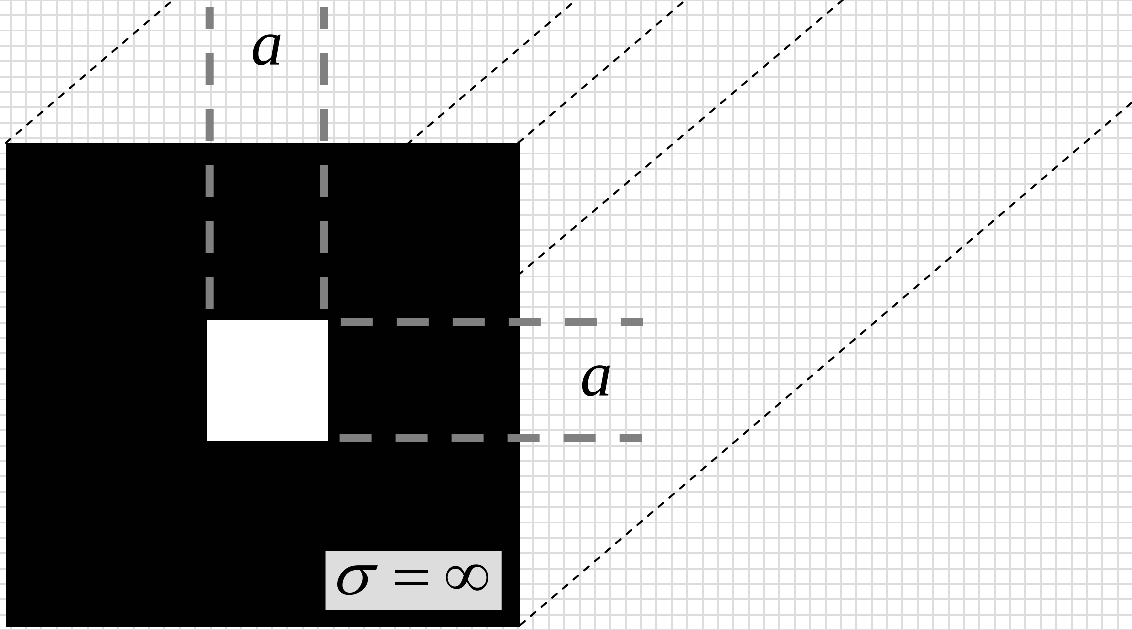
Example I

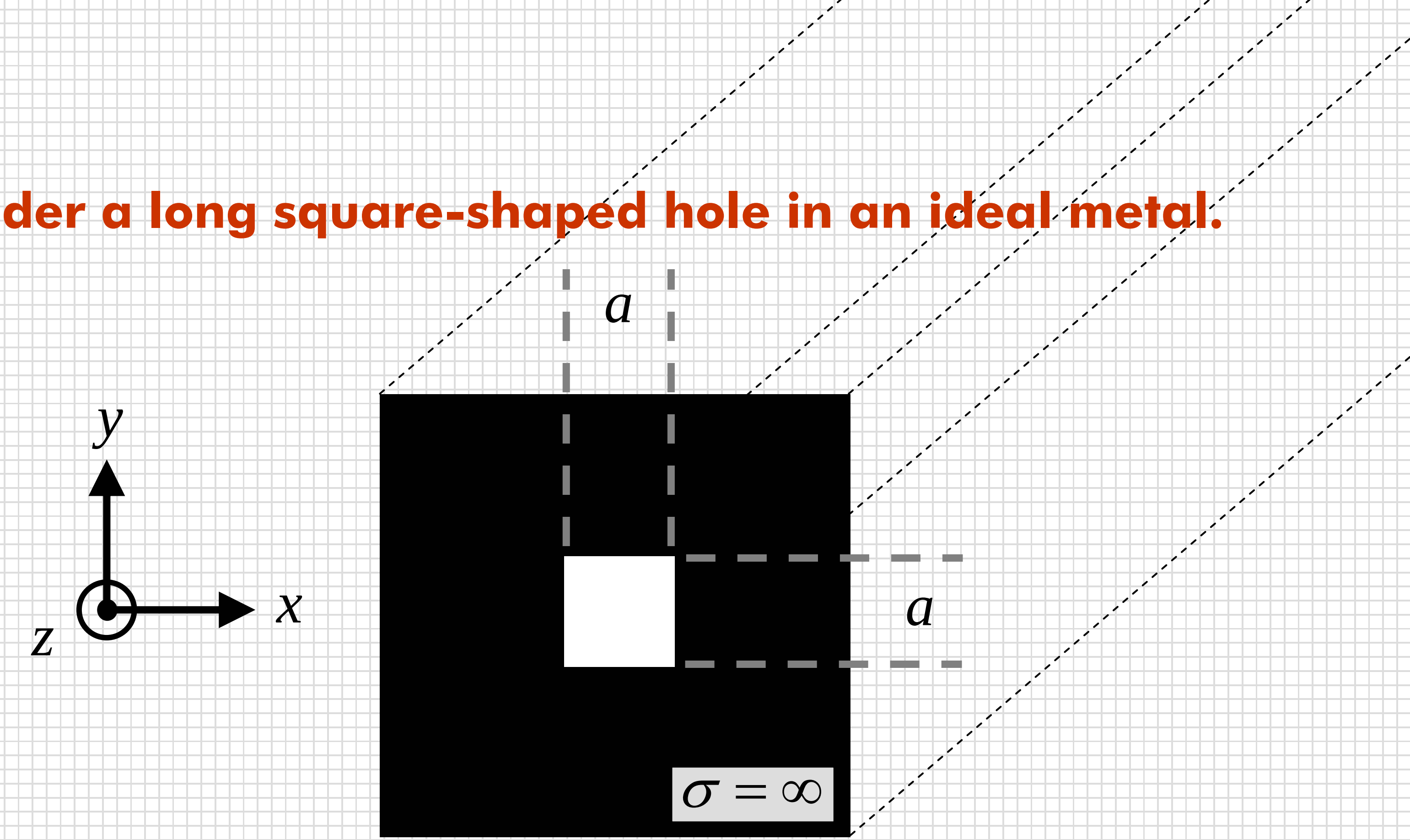
Consider a long square-shaped hole in an ideal metal.





$$c_0 = \frac{\omega}{|\vec{K}|} = \frac{\omega}{\sqrt{K_x^2 + K_y^2 + K_z^2}} = \frac{\omega}{\sqrt{\left(N_x \frac{\pi}{a}\right)^2 + \left(N_y \frac{\pi}{a}\right)^2 + K_z^2}}$$

Consider a long square-shaped hole in an ideal metal.

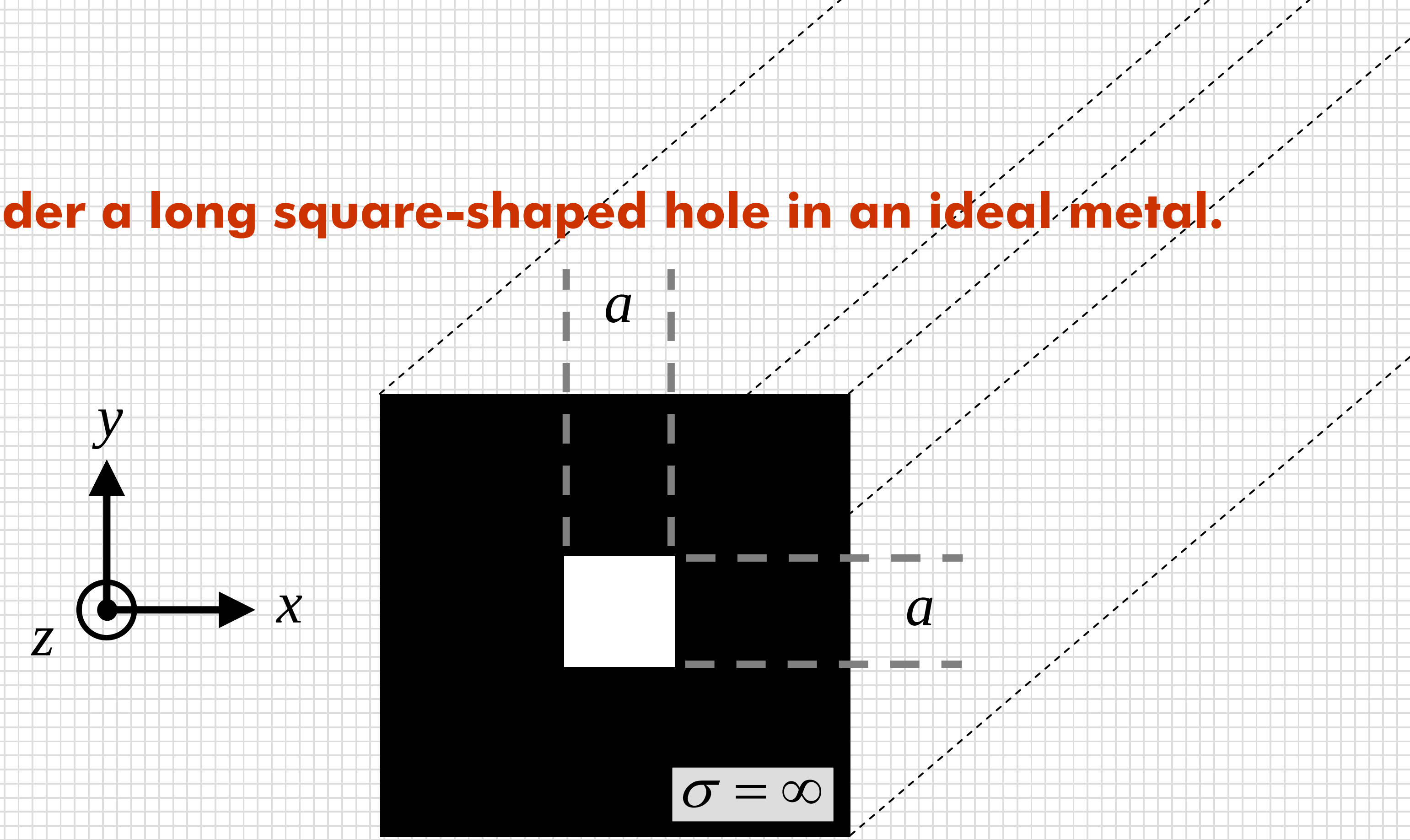


$$\Rightarrow K_z = \sqrt{\frac{\omega^2}{c_0^2} - \left(N_x \frac{\pi}{a}\right)^2 - \left(N_y \frac{\pi}{a}\right)^2}$$

This leads to evanescent waves for $\boxed{a < \lambda / \sqrt{2}}$ **.**

Martin Wegener

Consider a long square-shaped hole in an ideal metal.

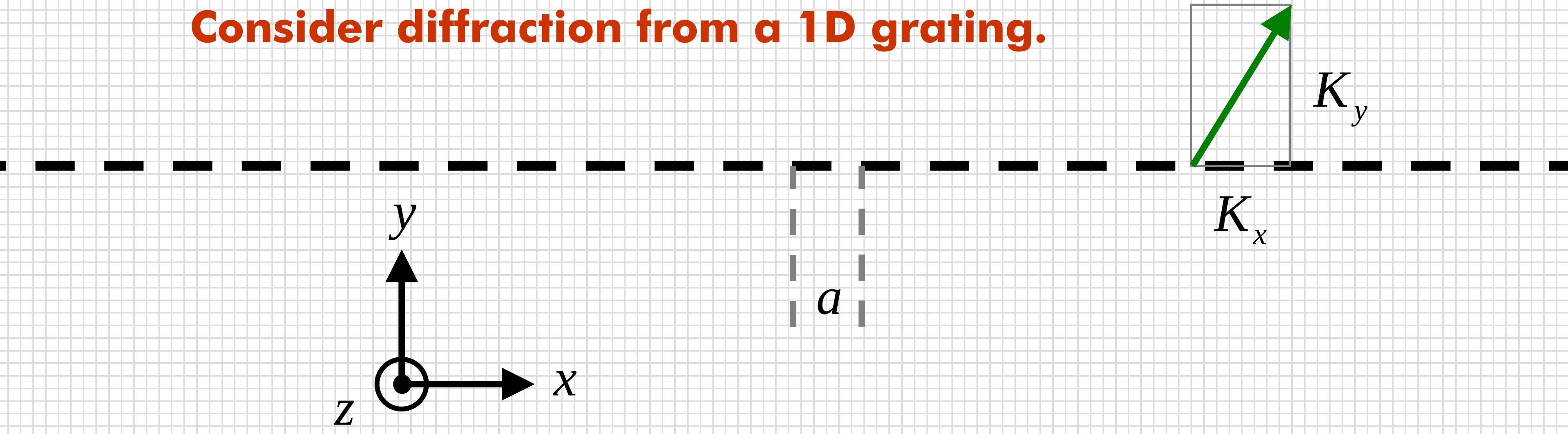


$$\Rightarrow \; K_z = \sqrt{\frac{\omega^2}{c_0^2} - \left(N_x \frac{\pi}{a}\right)^2 - \left(N_y \frac{\pi}{a}\right)^2}$$

With decreasing *a*, the decay is getting more rapid.

Example II

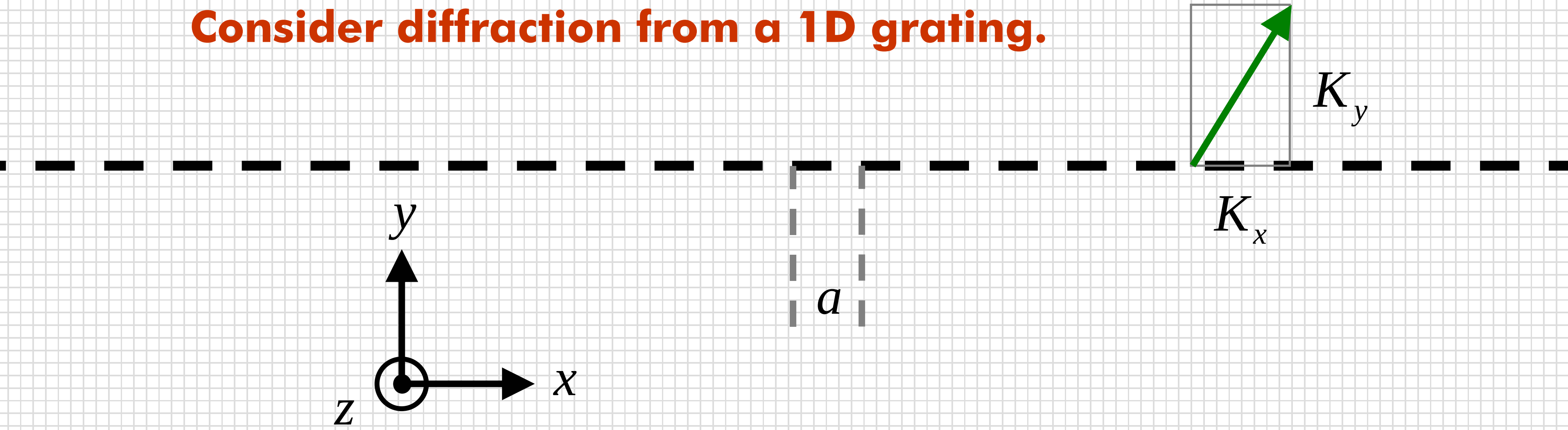
Consider diffraction from a 1D grating.



$$c_0 = \frac{\omega}{|\vec{K}|} = \frac{\omega}{\sqrt{K_x^2 + K_y^2 + K_z^2}} = \frac{\omega}{\sqrt{\left(N_x \frac{2\pi}{a}\right)^2 + K_y^2}}$$

reciprocal lattice vector

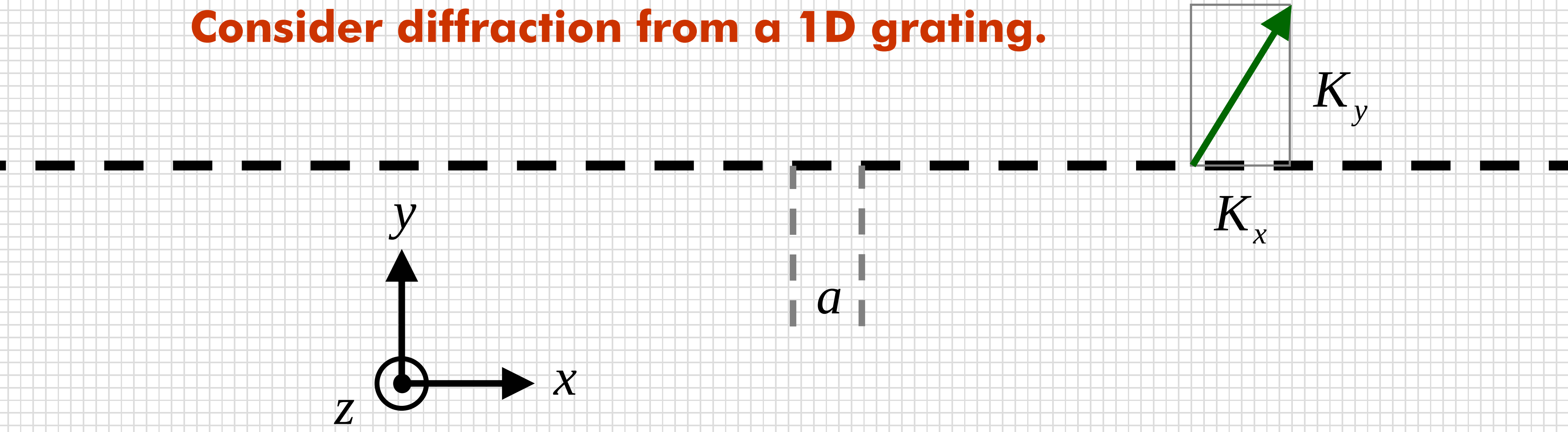
Consider diffraction from a 1D grating.



$$\Rightarrow K_y = \sqrt{\frac{\omega^2}{c_0^2} - \left(N_x \frac{2\pi}{a}\right)^2}$$

Lowest diffracted order becomes evanescent for $\boxed{a < \lambda}$ **.**

Consider diffraction from a 1D grating.



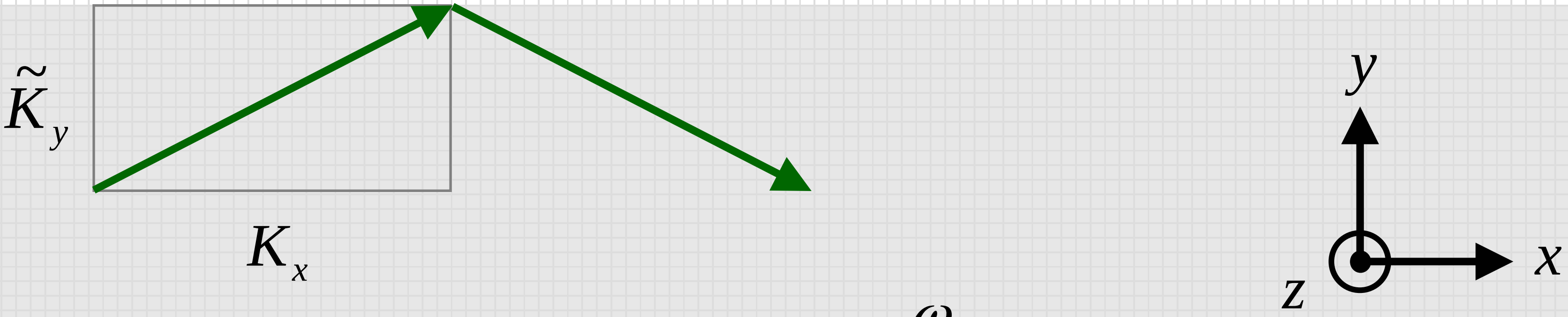
$$\Rightarrow K_y = \sqrt{\frac{\omega^2}{c_0^2} - \left(N_x \frac{2\pi}{a}\right)^2}$$

With decreasing a, the decay is getting more rapid.

Example III

Consider total internal reflection.

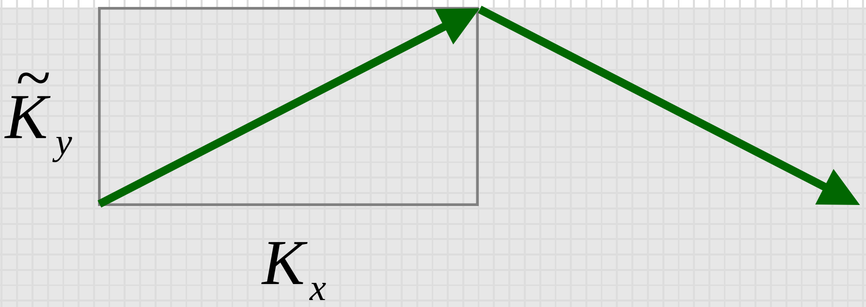
$$c_0 = \frac{\omega}{|\vec{K}|} = \frac{\omega}{\sqrt{K_x^2 + K_y^2}}$$

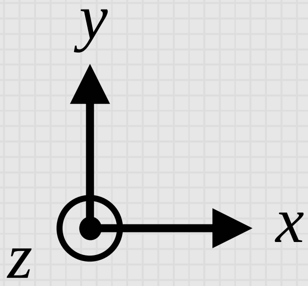


$$c_0 < c = \frac{\omega}{|\vec{K}|}$$

Consider total internal reflection.

$$\Rightarrow \; K_y^2 = \frac{\omega^2}{c_0^2} - K_x^2 < 0$$

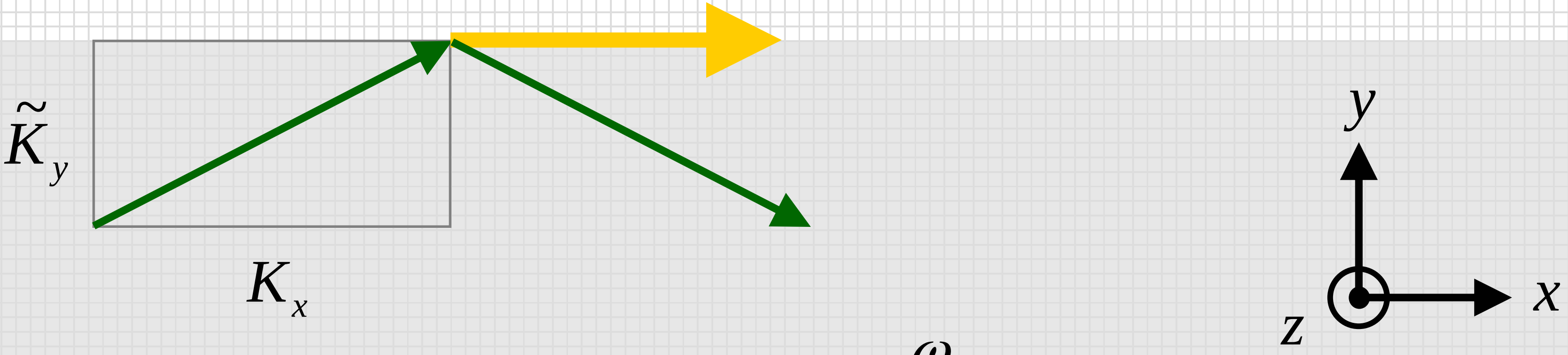




$$c_0 < c = \frac{\omega}{|\vec{K}|}$$

Consider total internal reflection.

evanescent surface wave



$$c_0 < c = \frac{\omega}{|\vec{K}|}$$

For very small length scales, i.e., for $\boxed{a << \lambda}$,

one can again apply the rules of electrostatics !

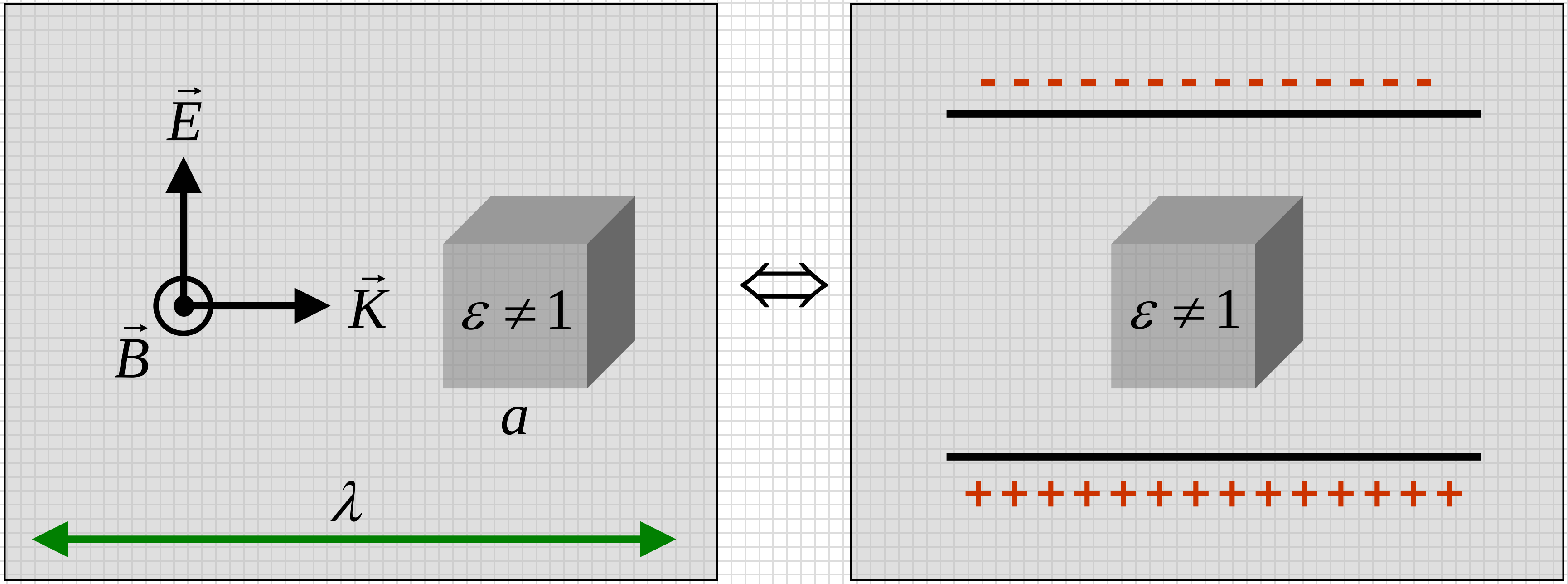
Here, the wavelength can be considered as infinitely large. According to the dispersion relation of light,

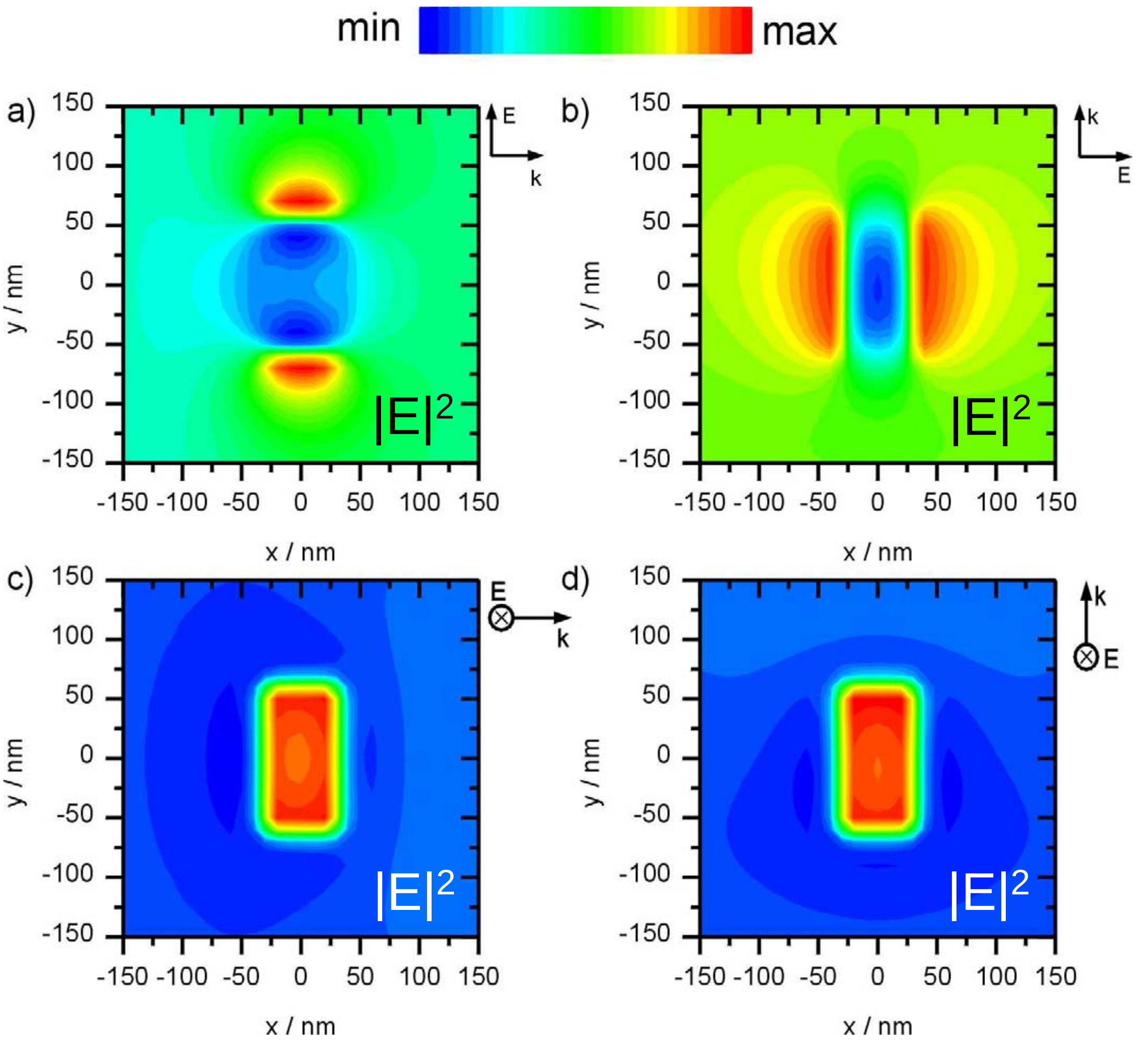
$$c_0 = \frac{\omega}{|\vec{K}|} = \frac{2\pi\ f}{2\pi\ /\lambda} = f\ \lambda$$

the frequency is approximately zero.

For **very small length scales**, i.e., for $\boxed{a << \lambda}$,

one can again apply the rules of **electrostatics** !





simulation for nano-brick with n=1.5, z=5nm, 60nm x 120nm x 40nm @ 633nm wavelength

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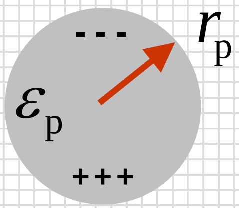
Consider scattering off a small dielectric sphere in air.

From a multipole expansion one gets, e.g., the scattering cross section (after a lenghty derivation)

$$C_n = \frac{2\pi}{K^2}\sum_{n=1}^{\infty}(2n+1)(|a_n|^2+|b_n|^2)$$

$$K = 2\pi/\lambda$$



with the coefficients

$$a_n = \frac{\sqrt{\varepsilon_\mathrm{p}}\,\psi_n(\sqrt{\varepsilon_\mathrm{p}}Kr_p)\,\psi'_n(Kr_p) - \psi_n(Kr_p)\,\psi'_n(\sqrt{\varepsilon_\mathrm{p}}Kr_p)}{\sqrt{\varepsilon_\mathrm{p}}\,\psi_n(\sqrt{\varepsilon_\mathrm{p}}Kr_p)\,\xi'_n(Kr_p) - \zeta_n(Kr_p)\,\psi'_n(\sqrt{\varepsilon_\mathrm{p}}Kr_p)}$$

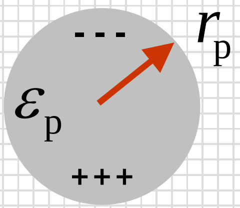
$$b_n = \ldots$$

***n*-th order Riccati-Bessel functions**

G. Mie, Ann. d. Physik 25, 377 (1908)

Consider scattering off a small dielectric sphere in air.

From a multipole expansion one gets the vectorial electric field as well ...



A readable discussion can be found in

- C.F. Bohren and D.R. Huffman,
  “Absorption and scattering of light by small particles“,
  John Wiley & Sons, 1983

- H.C. van de Hulst,
  “Light scattering by small particles“,
  Dover, 1981

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Consider a small metallic sphere in vacuum.

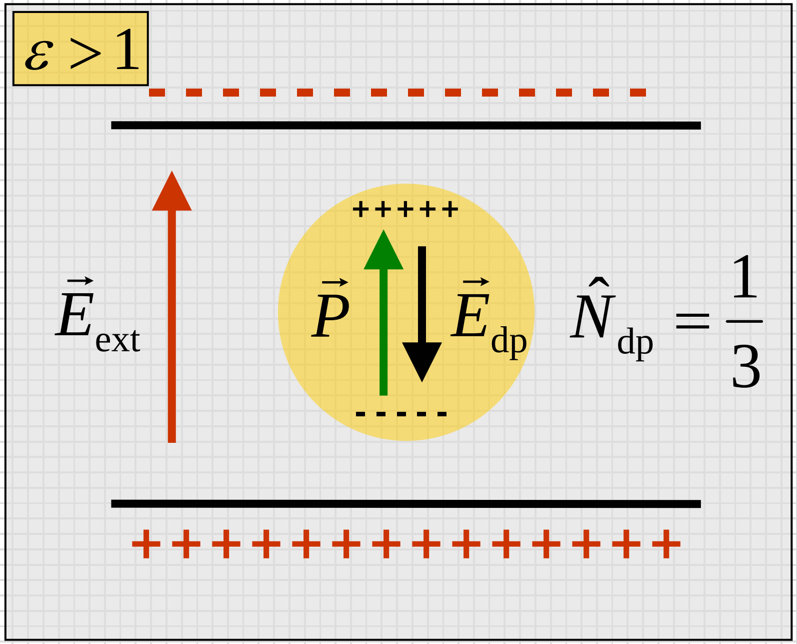
A reminder on the electrostatic depolarization factor:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \varepsilon_0 \chi \, \vec{E} = \varepsilon_0 \chi \, (\vec{E}_{\mathrm{ext}} + \vec{E}_{\mathrm{dp}})$$

$$\varepsilon_0 \vec{E}_{\mathrm{dp}} = -\hat{N}_{\mathrm{dp}} \vec{P} = -\frac{1}{3} \vec{P}$$

$$\boxed{\Rightarrow \vec{P} = 3\varepsilon_0 \frac{\varepsilon - 1}{\varepsilon + 2} \vec{E}_{\mathrm{ext}}}$$



“Clausius-Mossotti relation“ for polarizability
or “Lorenz-Lorenz relation“ if expressed via *n*

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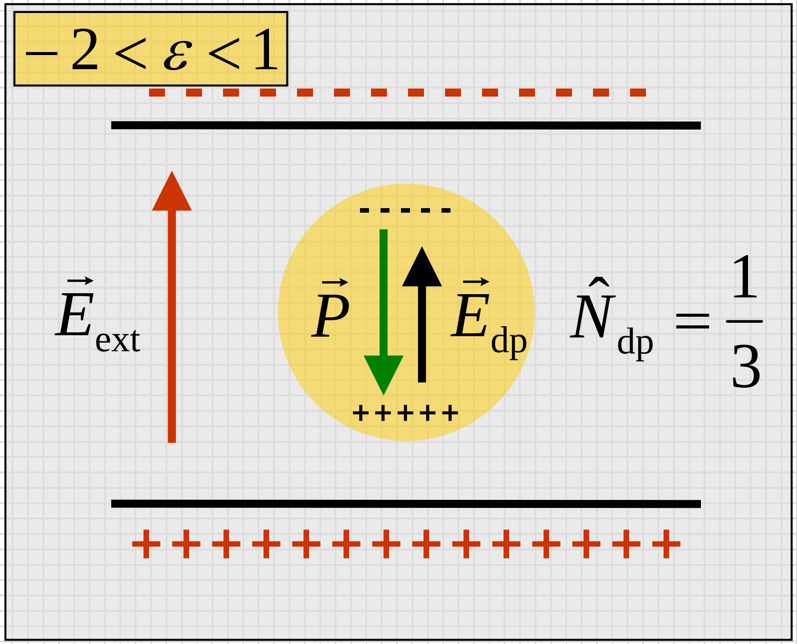

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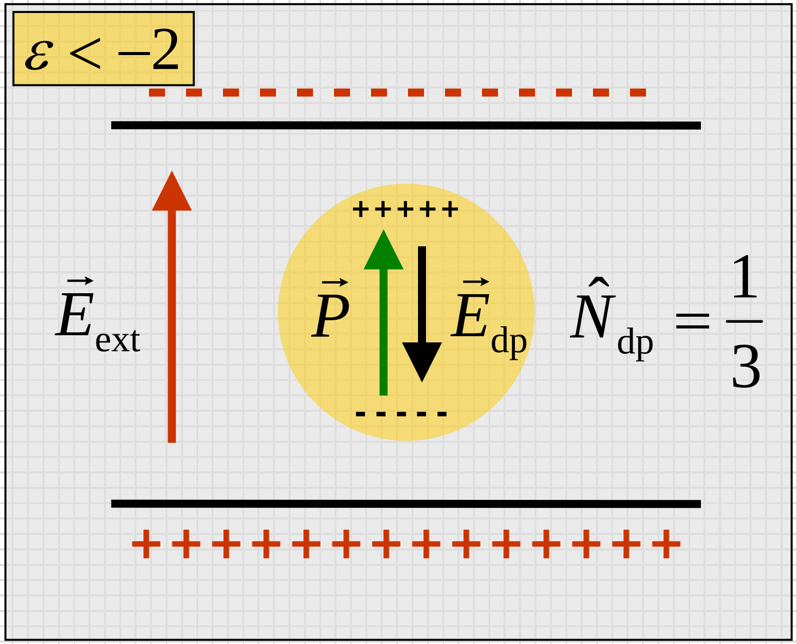


"Clausius-Mossotti relation" for polarizability or "Lorenz-Lorenz relation" if expressed via *n*

Consider a small metallic sphere in vacuum.

Light field excites within the long-wavelength limit:

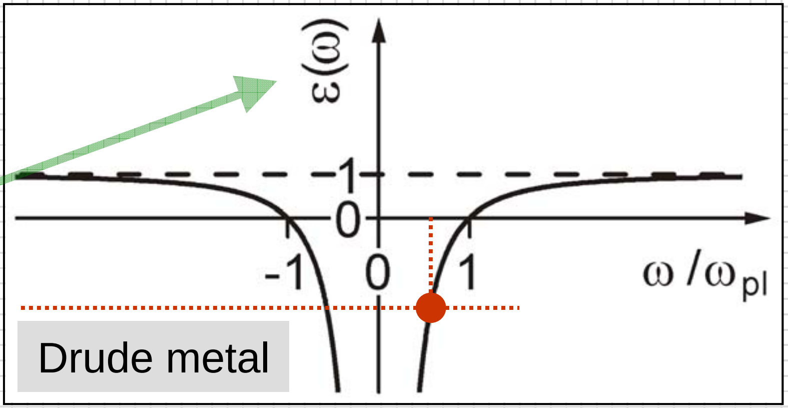
$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \varepsilon_0 \chi \vec{E} = \varepsilon_0 \chi \left(\vec{E}_{\mathrm{ext}} + \vec{E}_{\mathrm{dp}}\right)$$

$$\varepsilon_0 \vec{E}_{\mathrm{dp}} = -\hat{N}_{\mathrm{dp}} \vec{P} = -\frac{1}{3}\vec{P}$$

$$\boxed{\Rightarrow \vec{P} = 3\varepsilon_0 \frac{\varepsilon - 1}{\varepsilon + 2}\vec{E}_{\mathrm{ext}}}$$

resonance for $\boxed{\varepsilon(\omega) = -2}$



Mathematica graphics here

Consider a small metallic sphere in vacuum.

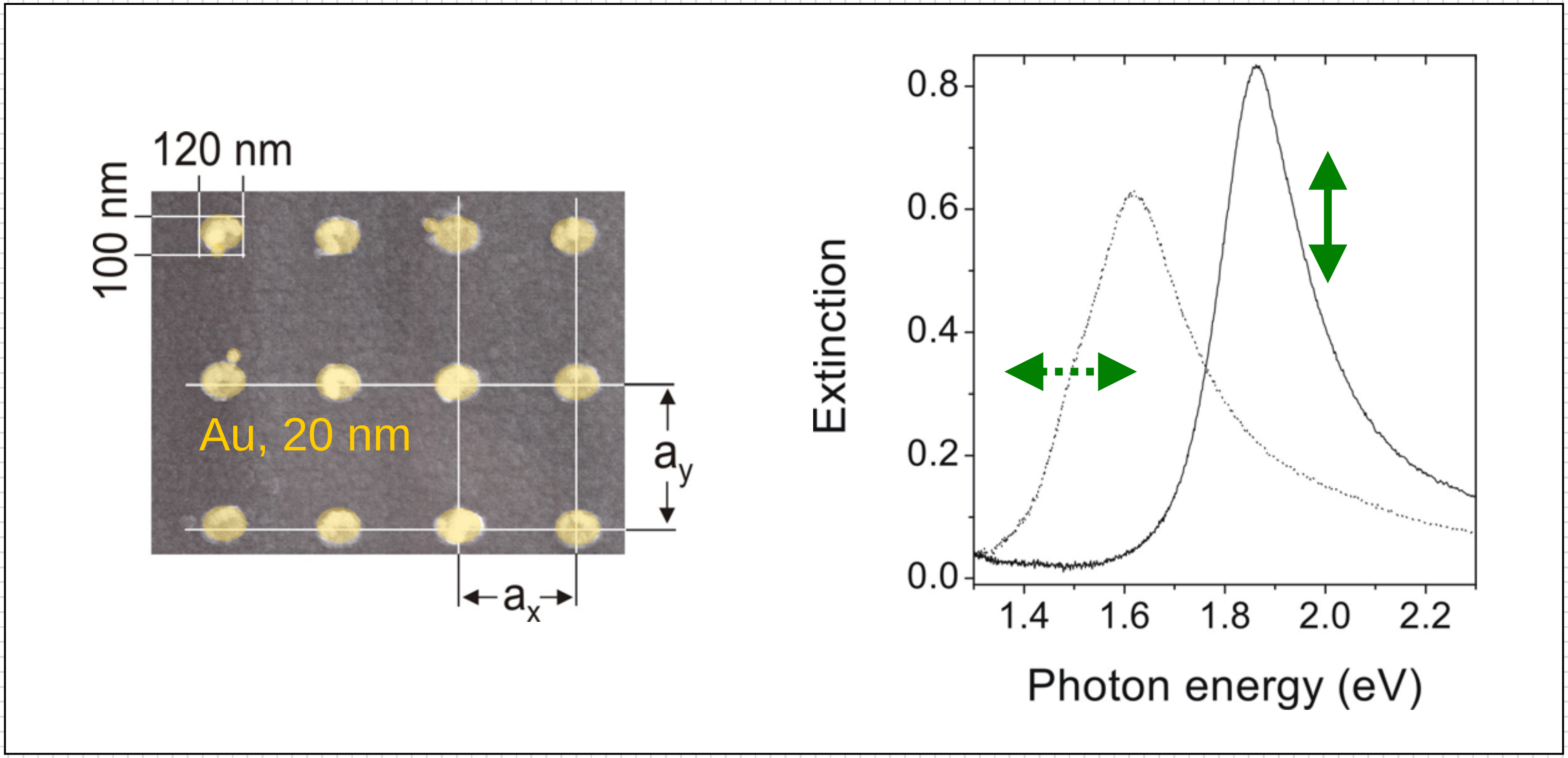
The optical response of a **bulk metal** can be described by the **Drude model**.

For a **metal nanoparticle**, one rather gets a **Lorentz oscillator** response.

The position of this **particle plasmon** resonance is also determined by the embedding dielectric medium and by the particle size & shape.

$$\boxed{\varepsilon(\omega) = -2\varepsilon_{\mathrm{med}}}$$!

Measurements on gold nanoparticles on ITO/glass:



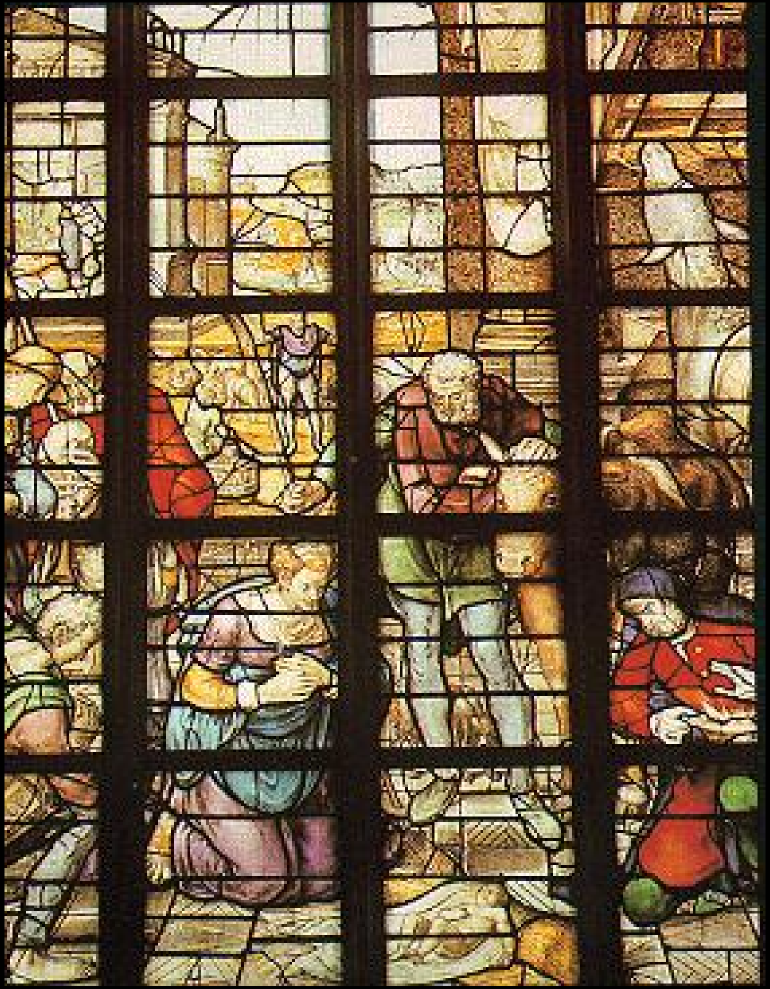
Lycurgus cup

British museum

Lycurgus cup

Ag and Au nanoparticles

Glass doped with metal nanocrystals



church window from Sint Jan in Gouda (Netherlands)

Exercise: Consider a thin (infinite) sheet of metal.

Where does the resonance occur?
For what polarization?

?

Exercise: Consider a thin (infinite) sheet of metal.

Where does the resonance occur?
For what polarization?

Solution: If the electric field is perpendicular to the plane of the sheet, we have

$$\hat{N}_{\mathrm{dp}} = 1 \Rightarrow \varepsilon(\omega) = 0 \Rightarrow \omega = \omega_{\mathrm{pl}}$$

If the electric field lies in the plane: No resonance, as

$$\hat{N}_{\mathrm{dp}} = 0$$

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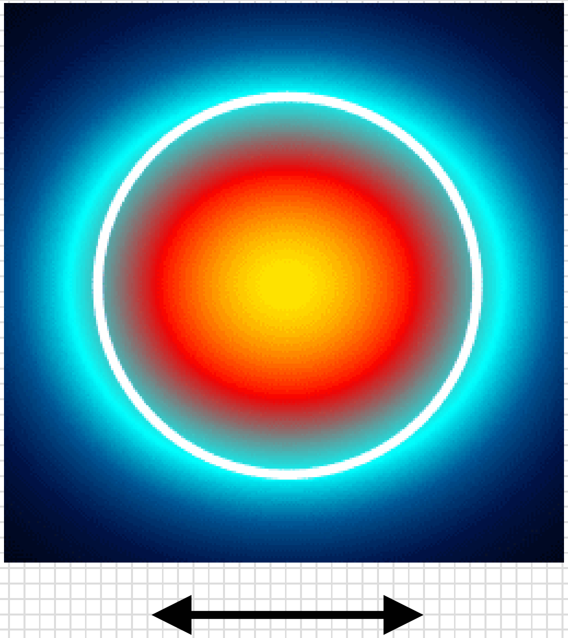
Consider a plane wave impinging onto a small circular hole in an infinitely thin film of an ideal metal.

This problem was solved by Bethe in 1944. In 1950, technical mistakes were eliminated by Bouwkamp. The **“Bethe-Bouwkamp theory“** is the exact analytical solution of this model problem.

It is relevant for scanning near-field optical microscopy (SNOM), which uses apertures in metal films.

H.E. Bethe, Phys. Rev. 66, 163 (1944) & C.J. Bouwkamp, Philips Res. Rep. 5, 321 (1950)

Consider a plane wave impinging onto a small circular hole in an infinitely thin film of an ideal metal.

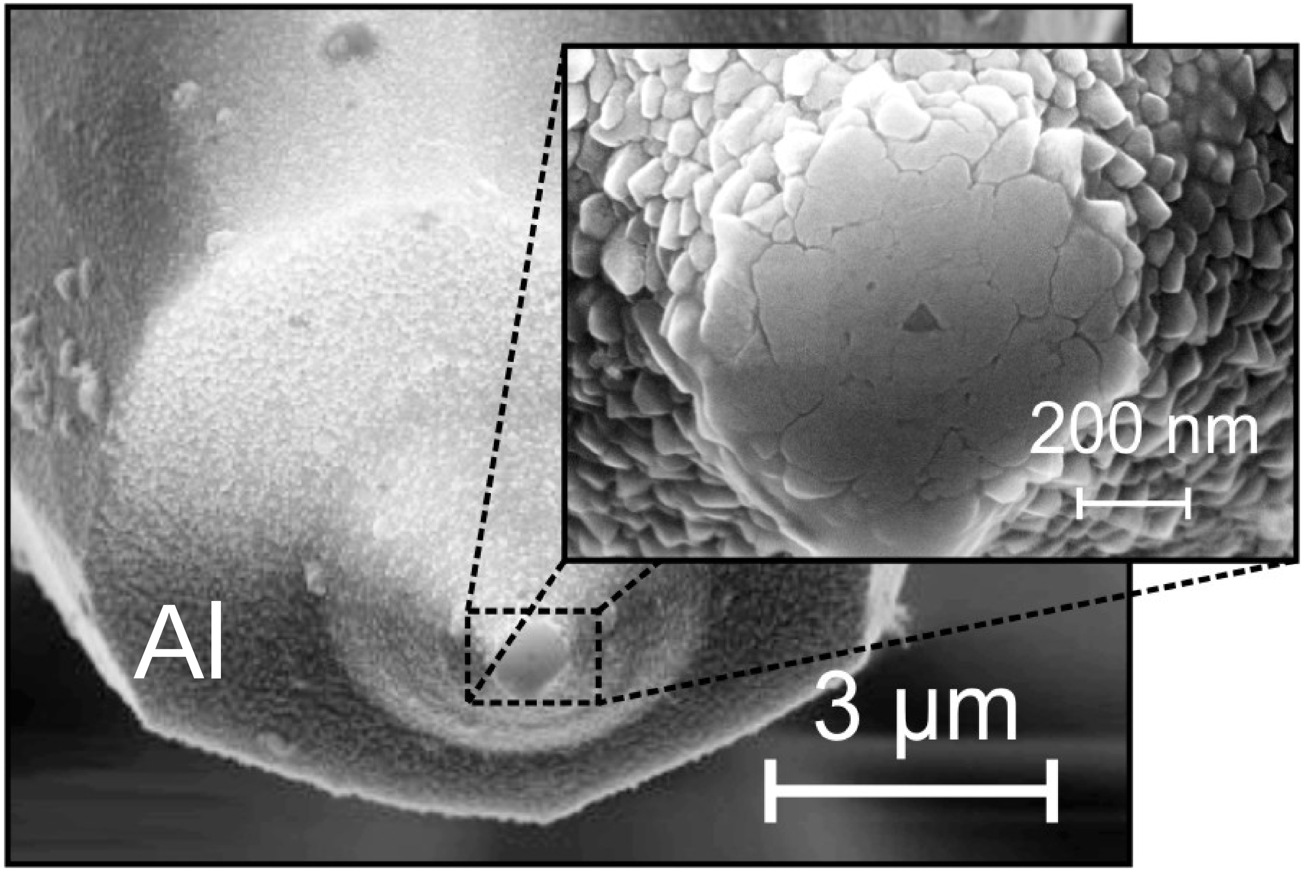


- aperture diameter = 50 nm
- wavelength = 500 nm
- movie from 50 nm to zero distance
- the square modulus of the electric field is shown on a false-color scale

- exponential decay versus z with a decay length given by 0.2 diameter
- transmitted power scales with the sixth power of the diameter

animation by Andreas Naber, Institut für Angewandte Physik

Actual SNOM tips



Ulrich Neuberth & Nicole Rau, Institut für Angewandte Physik

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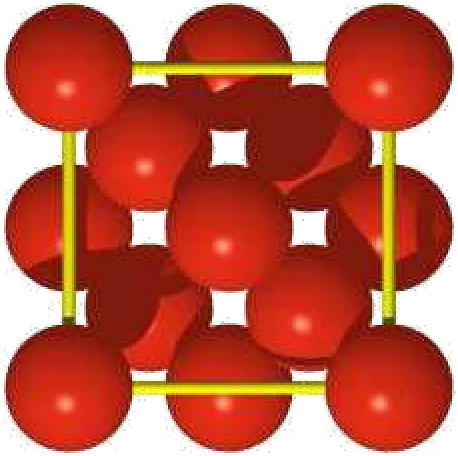
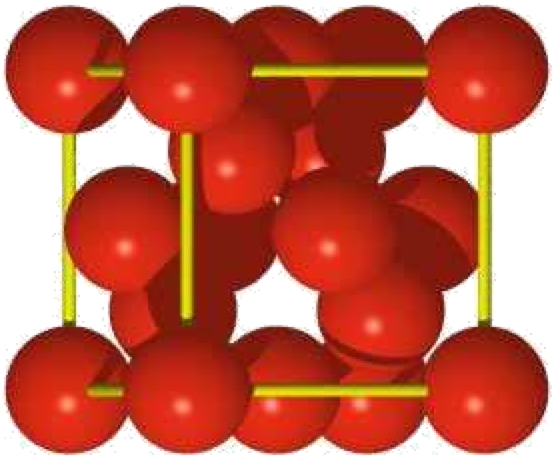
Silicon, a semiconductor crystal



Silicon, a semiconductor crystal



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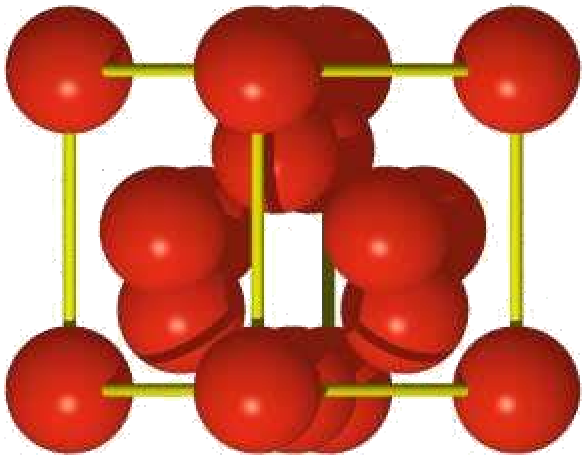

Is there such a thing as a "semiconductor for light" ?

S. John, Phys. Rev. Lett. 58, 2586 (1987)
E. Yablonovitch, Phys. Rev. Lett. 58, 2059 (1987)

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In the 1D case with piecewise-constant refractive index, the analogy between the band structure of electrons and that of light is strict (see 1.1.).

see: **Bloch waves** for electrons

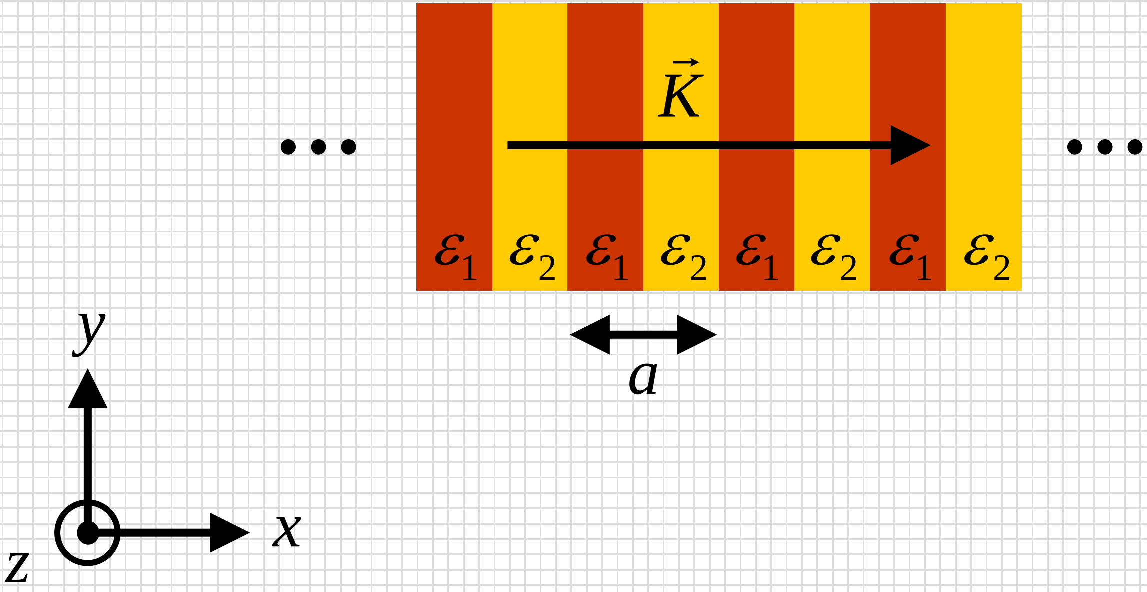
see: **Kronig-Penney model** for electrons

see: **tight-binding model** for electrons

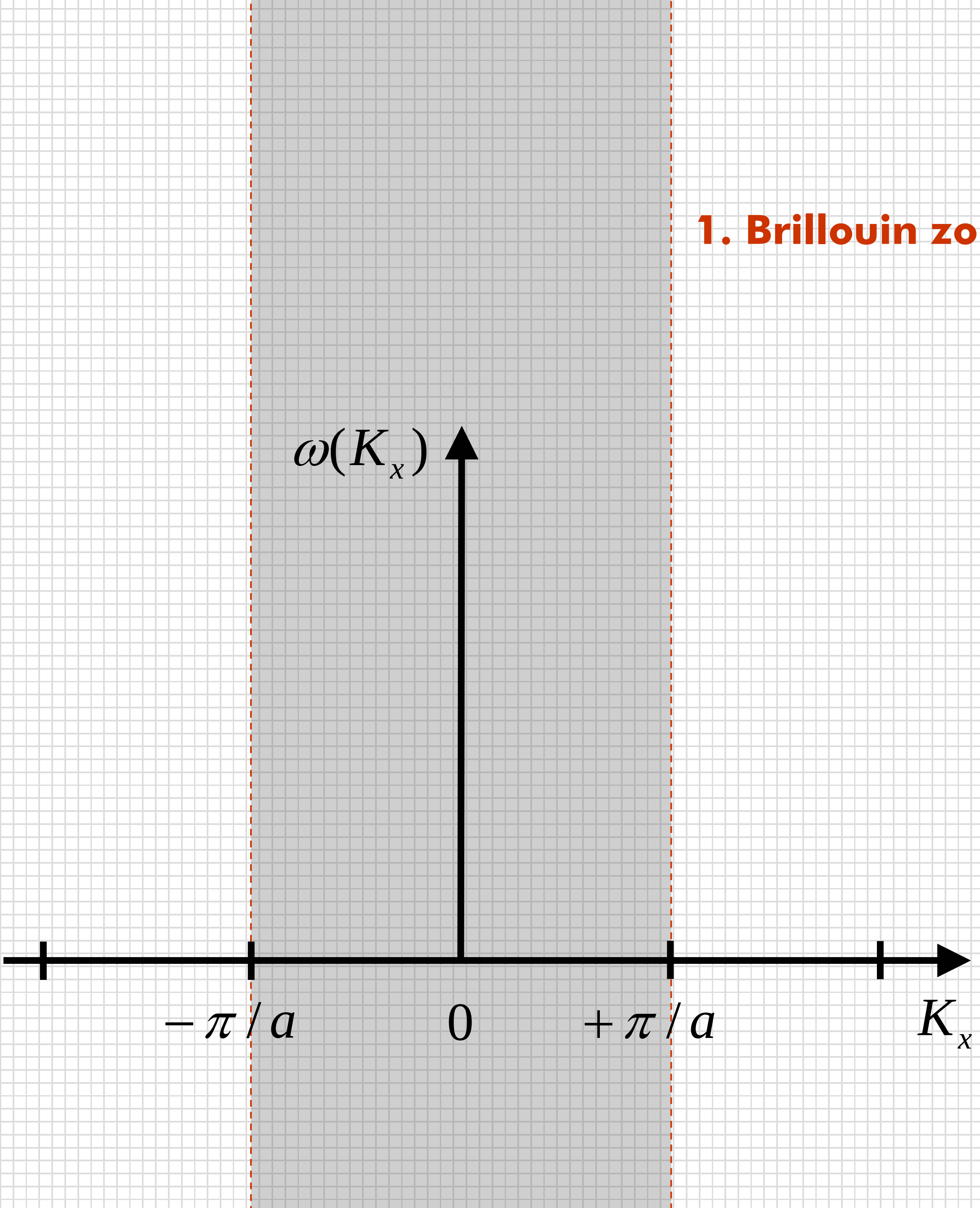
see: **nearly-free electron model** for electrons

N.W. Ashcroft and N.D. Mermin, “Solid State Physics“, Harcourt College Publishers

Consider a 1D Photonic Crystal (a dielectric mirror).

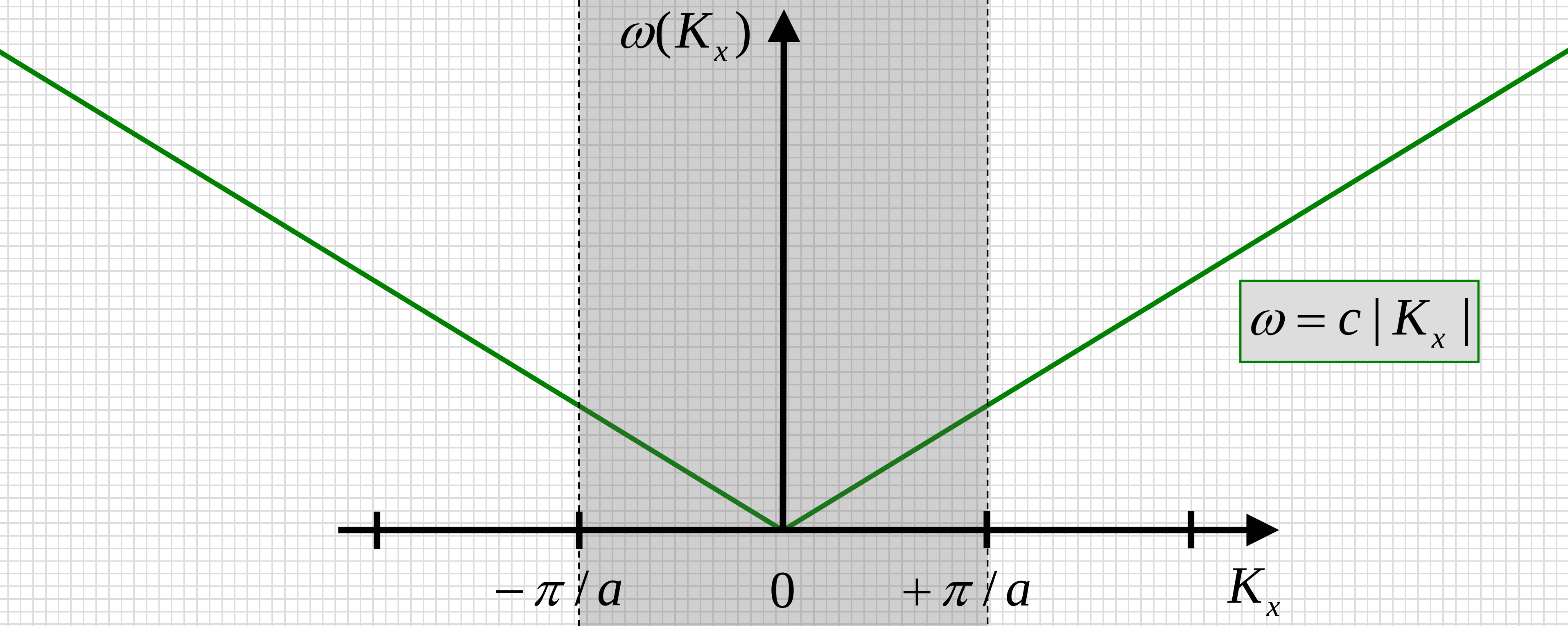


Martin Wegener



1. Brillouin zone

$\varepsilon_1 \approx \varepsilon_2$



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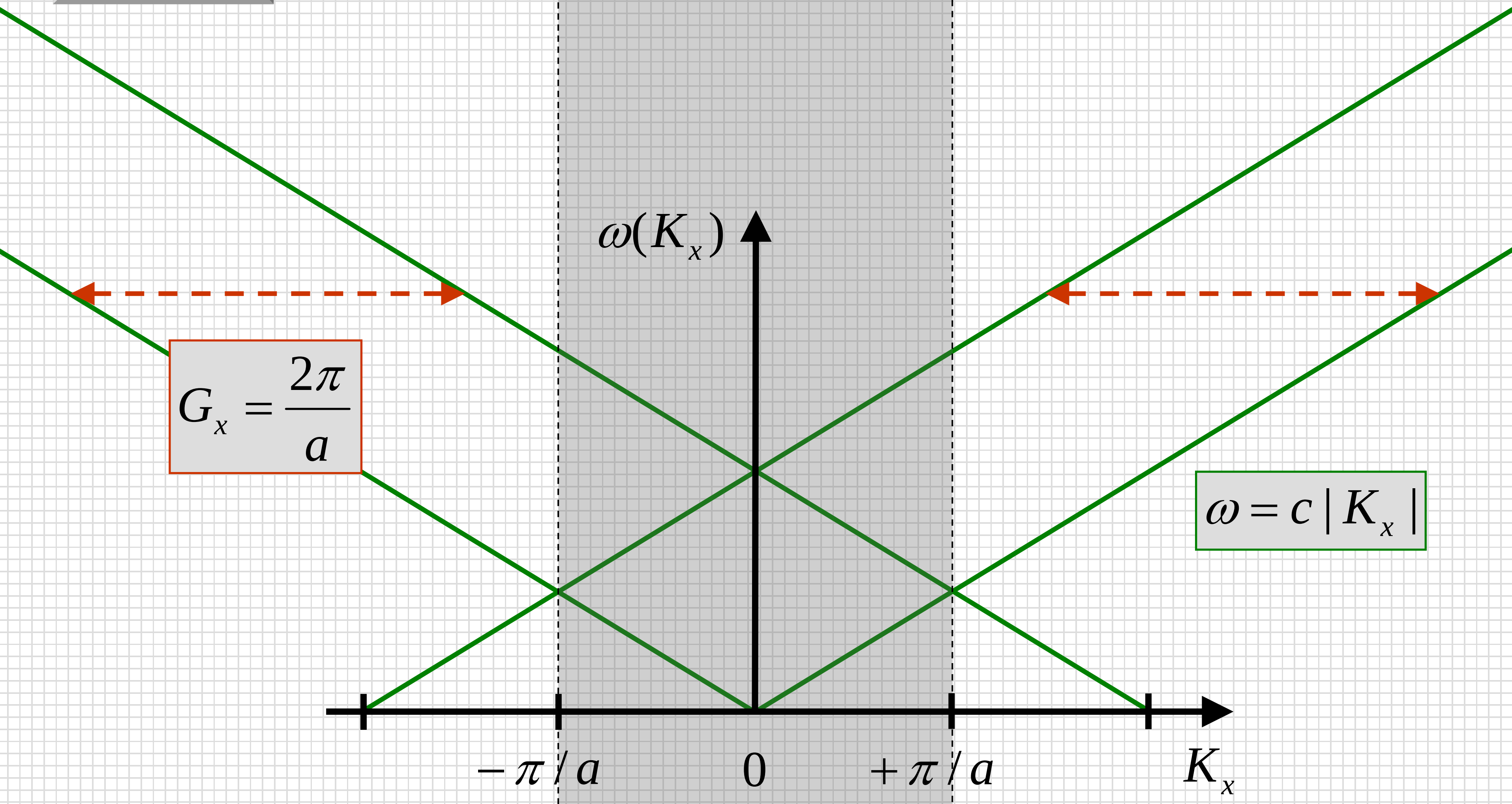
$\varepsilon_1 \approx \varepsilon_2$



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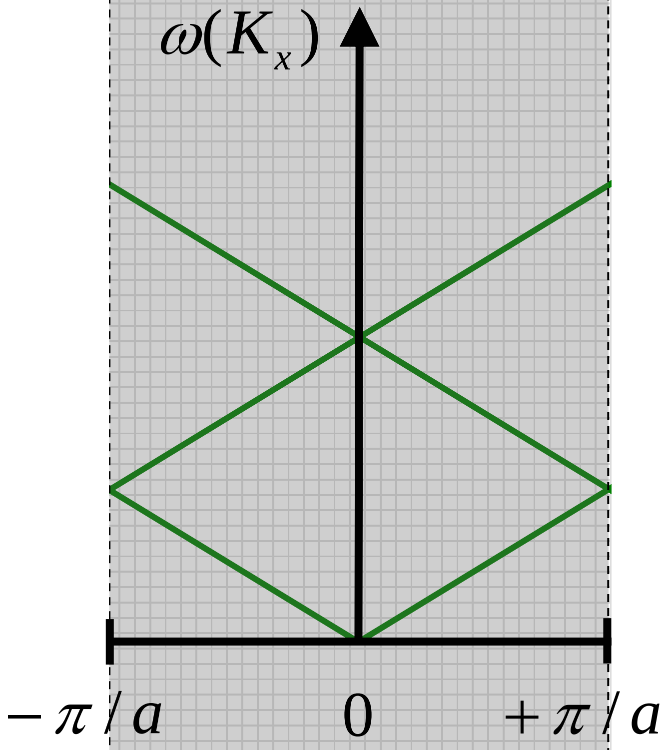
$\varepsilon_1 \approx \varepsilon_2$



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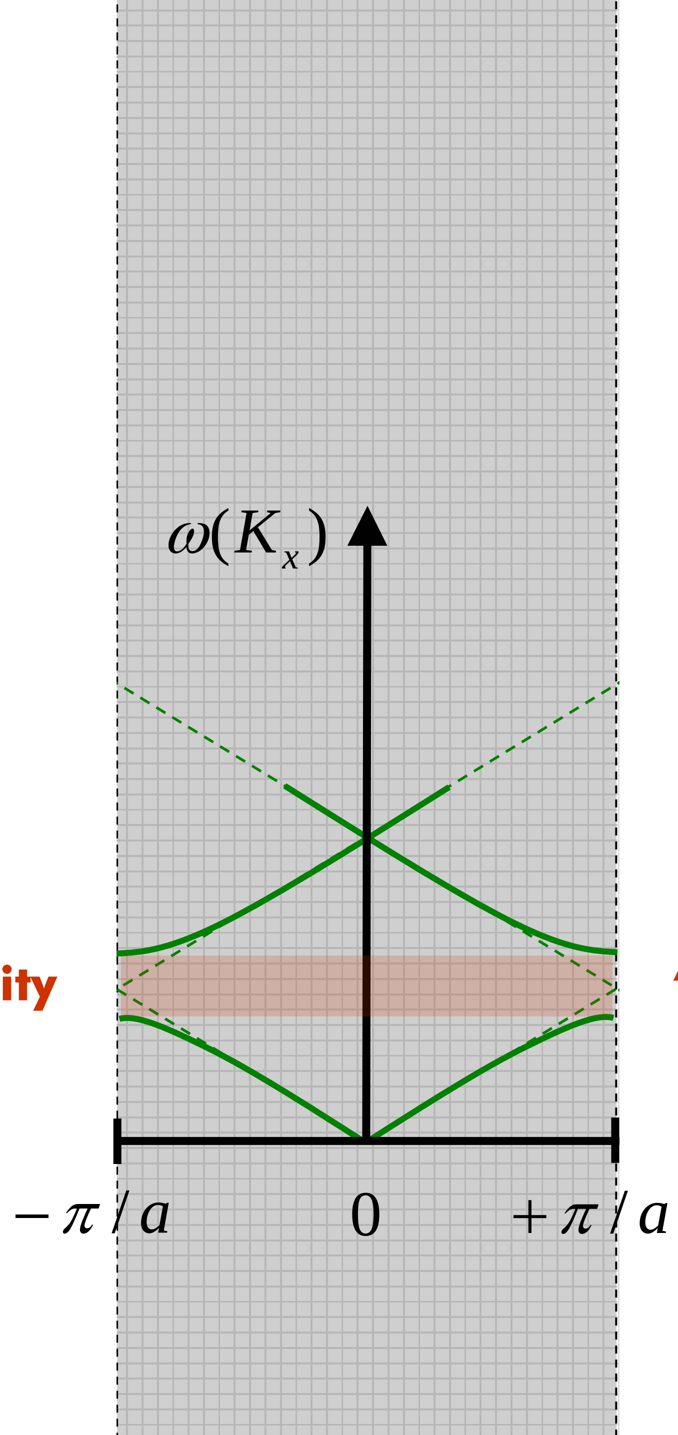
$\varepsilon_1 < \varepsilon_2$



high reflectivity

“photonic band gap“

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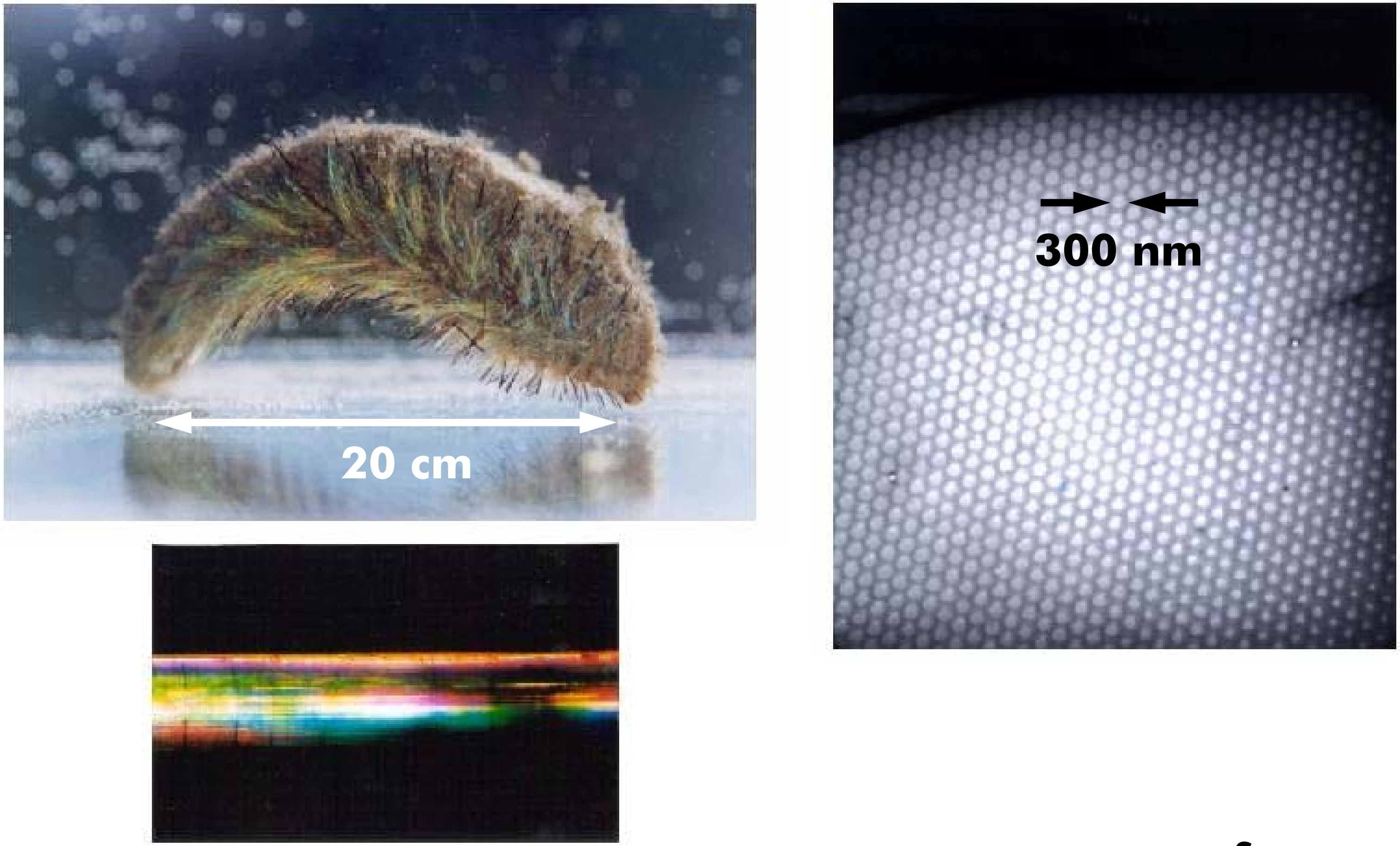
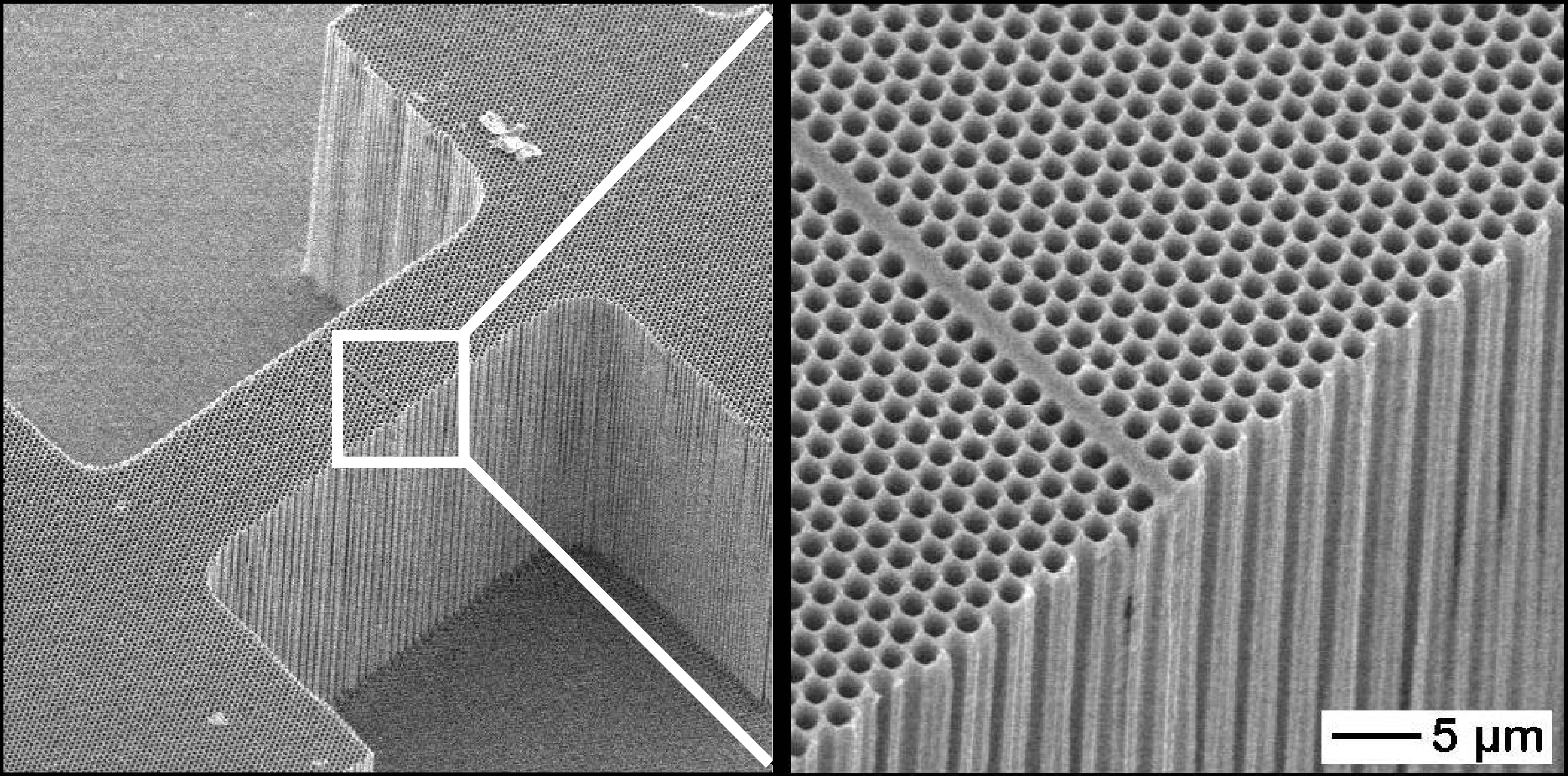
2D Photonic Crystals in nature



Sea-mouse

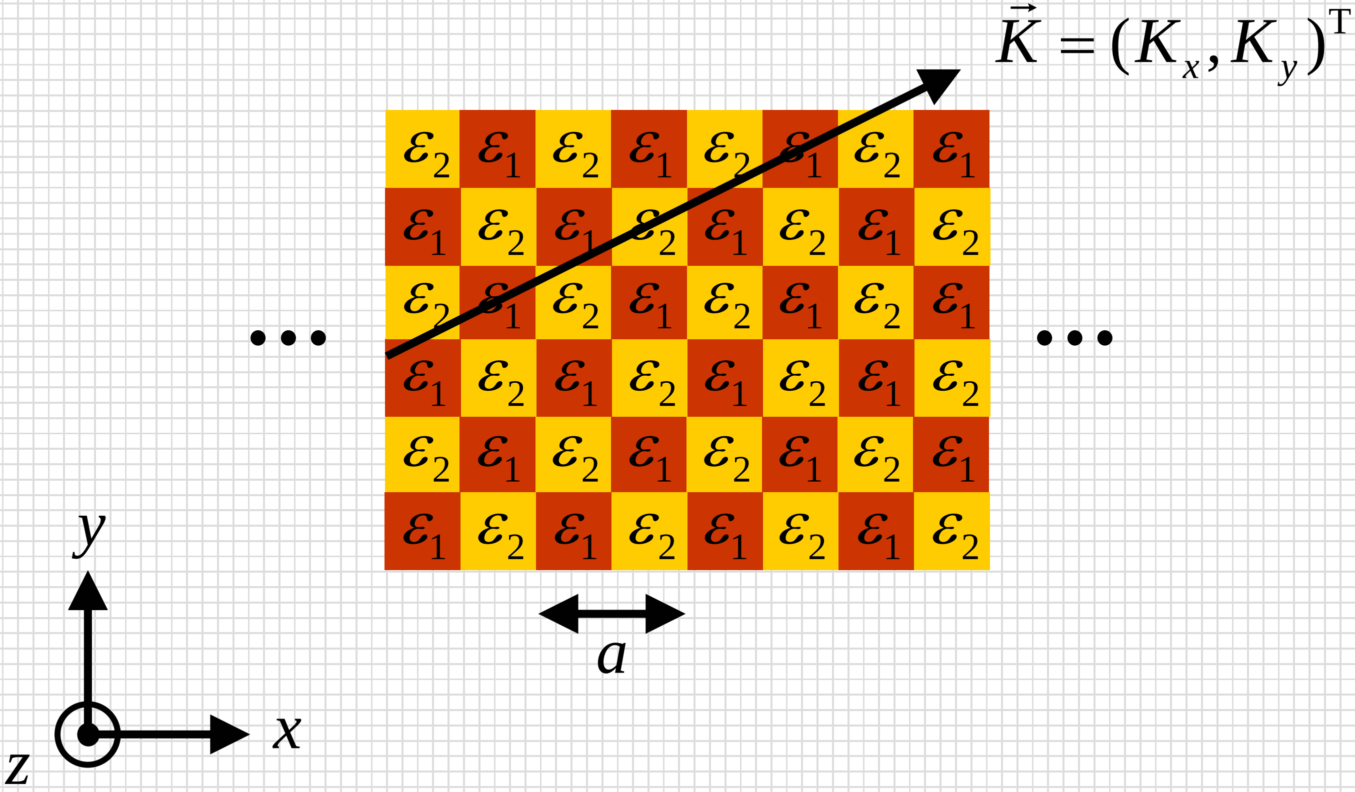
Taken from: A.R. Parker et al., Nature 409, 36 (2001)

2D hexagonal PC structure

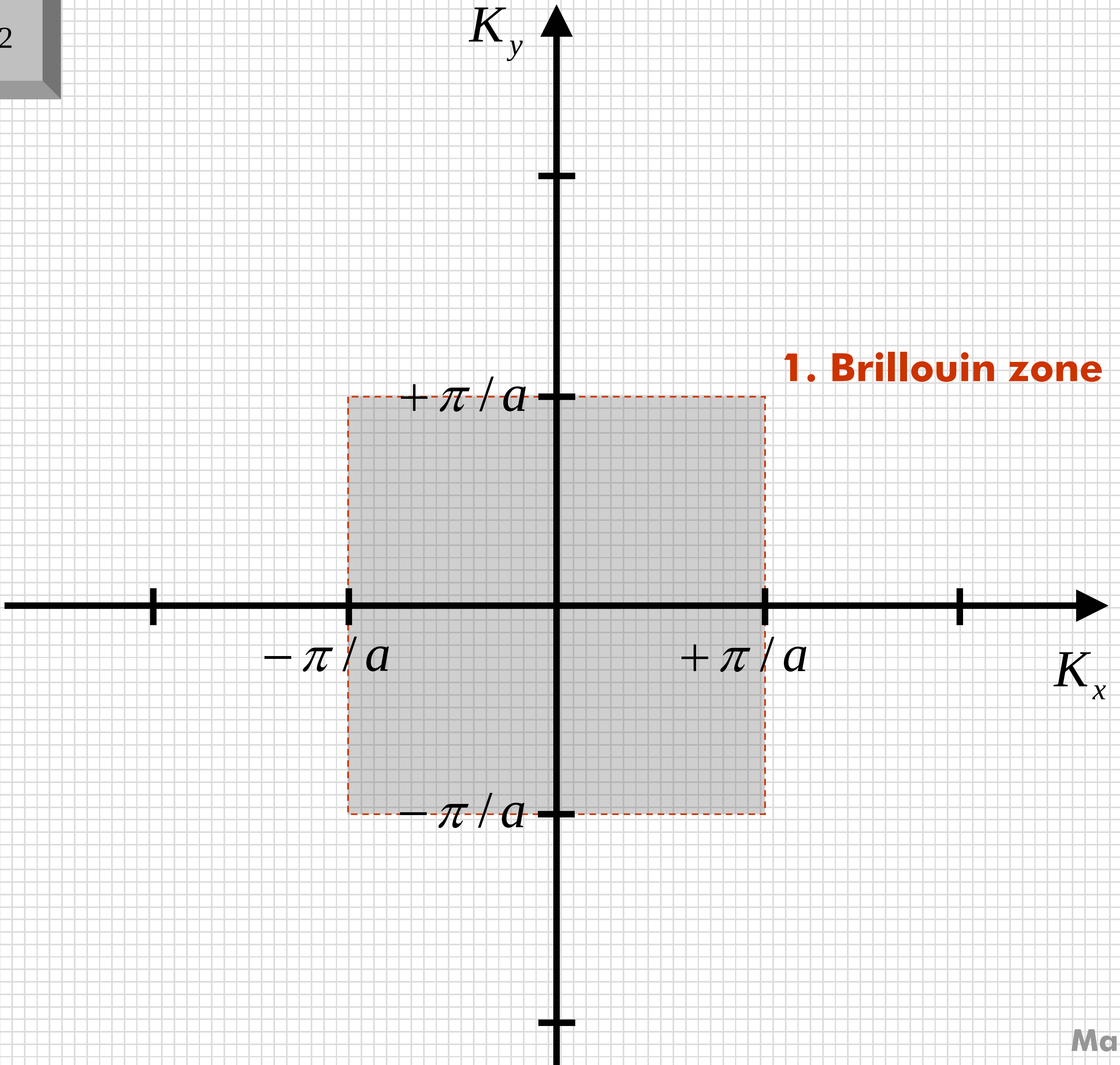


Taken from: R. Wehrspohn, U. Gösele et al., MPI in Halle

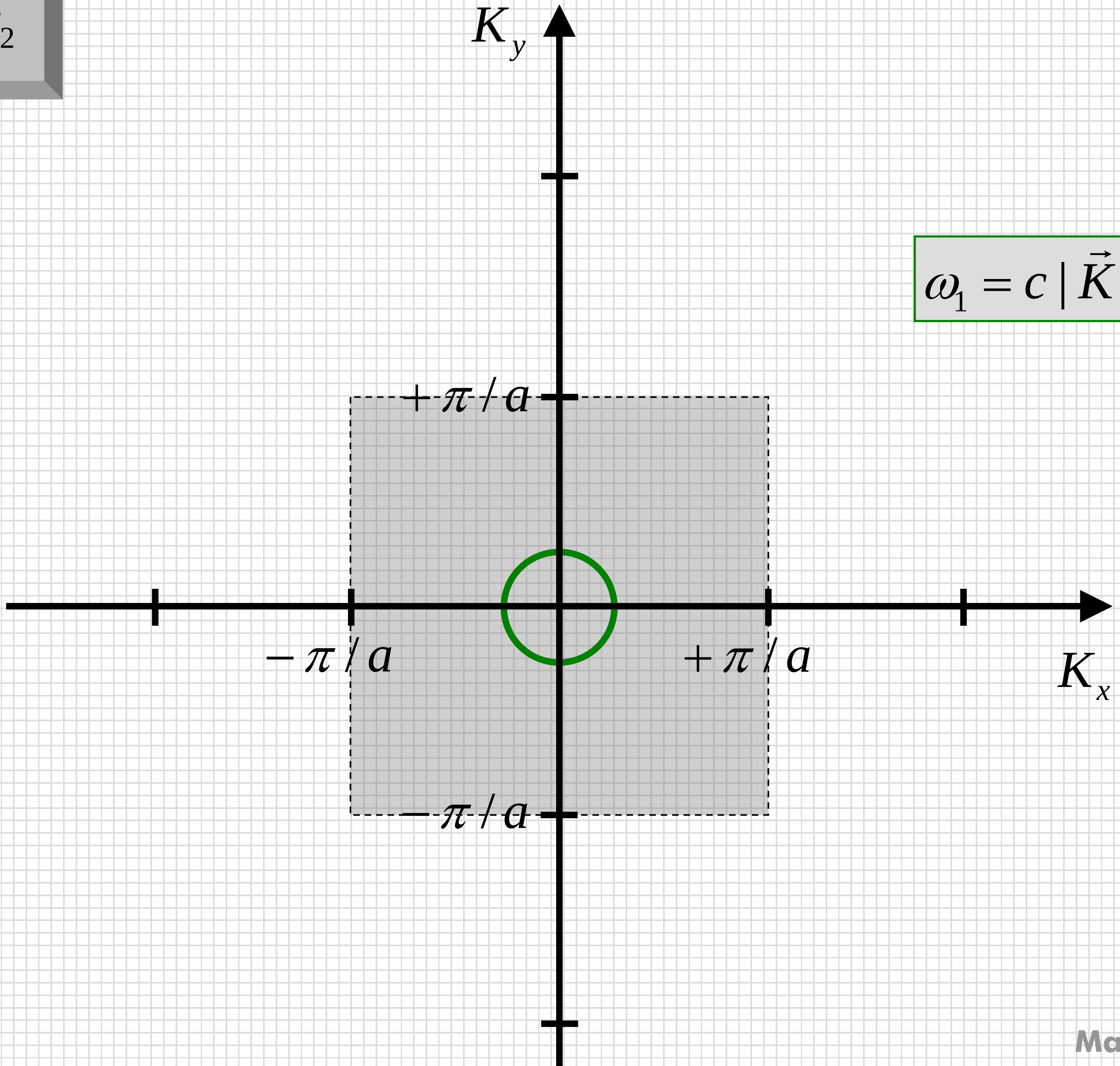
Consider a Photonic Crystal with a 2D square lattice.



$\varepsilon_1 \approx \varepsilon_2$

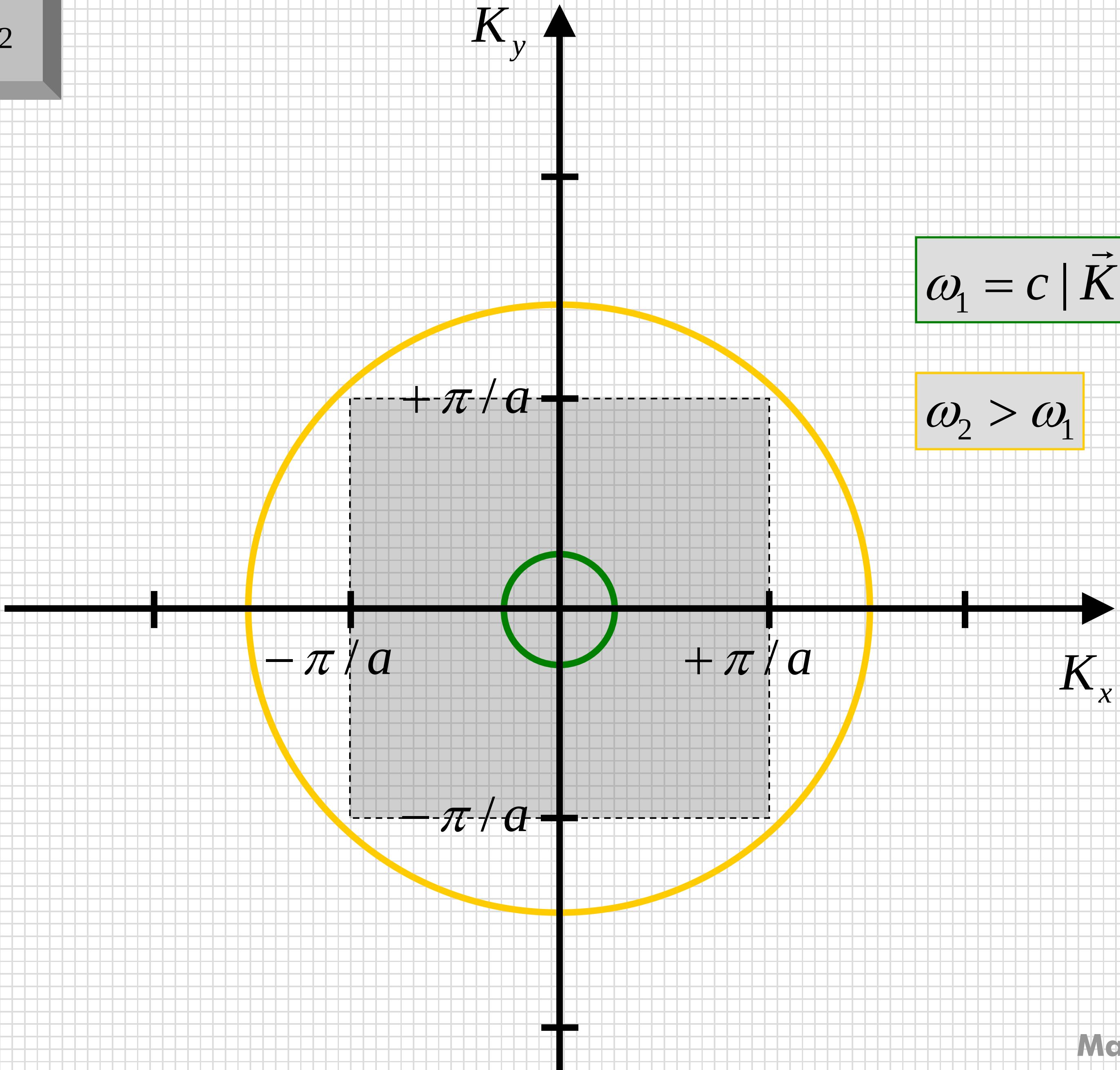


$\varepsilon_1 \approx \varepsilon_2$



$\omega_1 = c\,|\vec{K}\,| = \text{const.}$

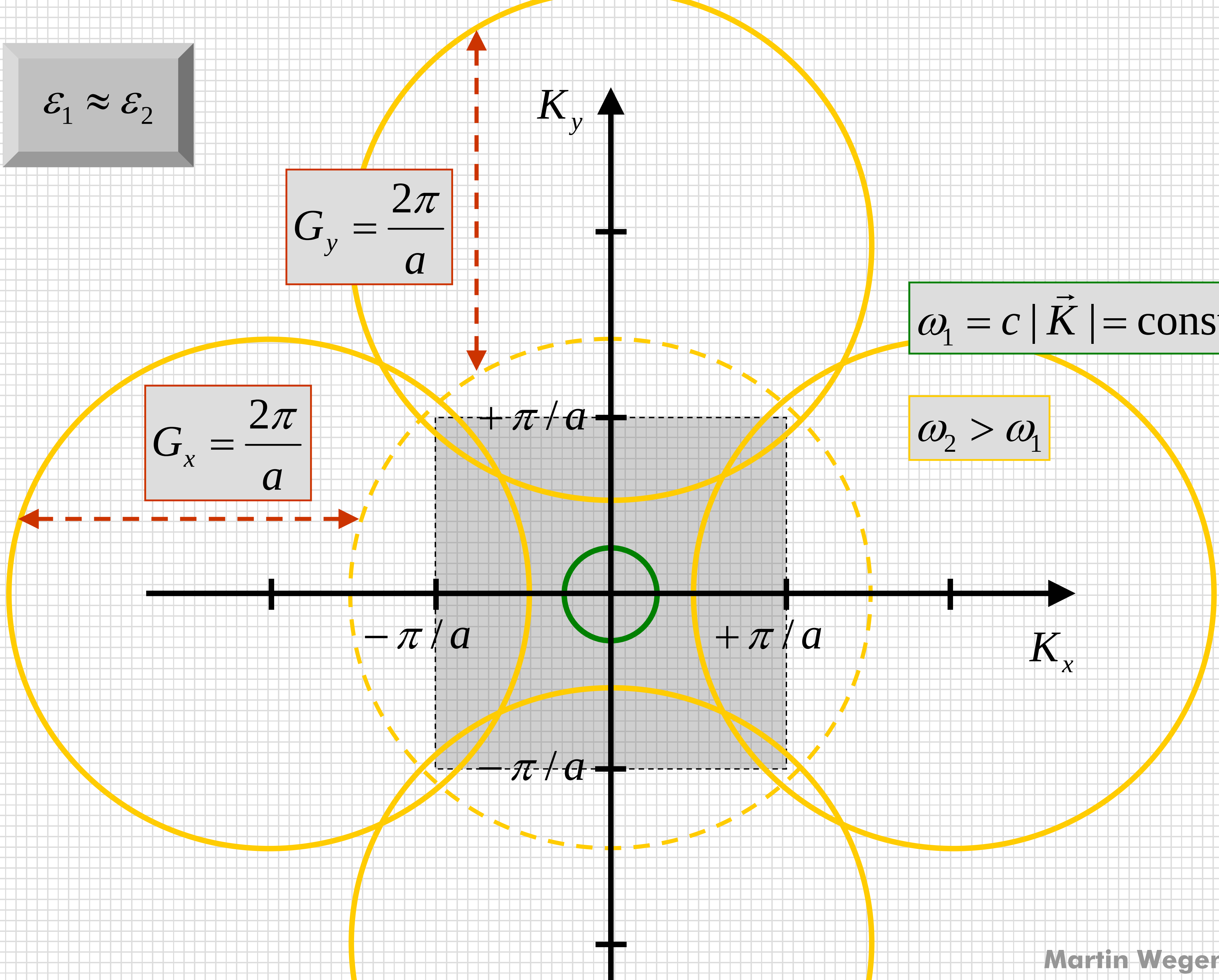
$\varepsilon_1 \approx \varepsilon_2$



$\omega_1 = c\,|\vec{K}| = \text{const.}$

$\omega_2 > \omega_1$

$\varepsilon_1 \approx \varepsilon_2$



$$G_y = \frac{2\pi}{a}$$

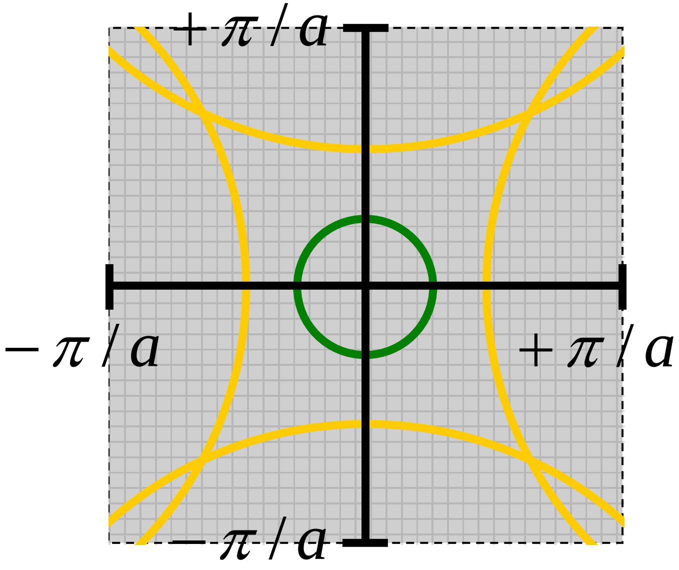
$$G_x = \frac{2\pi}{a}$$

$$\omega_1 = c\,|\vec{K}\,| = \mathrm{const.}$$

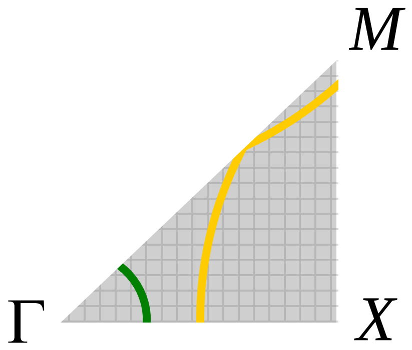
$$\omega_2 > \omega_1$$

Martin Wegener

One gets complex iso-frequency curves – especially for the higher bands at larger frequencies.



One gets complex iso-frequency curves – especially for the higher bands at larger frequencies.



for band structure calculations

One gets complex iso-frequency curves – especially for the higher bands at larger frequencies.

This is simply the result of
back-folding into the 1. Brillouin zone.

At the same time, one gets flat bands with small group velocities (e.g., along the ΓM direction)

$$\boxed{\vec{v}_{\text{group}} = \vec{\nabla}_{\vec{K}}\, \omega}$$

Actual band structure calculations have to solve the eigenvalue problem (see 1.1.).

$$\frac{1}{\mu(\vec{r})}\vec{\nabla}\times\left(\frac{1}{\varepsilon(\vec{r})}\vec{\nabla}\times\vec{H}(\vec{r})\right)=\hat{O}(\vec{r})\vec{H}(\vec{r})=\frac{\omega^2}{c_0^2}\vec{H}(\vec{r})$$

Translational symmetry means that

$$\varepsilon(\vec{r})=\varepsilon(\vec{r}+\vec{T})$$

and

$$\mu(\vec{r})=\mu(\vec{r}+\vec{T})$$

often = 1

hence

$$\hat{O}(\vec{r})=\hat{O}(\vec{r}+\vec{T})$$

Actual band structure calculations have to solve the eigenvalue problem (see 1.1.).

$$\frac{1}{\mu(\vec{r})}\vec{\nabla}\times\left(\frac{1}{\varepsilon(\vec{r})}\vec{\nabla}\times\vec{H}(\vec{r})\right)=\hat{O}(\vec{r})\vec{H}(\vec{r})=\frac{\omega^2}{c_0^2}\vec{H}(\vec{r})$$

Translational symmetry delivers the Bloch theorem

$$\vec{H}(\vec{r})=\vec{h}_{\vec{K}}(\vec{r})\exp(\mathrm{i}\vec{K}\cdot\vec{r})\neq\vec{H}(\vec{r}+\vec{T})$$

$$\vec{h}_{\vec{K}}(\vec{r})=\vec{h}_{\vec{K}}(\vec{r}+\vec{T})$$

Use, e.g., plane-wave expansion method $\Rightarrow\omega(\vec{K})$

Actual band structure calculations using the plane-wave expansion method can be performed by everybody using the **"MIT package"**.

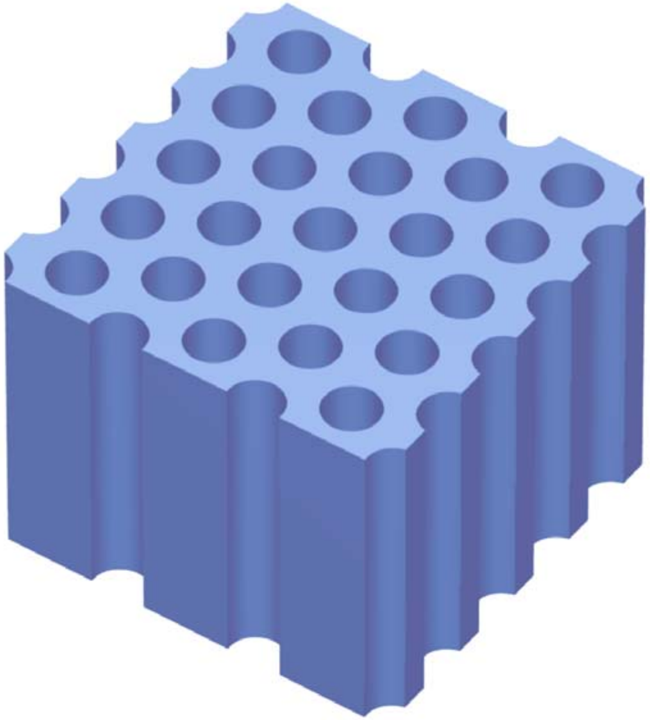
It can be downloaded (free of cost) from

http://ab-initio.mit.edu/mpb/

S.G. Johnson and J.D. Joannopoulos, Optics Express 8, 173 (2001)

Consider a 2D hexagonal Photonic Crystal.

For example, air cylinders in a dielectric, or vice versa (see sea-mouse or "Halle" structures).

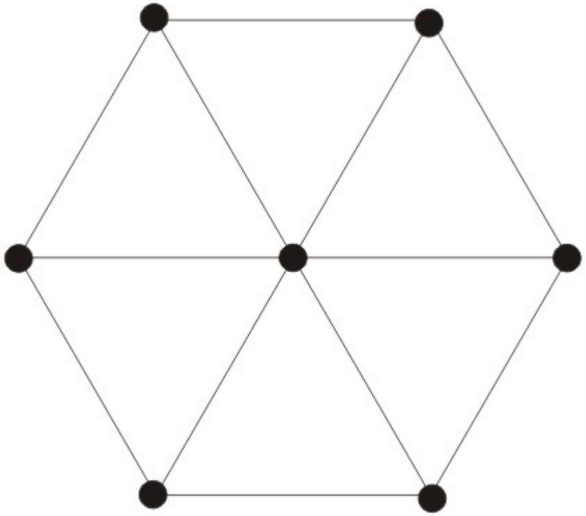


Important parameters:

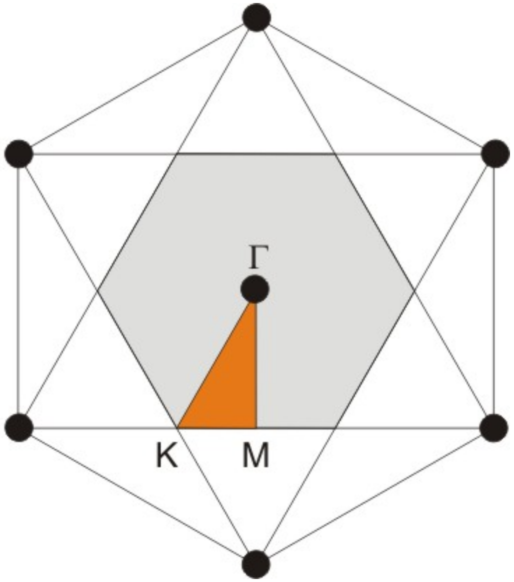
- r/a ratio
- refractive index (here 1.6)

Consider a 2D hexagonal Photonic Crystal.

(a) **Real space lattice**

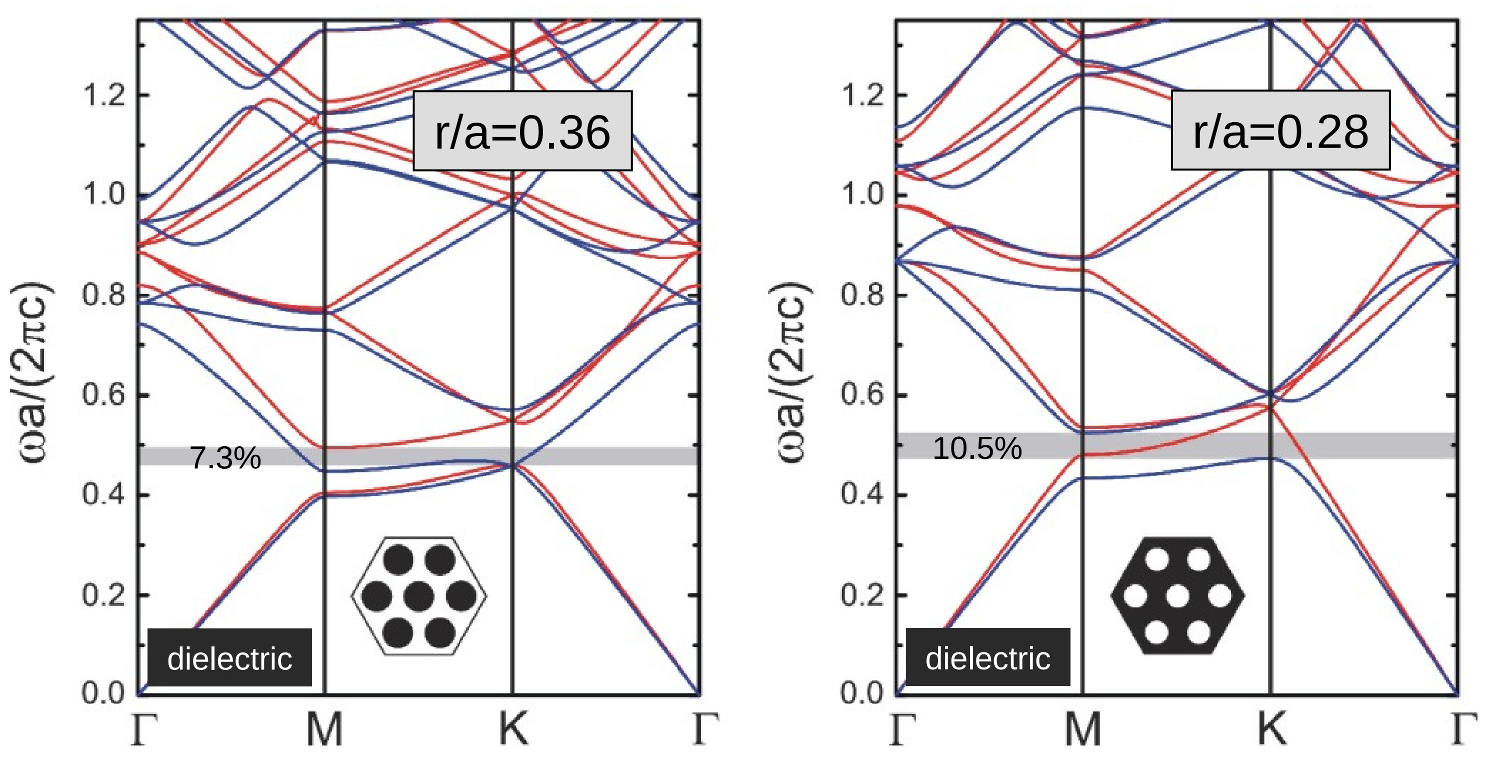


(b) **1. Brillouin zone**



Consider a 2D hexagonal Photonic Crystal.

Calculated band structure for H and for E-polarisation



Martin Hermatschweiler & the MIT package

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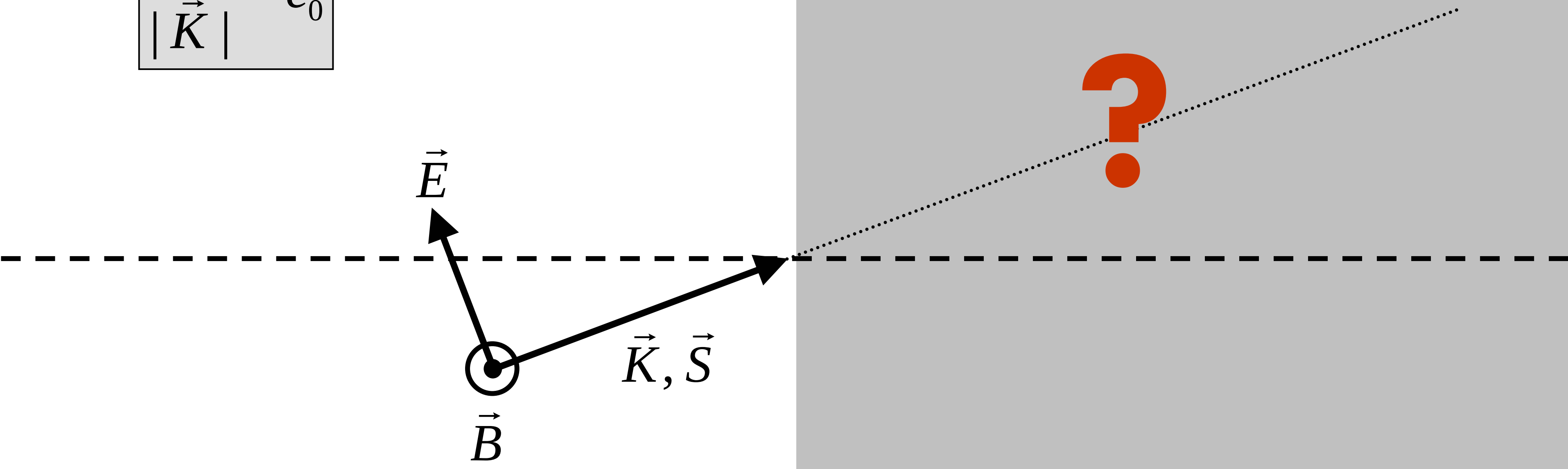
Refraction at an interface

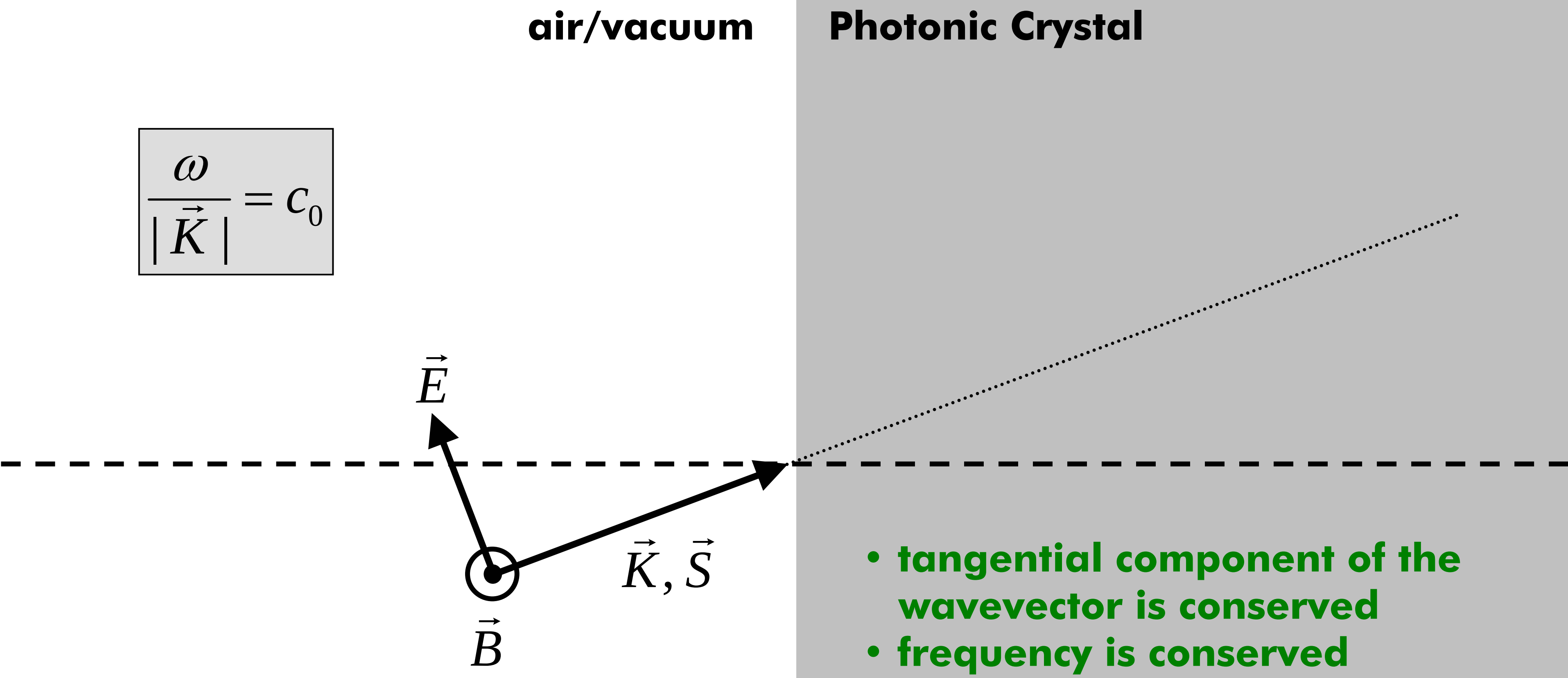
air/vacuum **Photonic Crystal**

$$\boxed{\frac{\omega}{|\vec{K}|} = c_0}$$



exercise

Refraction at an interface

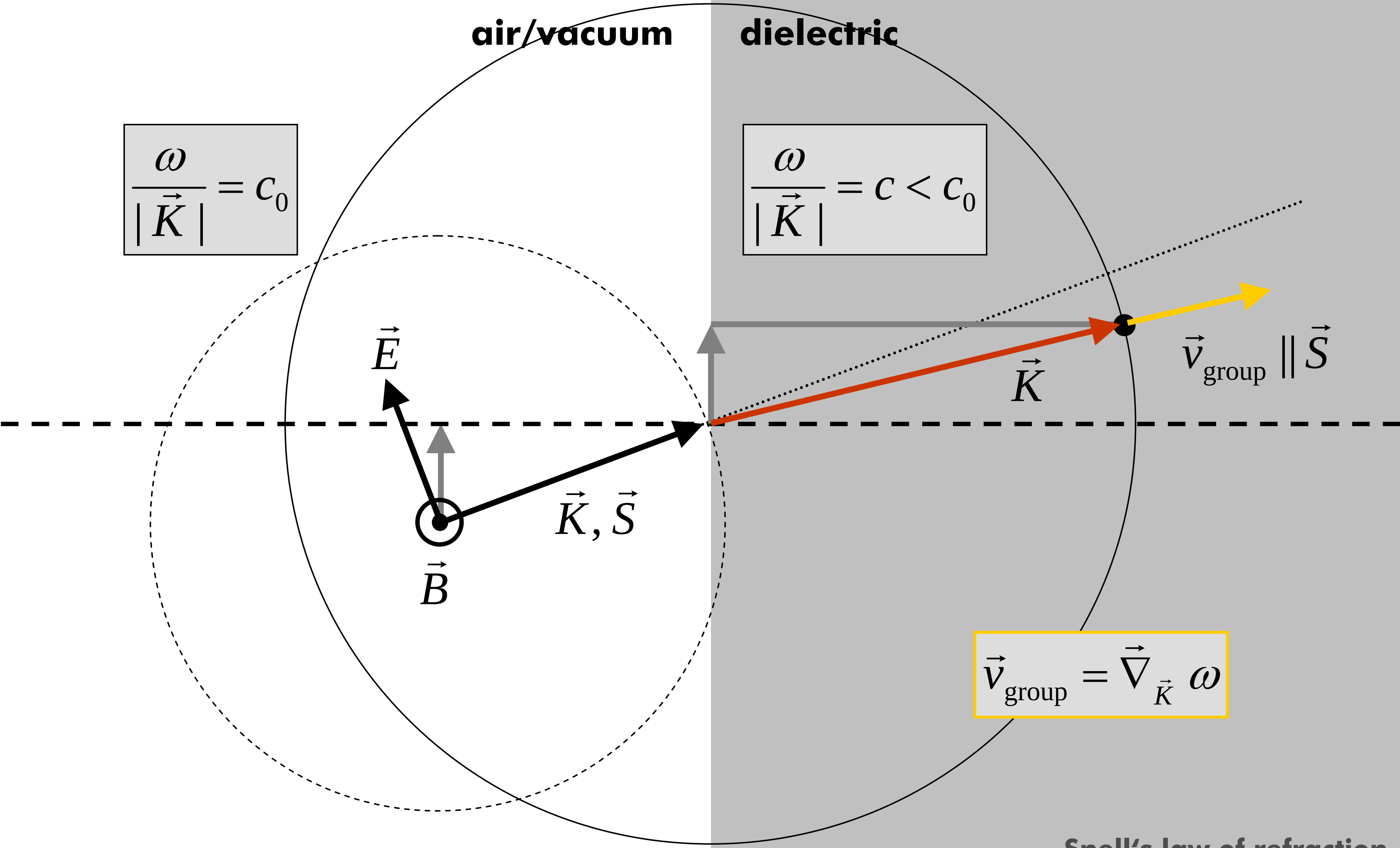


$$\boxed{\frac{\omega}{|\vec{K}|} = c_0}$$

- tangential component of the wavevector is conserved
- frequency is conserved
- look at corresponding iso-frequency curve (analogy: Fermi surface)

Martin Wegener

Refraction at an interface



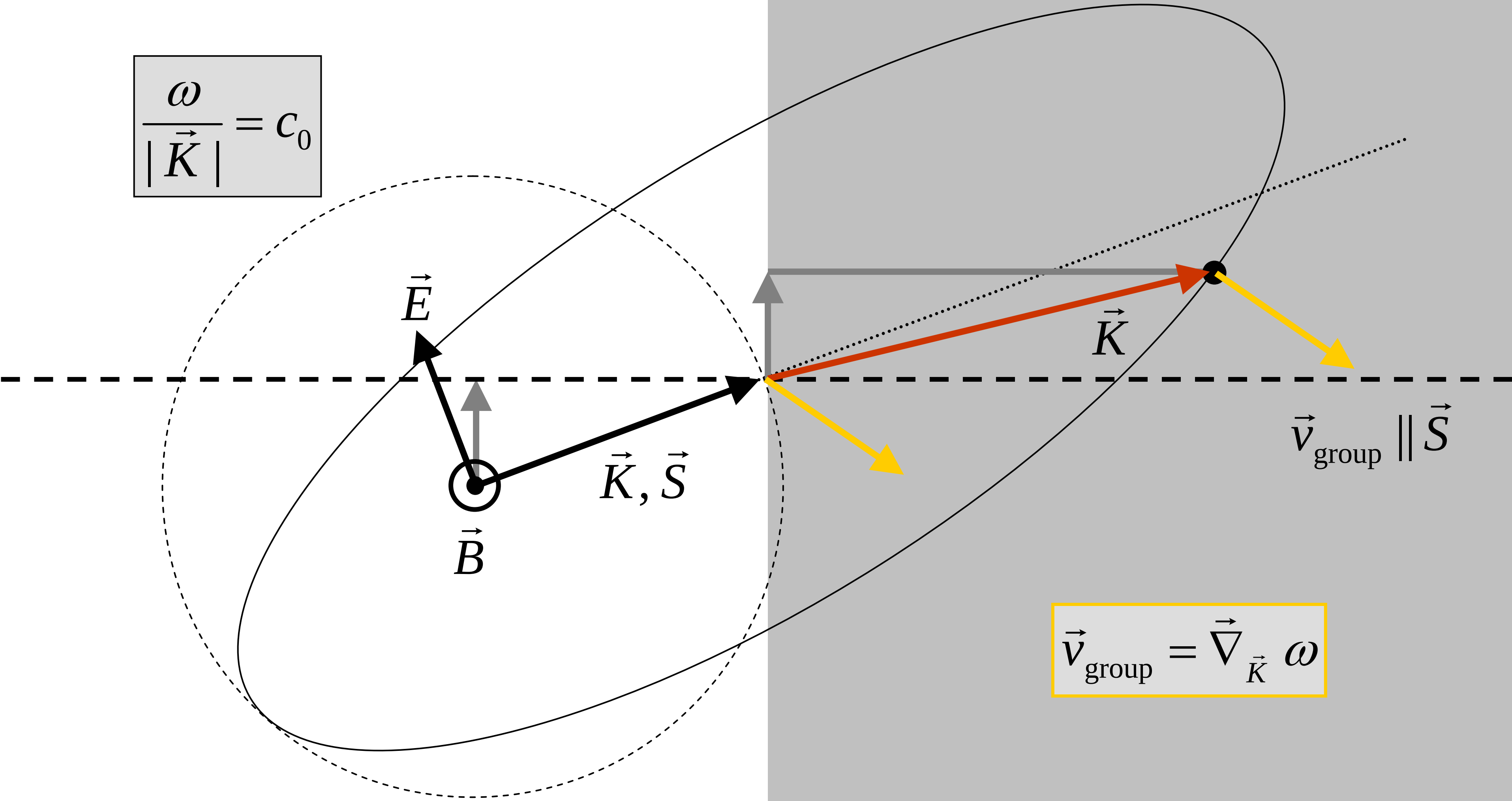
$$\frac{\omega}{|\vec{K}|} = c_0$$

$$\frac{\omega}{|\vec{K}|} = c < c_0$$

$$\vec{v}_{\text{group}} = \vec{\nabla}_{\vec{K}}\, \omega$$

Refraction at an interface

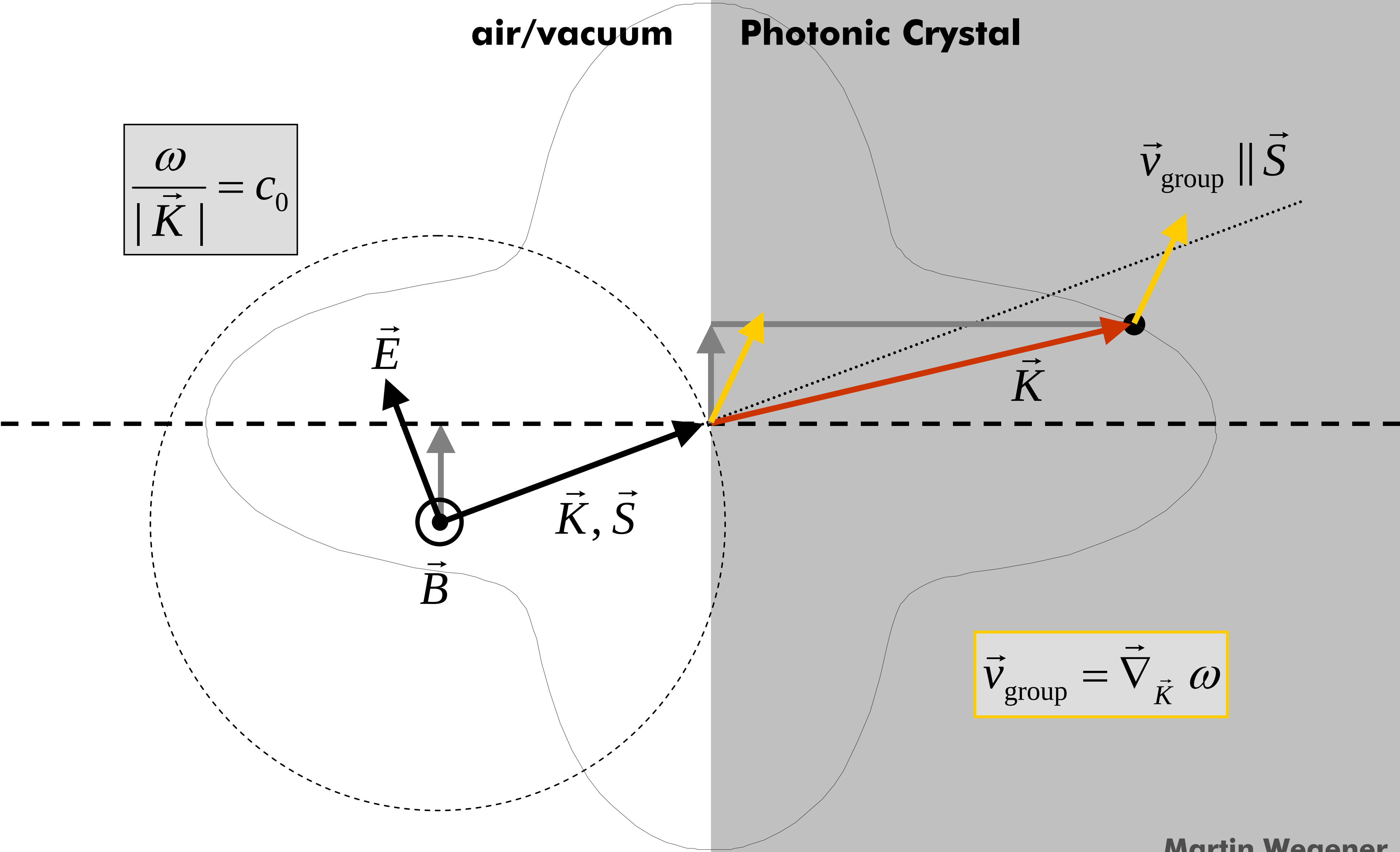
air/vacuum **birefringent dielectric**



$$\frac{\omega}{|\vec{K}|} = c_0$$

$$\vec{v}_{\text{group}} = \vec{\nabla}_{\vec{K}}\,\omega$$

Refraction at an interface



$$\frac{\omega}{|\vec{K}|} = c_0$$

$$\vec{v}_{\text{group}} = \vec{\nabla}_{\vec{K}}\, \omega$$

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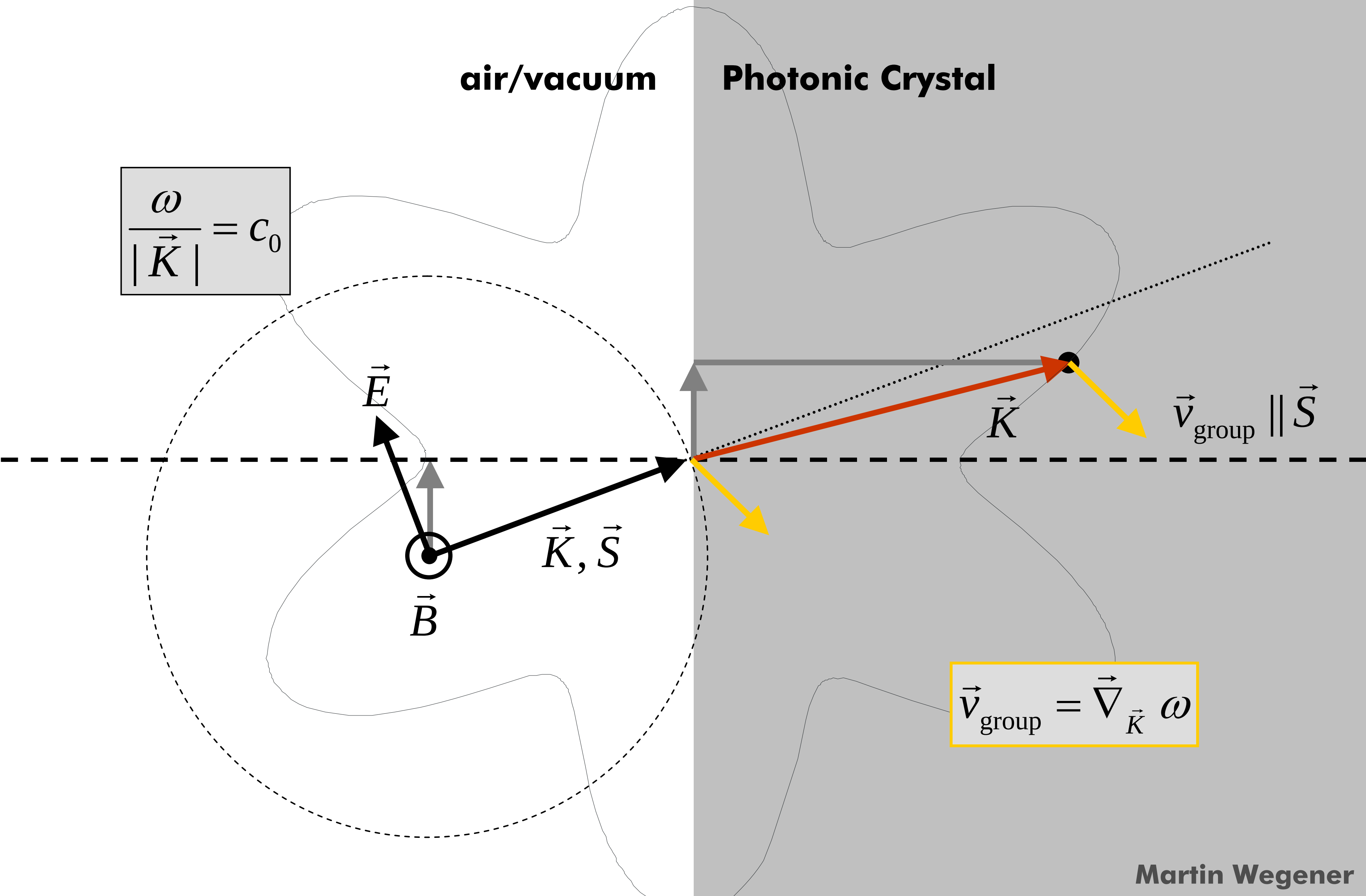
Refraction at an interface



$$\frac{\omega}{|\vec{K}|} = c_0$$

$$\vec{v}_{\text{group}} = \vec{\nabla}_{\vec{K}}\, \omega$$

Martin Wegener

The result is negative refraction, i.e., refraction that looks as if the refractive index in Snell's law would be negative.

$$\boxed{\frac{\sin(\alpha_{\mathrm{vac}})}{\sin(\alpha_{\mathrm{med}})} = n}$$

The angle inside the medium can be a very sensitive function of the incident (vacuum) angle.

The angle inside the PC also sensitively depends on the frequency via the dependence of the shape of the iso-frequency curve on frequency.

The latter effect can be used as a "superprism".

H. Kosaka et al., Phys. Rev. B 58, R10096 (1998)

The phenomenon of negative refraction …

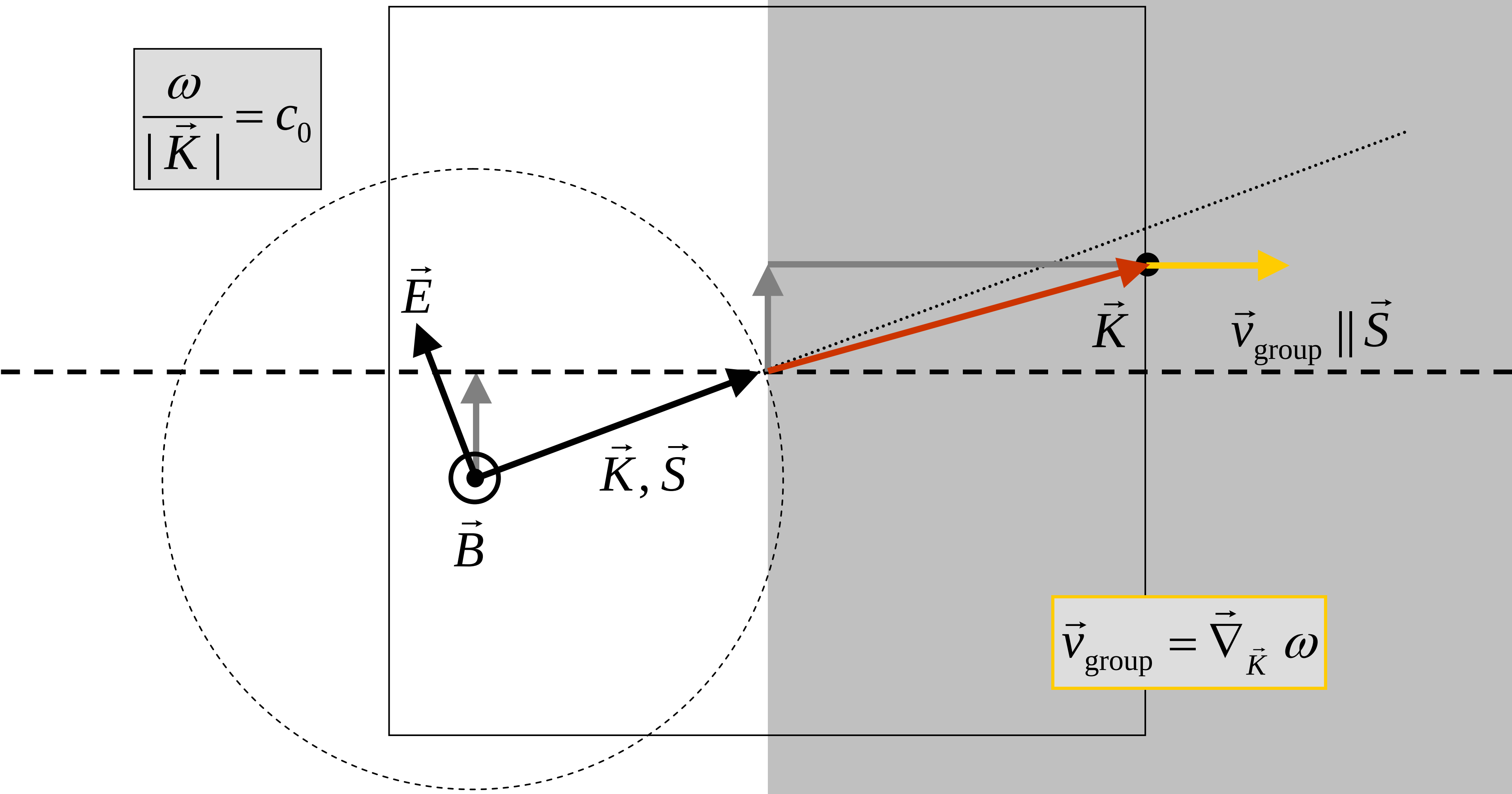
… can occur in **Photonic Crystals** with
positive permittivity and unity permeability.
It is a result of Bragg reflection (this section).

… can occur in **left-handed metamaterials** with
negative permittivity and negative permeability.
Bragg reflection plays no role (section 3.1.).

Self-collimation

air/vacuum **Photonic Crystal**

$$\frac{\omega}{|\vec{K}|} = c_0$$

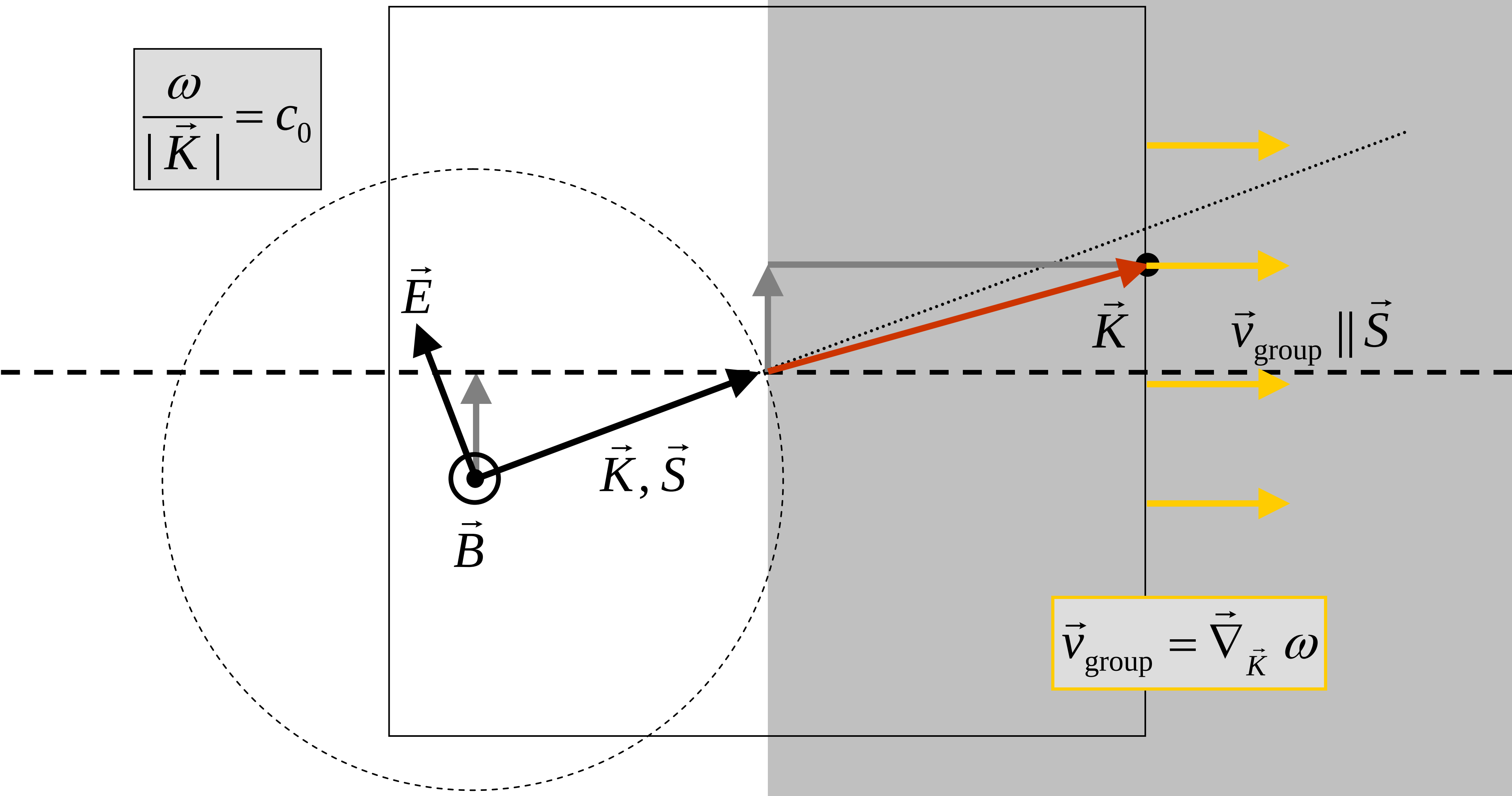


$$\vec{v}_{\text{group}} = \vec{\nabla}_{\vec{K}}\, \omega$$

Martin Wegener

Self-collimation

air/vacuum **Photonic Crystal**



$$\frac{\omega}{|\vec{K}|} = c_0$$

$$\vec{v}_{\text{group}} = \vec{\nabla}_{\vec{K}}\,\omega$$

Martin Wegener

For example, for a tight-binding band structure and for a frequency in the middle of the band, the 2D iso-frequency contour can be a square.

A tight focus on the PC surface corresponds to a large spread of wave vectors of light (and of the Poynting vectors) in air.

Inside the PC, all Poynting vectors point in one direction – while the tight spatial focus remains (not possible in air due to diffraction).

This remarkable phenomenon is often refered to as self-collimation or as diffraction-less flow of light.

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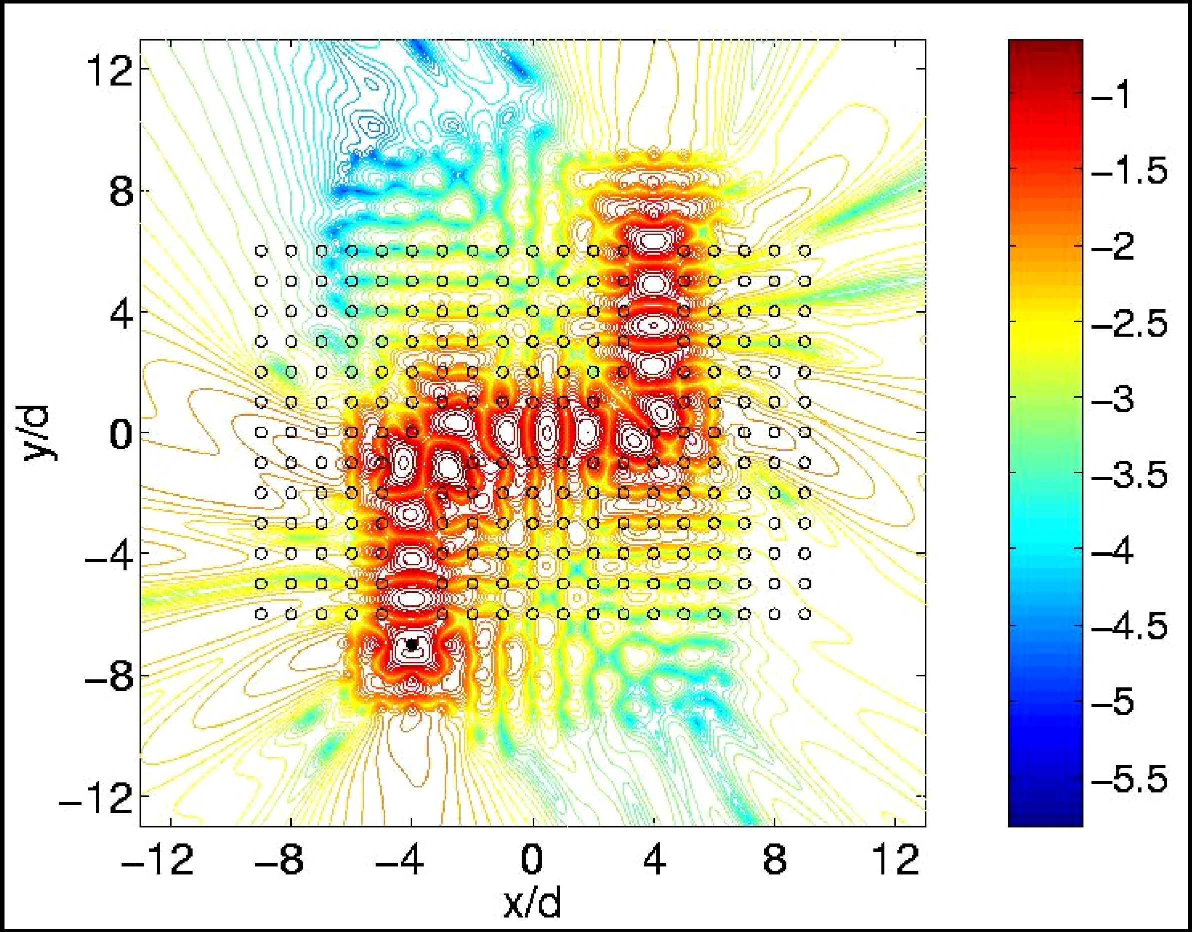
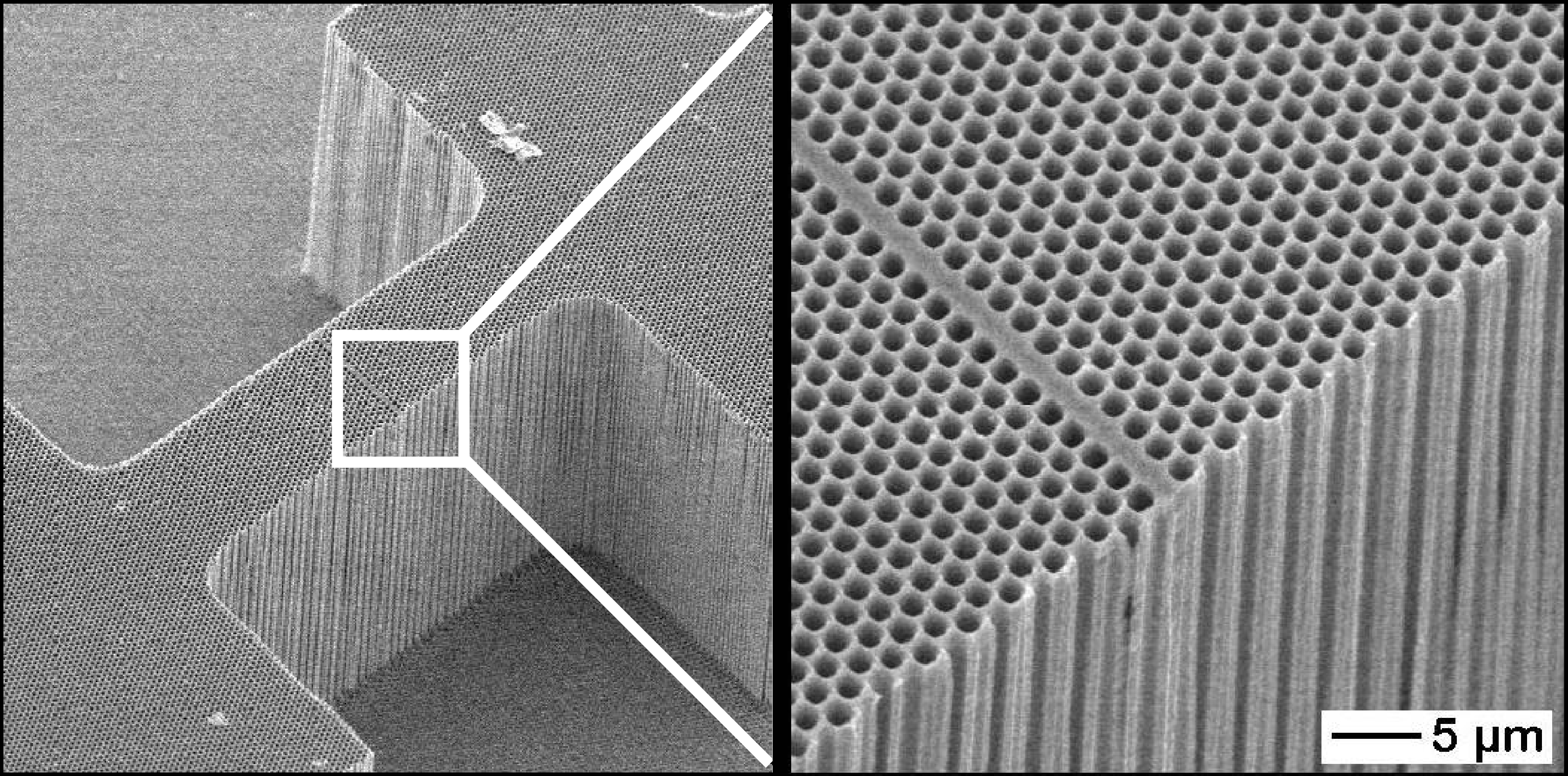
Guiding light around the corner



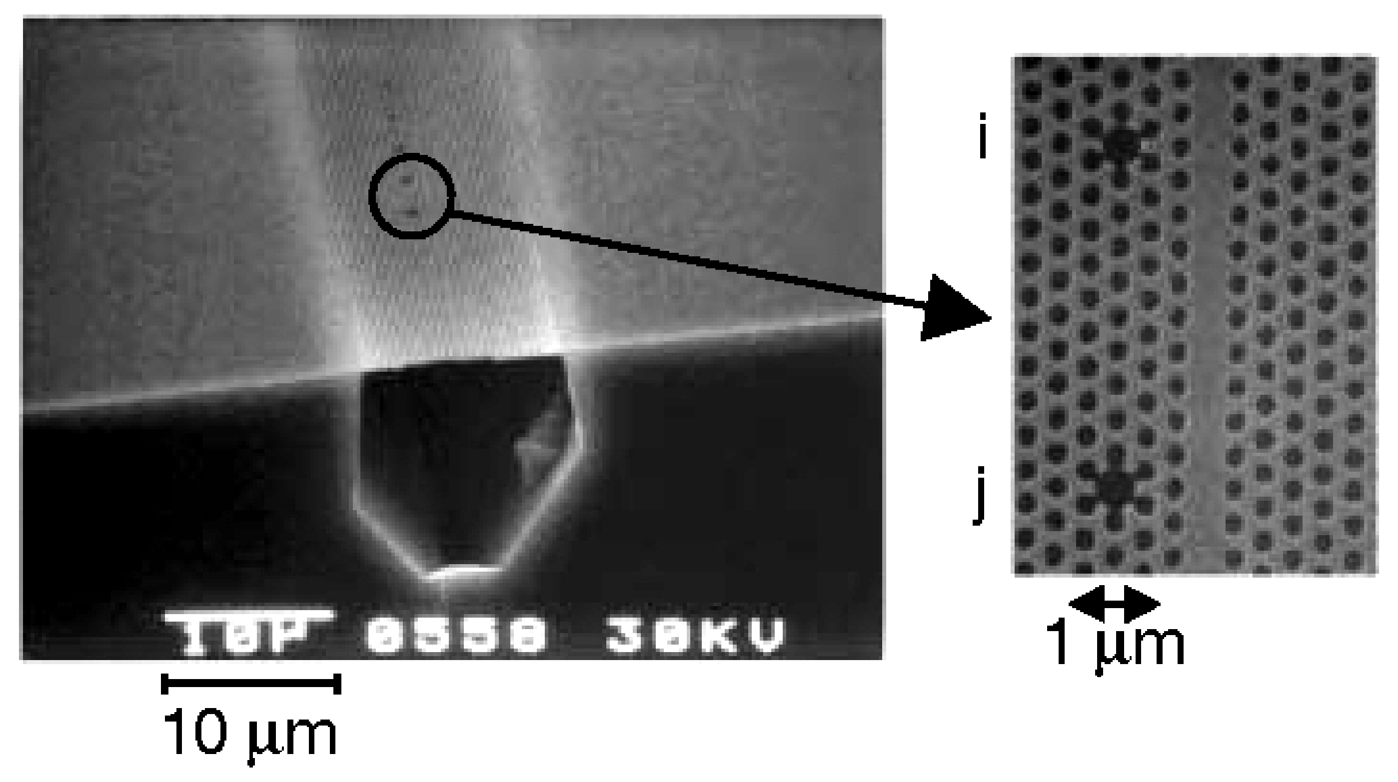
Kurt Busch

2D hexagonal PC structure



Taken from: R. Wehrspohn, U. Gösele et al., MPI in Halle

2D PC slab waveguides



Taken from: S. Noda et al., Nature 407, 608 (2000)

Note that the physics of 2D PC slab waveguides …

… is different from that of an ideal 2D PC as some of the modes can escape from the slab, others cannot (total internal reflection).

… is important because such structures are much more relevant for applications (light is guided in the third dimension as well).

J.D. Joannopoulos et al., "Photonic Crystals", Princeton University Press

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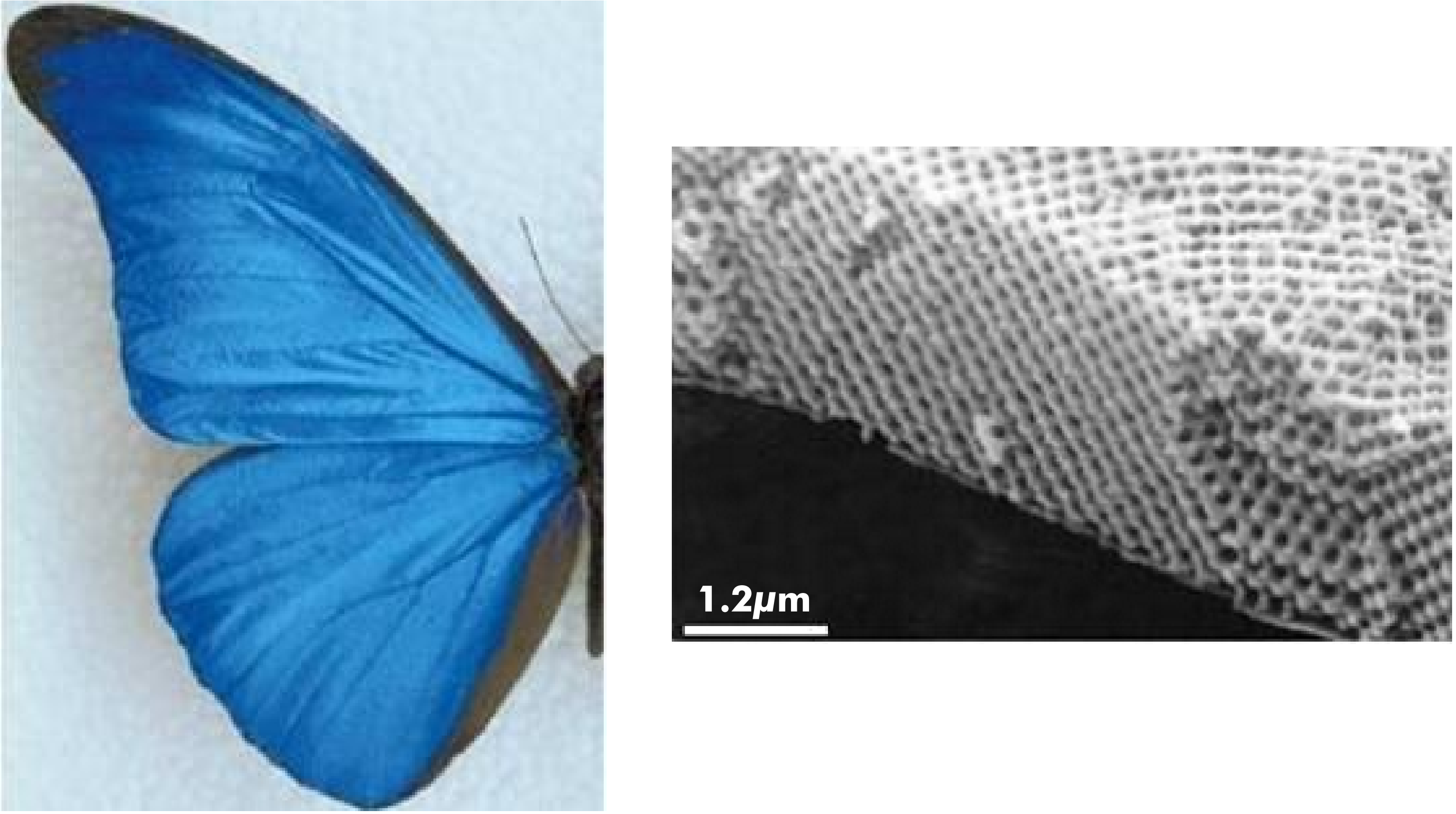
3D Photonic Crystals in nature



Morpho Rhetenor und Parides Sesostris

Overview: P. Vukusic and J.R. Sambles, Nature 424, 852 (2003)

3D Photonic Crystals in nature

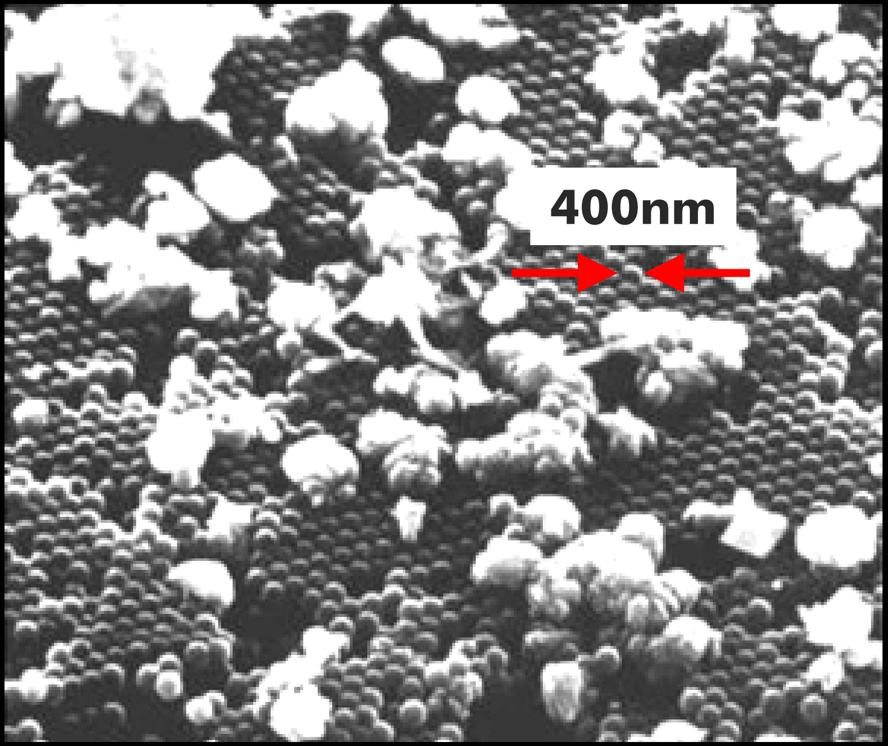
Pachyrhynchus Argus

Overview: P. Vukusic and J.R. Sambles, Nature 424, 852 (2003)

Opals: 3D Photonic Crystals

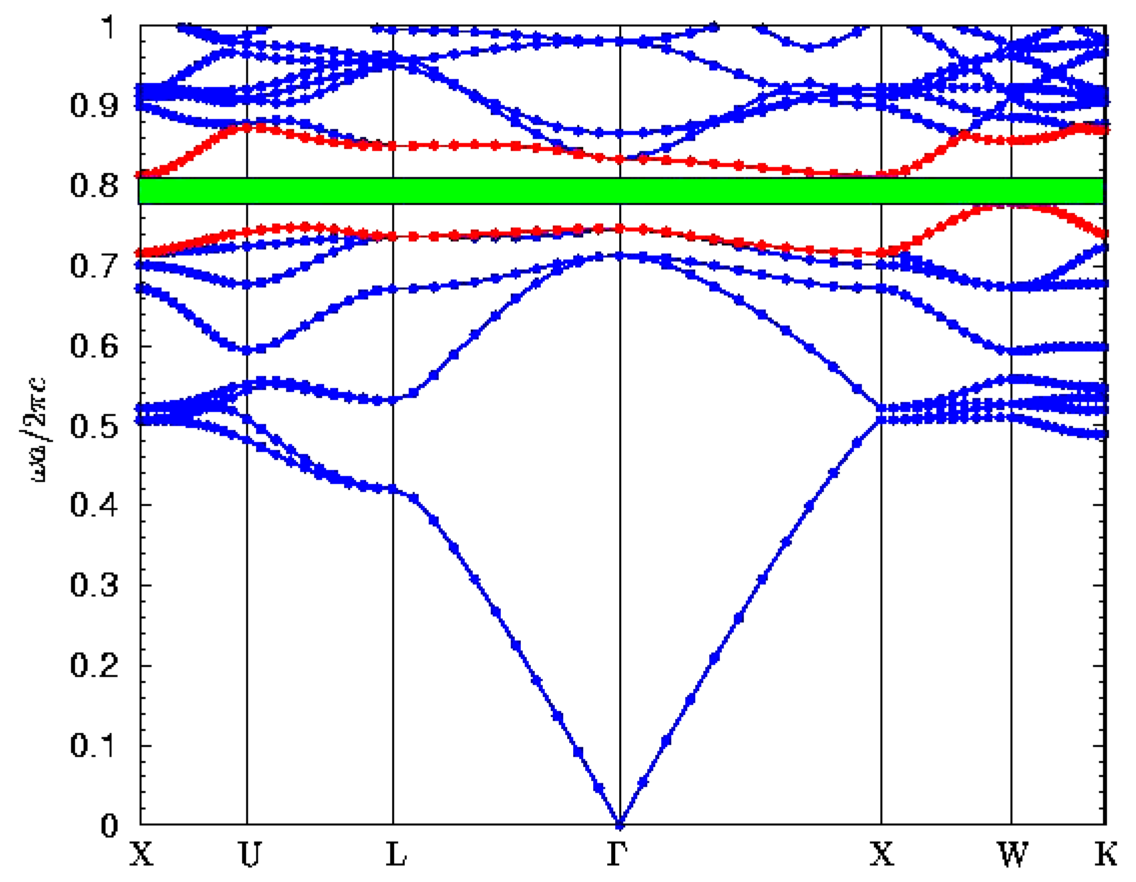
Taken from: eBay.com

A closer look at an Opal



Taken from: J.B. Pendry, Current Science 76, 1311 (1999)

Silicon-based inverse Opals



Kurt Busch

Actual band structure calculations have to solve the eigenvalue problem (see 1.1. and 2.2.).

$$\frac{1}{\mu(\vec{r})}\vec{\nabla}\times\left(\frac{1}{\varepsilon(\vec{r})}\vec{\nabla}\times\vec{H}(\vec{r})\right)=\hat{O}(\vec{r})\vec{H}(\vec{r})=\frac{\omega^2}{c_0^2}\vec{H}(\vec{r})$$

Translational symmetry delivers the Bloch theorem

$$\vec{H}(\vec{r})=\vec{h}_{\vec{K}}(\vec{r})\exp(\mathrm{i}\vec{K}\cdot\vec{r})\neq\vec{H}(\vec{r}+\vec{T})$$

$$\vec{h}_{\vec{K}}(\vec{r})=\vec{h}_{\vec{K}}(\vec{r}+\vec{T})$$

Use, e.g., plane-wave expansion method $\Rightarrow \omega(\vec{K})$

Same mathematics as in 2D

Today, complete three-dimensional photonic band gaps have been found for the following crystal symmetries:

- simple cubic (sc)
- base centered cubic (bcc)
- face centered cubic (fcc)
- diamond
- rhombohedral

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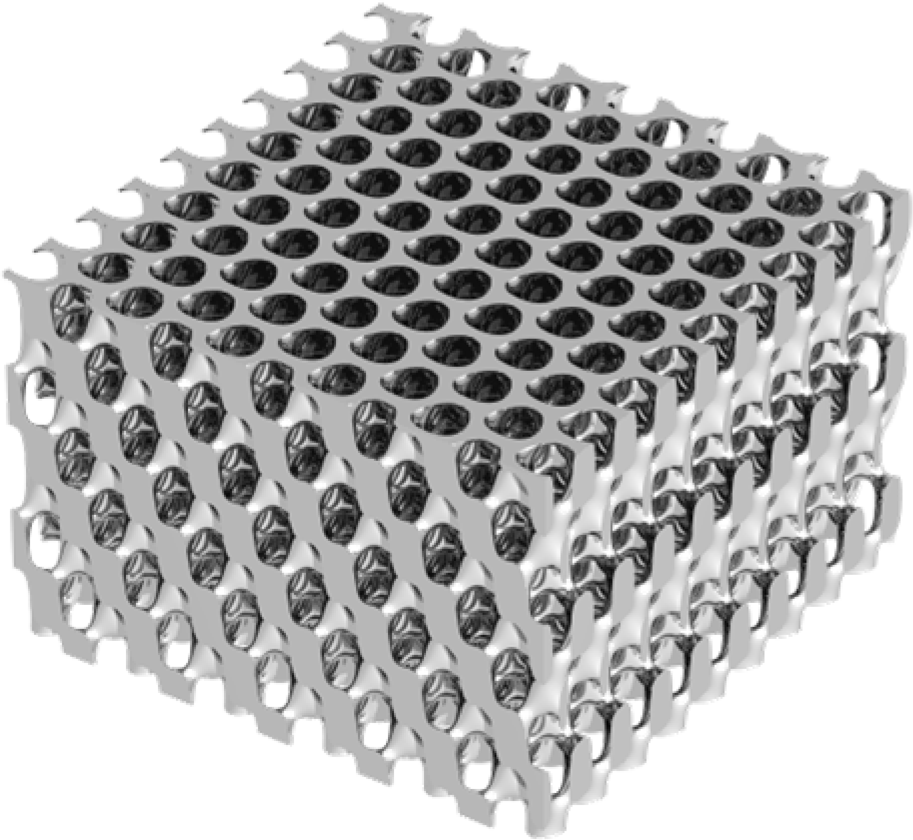
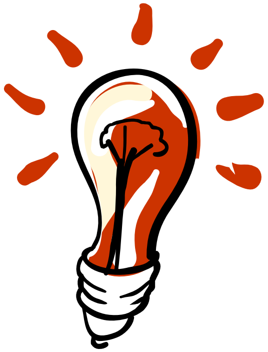
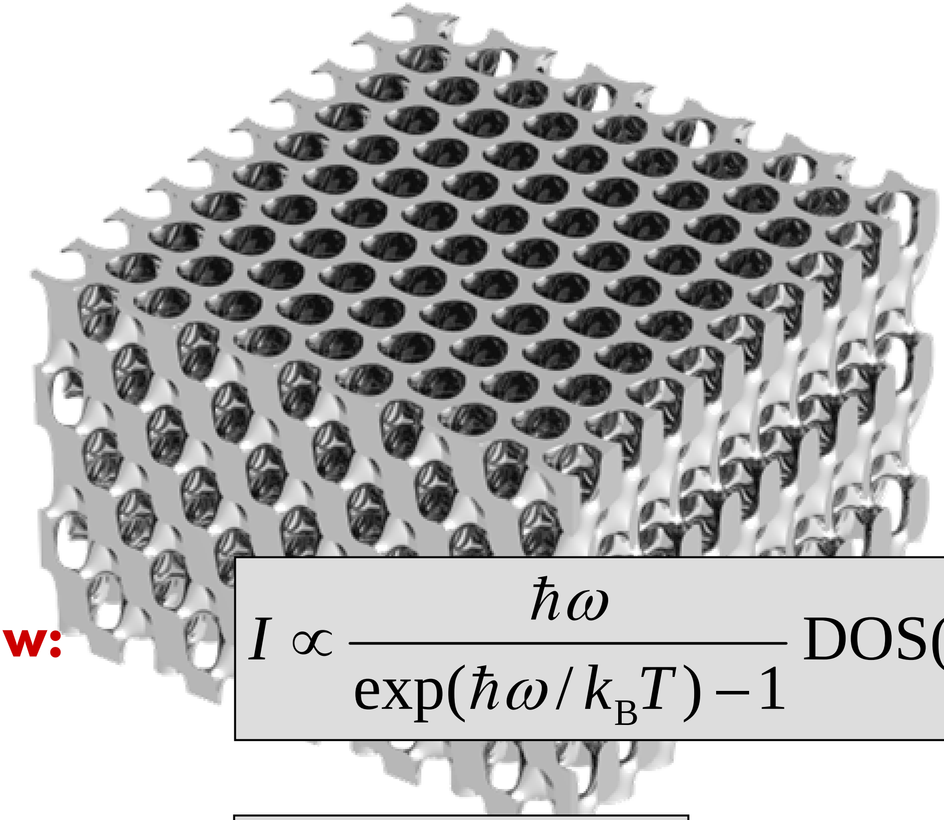
3D metallic PC as light emitters





Idea & IR experiment: J.G. Fleming et al., Nature 417, 52 (2002)

3D metallic PC as light emitters



Planck's law:

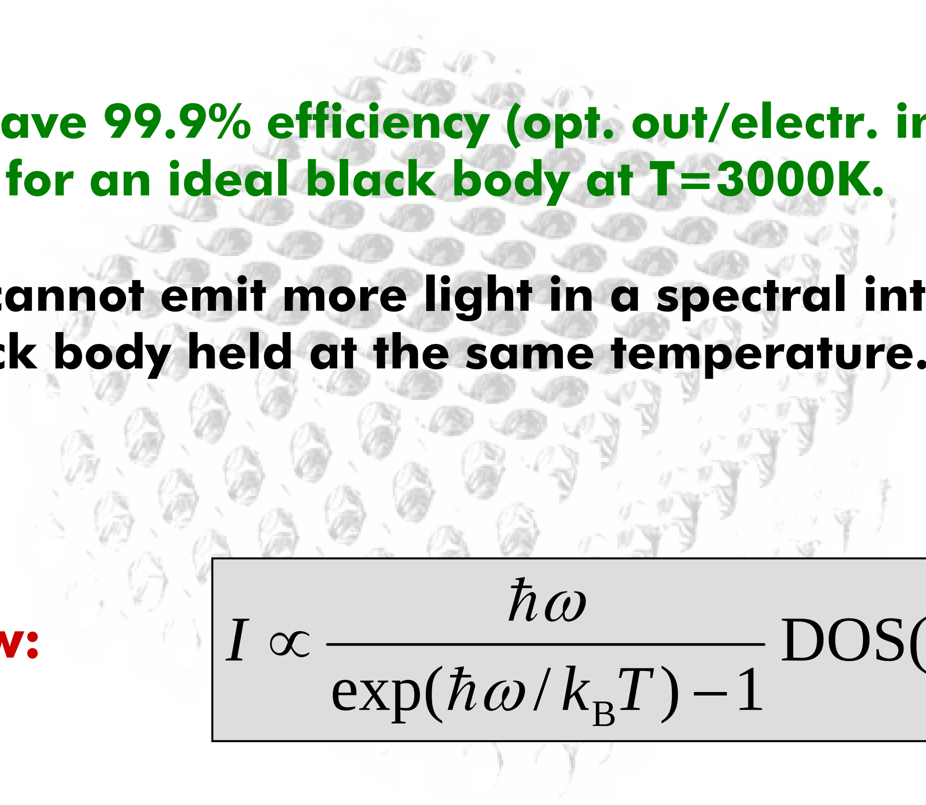
$$\boxed{I \propto \frac{\hbar\omega}{\exp(\hbar\omega/k_{\mathrm{B}}T)-1}\,\mathrm{DOS}(\omega)}$$

Free 3D space:

$$\boxed{\mathrm{DOS}(\omega) \propto (\hbar\omega)^2}$$

Idea & IR experiment: J.G. Fleming et al., Nature 417, 52 (2002)

2. law of thermodynamics …



The PC can have 99.9% efficiency (opt. out/electr. in) rather than just 3% for an ideal black body at T=3000K.

However, it cannot emit more light in a spectral interval than an ideal black body held at the same temperature.

Planck's law:

$$\boxed{I \propto \frac{\hbar\omega}{\exp(\hbar\omega/k_{\mathrm{B}}T)-1}\,\mathrm{DOS}(\omega)}$$

Free 3D space:

$$\boxed{\mathrm{DOS}(\omega) \propto (\hbar\omega)^2}$$

P. Würfel et al., Appl. Phys. Lett. 84, 1997 (2004)

Consider a **3D sc lattice of air cubes**, each coated with a thin film of an ideal metal.

Solutions are standing waves within the cubes

$$\mathrm{DOS}(\omega) \propto \sum_{N_x\, N_y\, N_z} \delta(\omega - \omega_{N_x\, N_y\, N_z})$$

Eigenfrequencies from the dispersion relation of light:

$$c_0 = \frac{\omega_{N_x\, N_y\, N_z}}{|\vec{K}|} = \frac{\omega_{N_x\, N_y\, N_z}}{\sqrt{\left(N_x \frac{\pi}{a}\right)^2 + \left(N_y \frac{\pi}{a}\right)^2 + \left(N_z \frac{\pi}{a}\right)^2}}$$

Consider a 3D sc lattice of air cubes, each coated with a thin film of an ideal metal.

Solutions are standing waves within the cubes

$$\mathrm{DOS}(\omega) \propto \sum_{N_x N_y N_z} \delta(\omega - \omega_{N_x N_y N_z})$$

Eigenfrequencies:

$$\omega_{111} = \sqrt{3}\, c_0 \frac{\pi}{a} \quad ; \quad \omega_{112} = \omega_{121} = \omega_{211} = \sqrt{6}\, c_0 \frac{\pi}{a} \quad \ldots$$

Consider a 3D sc lattice of air cubes, each coated with a thin film of an ideal metal.

Solutions are standing waves within the cubes

$$\boxed{\mathrm{DOS}(\omega) \propto \sum_{N_x N_y N_z} \delta(\omega - \omega_{N_x N_y N_z})}$$

Eigenfrequencies, e.g., in the visible (red):

$$\hbar\omega_{111} = 2\,\mathrm{eV}\ (\lambda = 620\,\mathrm{nm}) \Rightarrow \quad a = \frac{\lambda\sqrt{3}}{2} = 536\,\mathrm{nm}$$

Consider a 3D sc lattice of air cubes, each coated with a thin film of an ideal metal.

Solutions are standing waves within the cubes

$$\mathrm{DOS}(\omega) \propto \sum_{N_x\, N_y\, N_z} \delta(\omega - \omega_{N_x\, N_y\, N_z})$$

Eigenfrequencies, e.g., in the visible (red):

$$\hbar\omega_{111} = 2\,\mathrm{eV} \;\Rightarrow\; \hbar\omega_{112} = 2.8\,\mathrm{eV}$$

$$\Rightarrow \exp\left(-\frac{\hbar\omega_{112} - \hbar\omega_{111}}{k_{\mathrm{B}}\, 3000\,\mathrm{K}} \right) \approx 0.046 << 1$$

typical light bulb

i.e., monochromatic emission!

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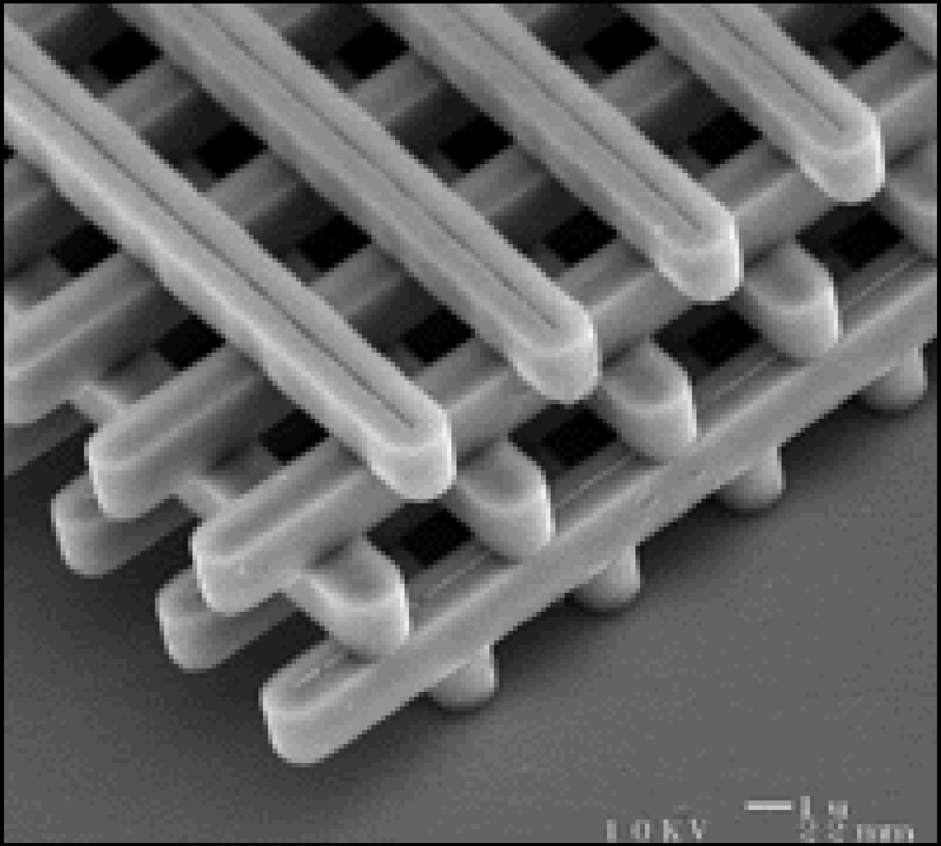
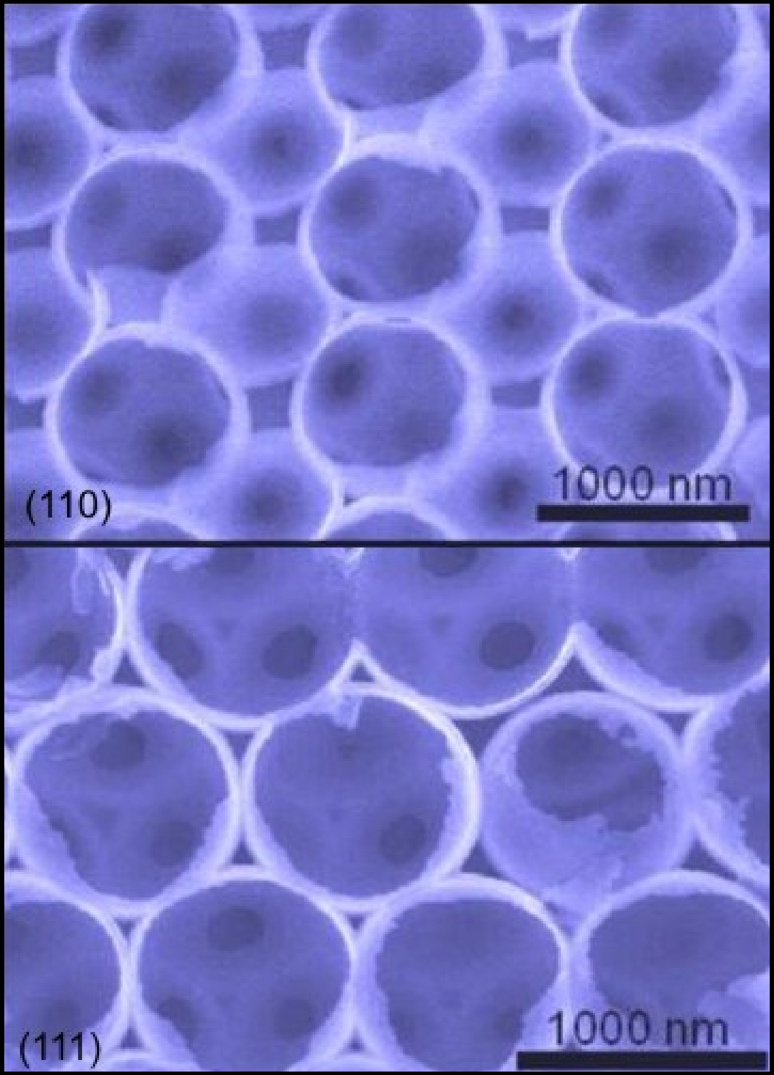
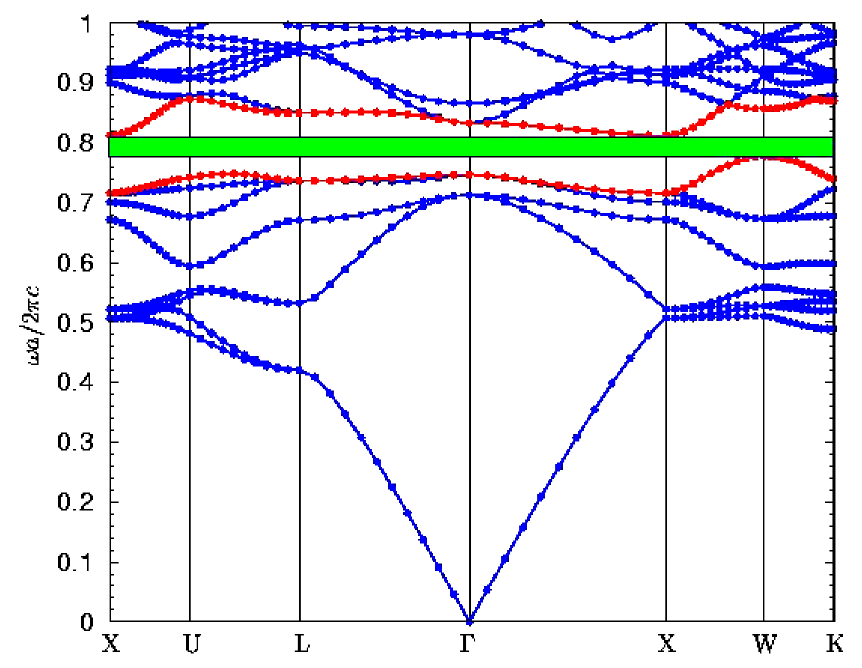
3D woodpile structure



Taken from: J.G. Fleming et al., Opt. Lett. 24, 49 (1999)

3D silicon-based inverted opal





Taken from: A. Blanco et al., Nature 405, 437 (2000) & K. Busch

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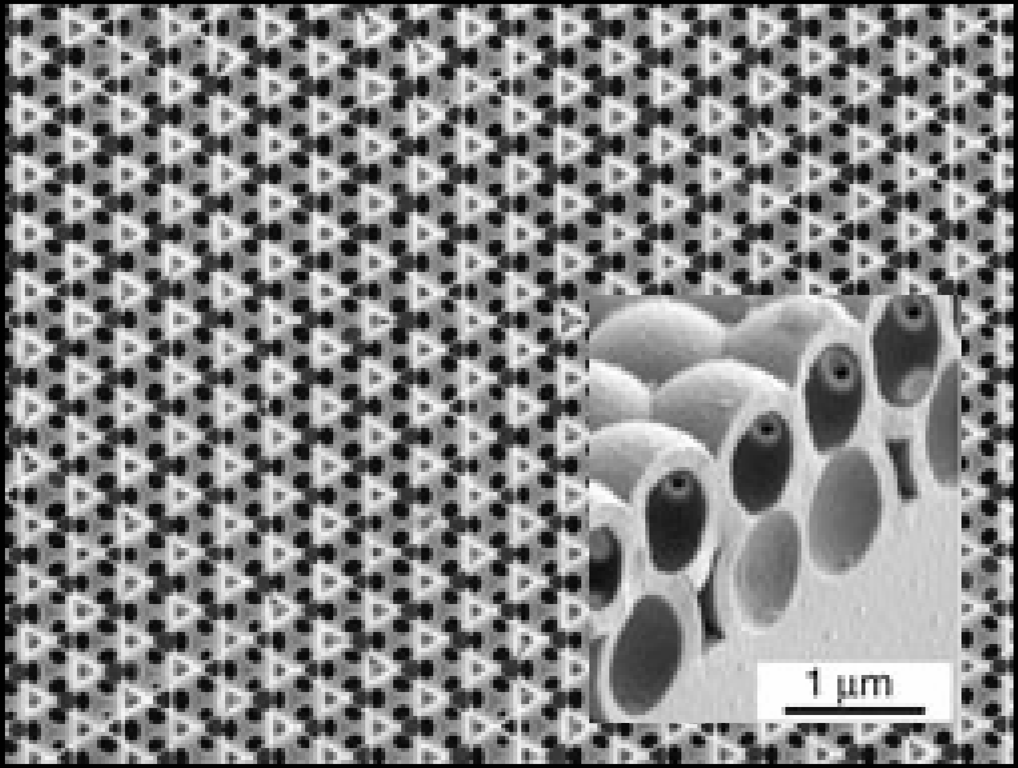
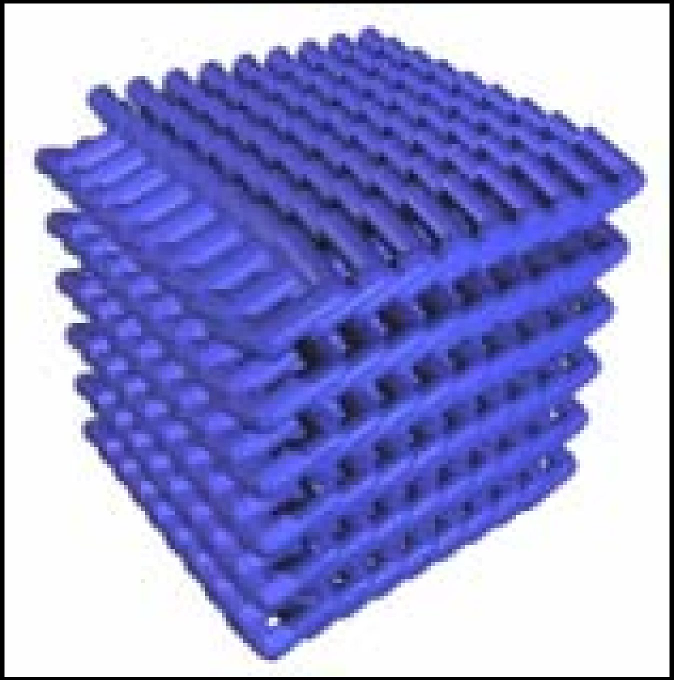


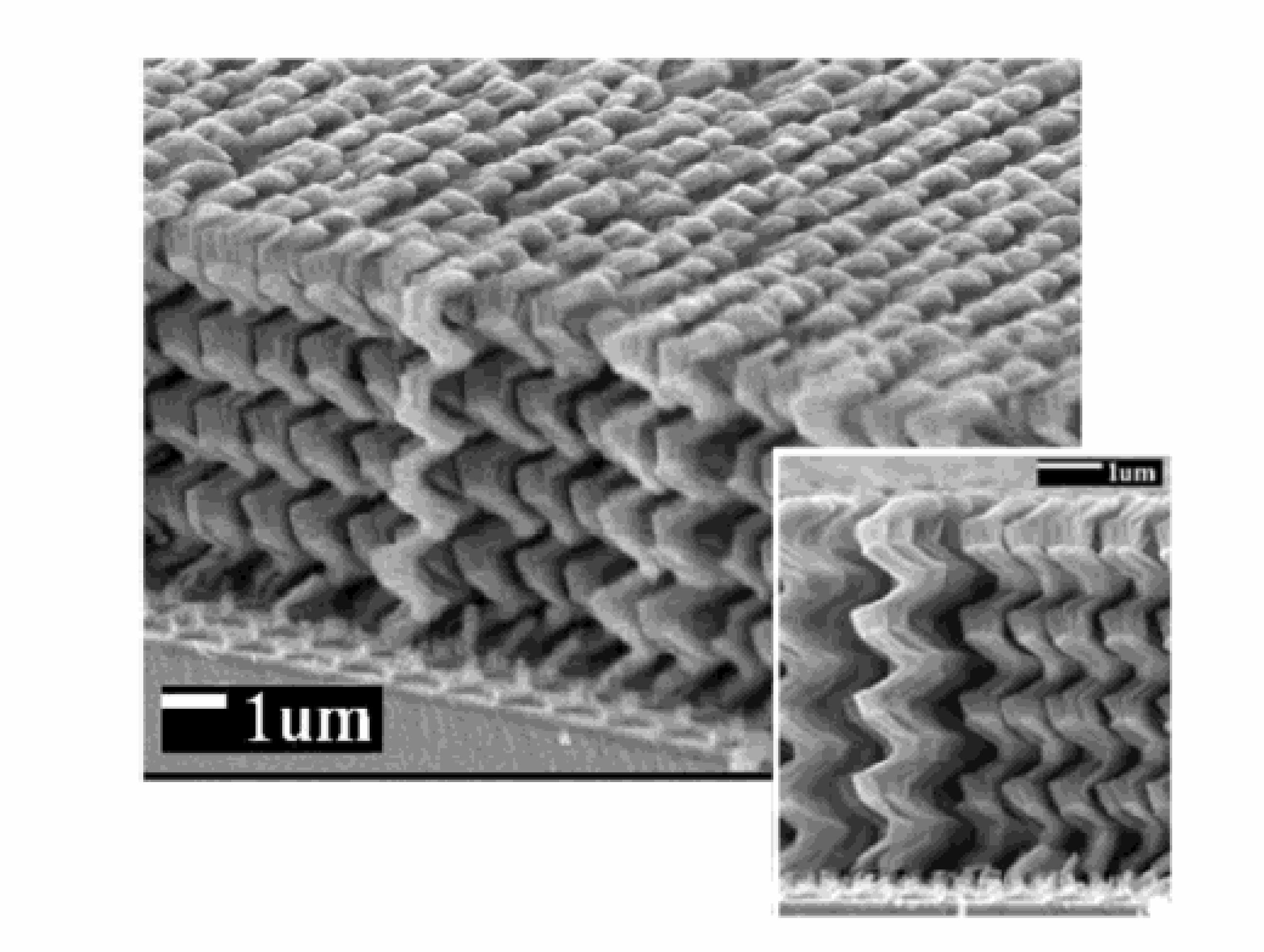
Taken from: D.J. Norris et al., Nature 414, 289 (2001)

3D square-spiral structure



Taken from: S. John et al., Science 292, 1133 (2001)

3D square-spiral structure



Taken from: S.R. Kennedy, S. John et al., Nano Letters 2, 59 (2002)

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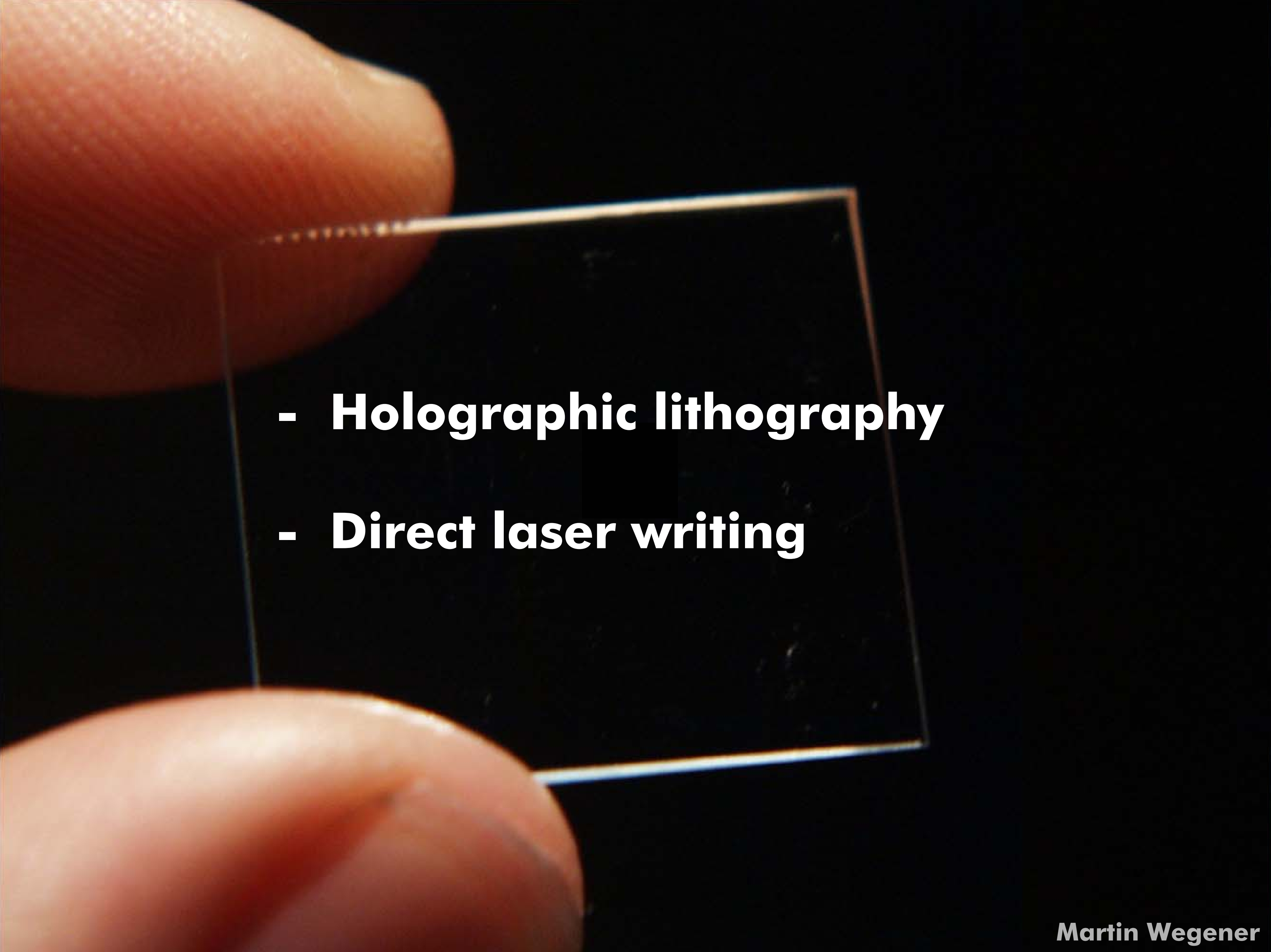
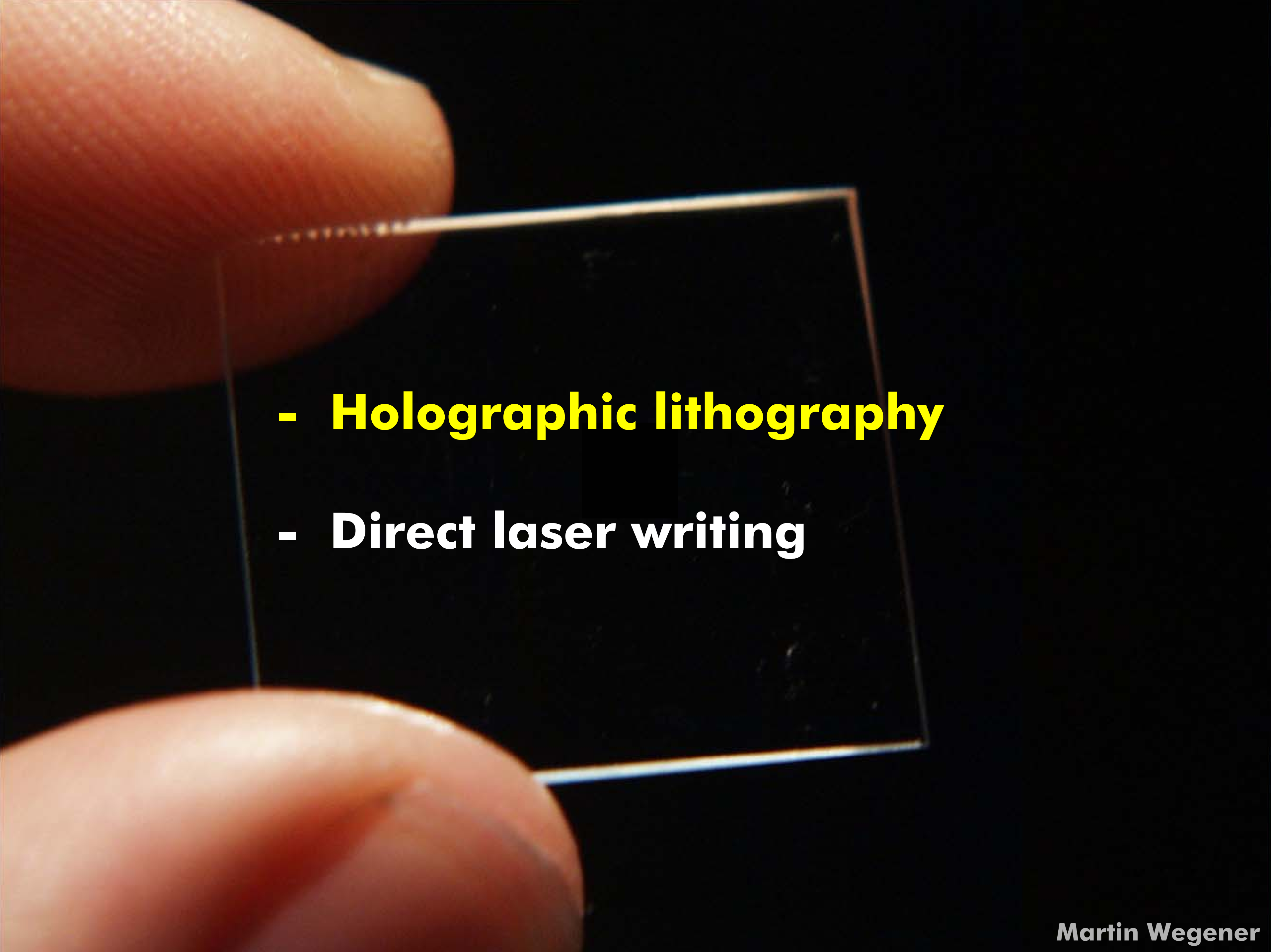


- Holographic lithography
- Direct laser writing

Martin Wegener



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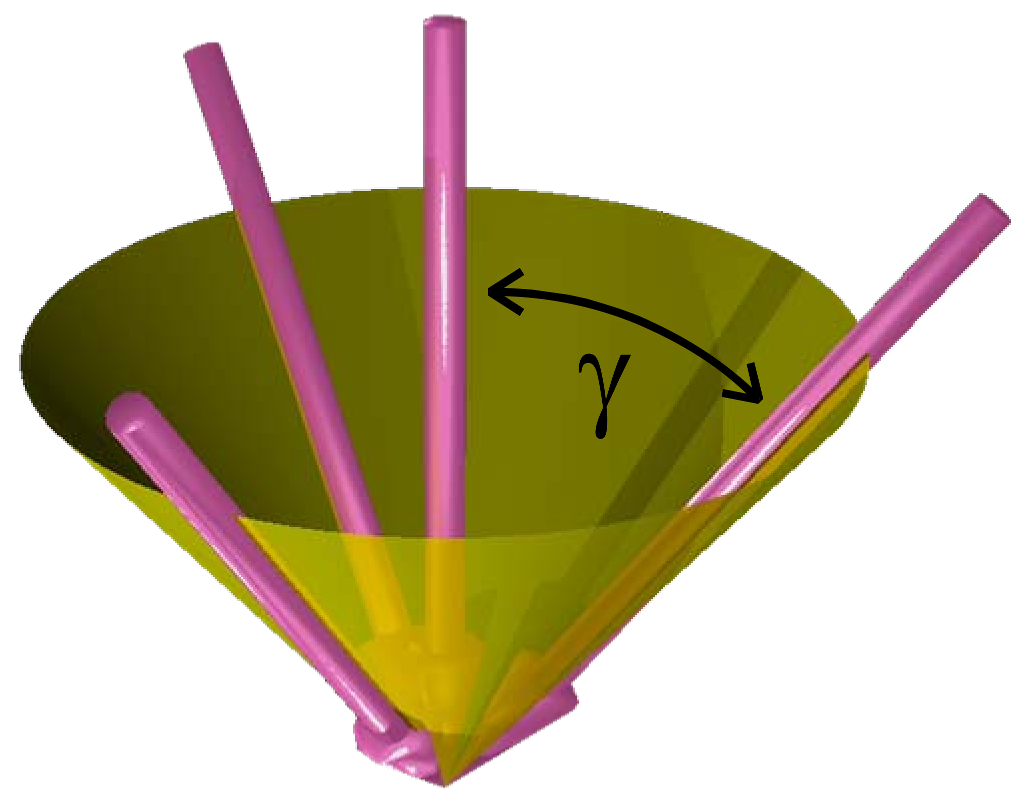

G. Grynberg et al., Phys. Rev. Lett. 70, 2249 (1993)

Holographic lithography

$$I(\vec{r}) \propto \left| \sum_{n=1}^{4} \vec{E}_n \exp(\mathrm{i}(\vec{k}_n \cdot \vec{r} - \omega t)) \right|^2$$

Holographic lithography

$$I(\vec{r}) \propto \left| \sum_{n=1}^{4} \vec{E}_n \exp(\mathrm{i}(\vec{k}_n \cdot \vec{r} - \omega\, t)) \right|^2$$

$$= \sum_{m=1}^{4} \sum_{l=1}^{4} a_{ml} \exp(\mathrm{i}\, \vec{G}_{ml} \cdot \vec{r})$$

$$a_{ml} = \vec{E}_m \cdot \vec{E}_l^{\,*}$$

$$\vec{G}_{ml} = \vec{k}_m - \vec{k}_l$$

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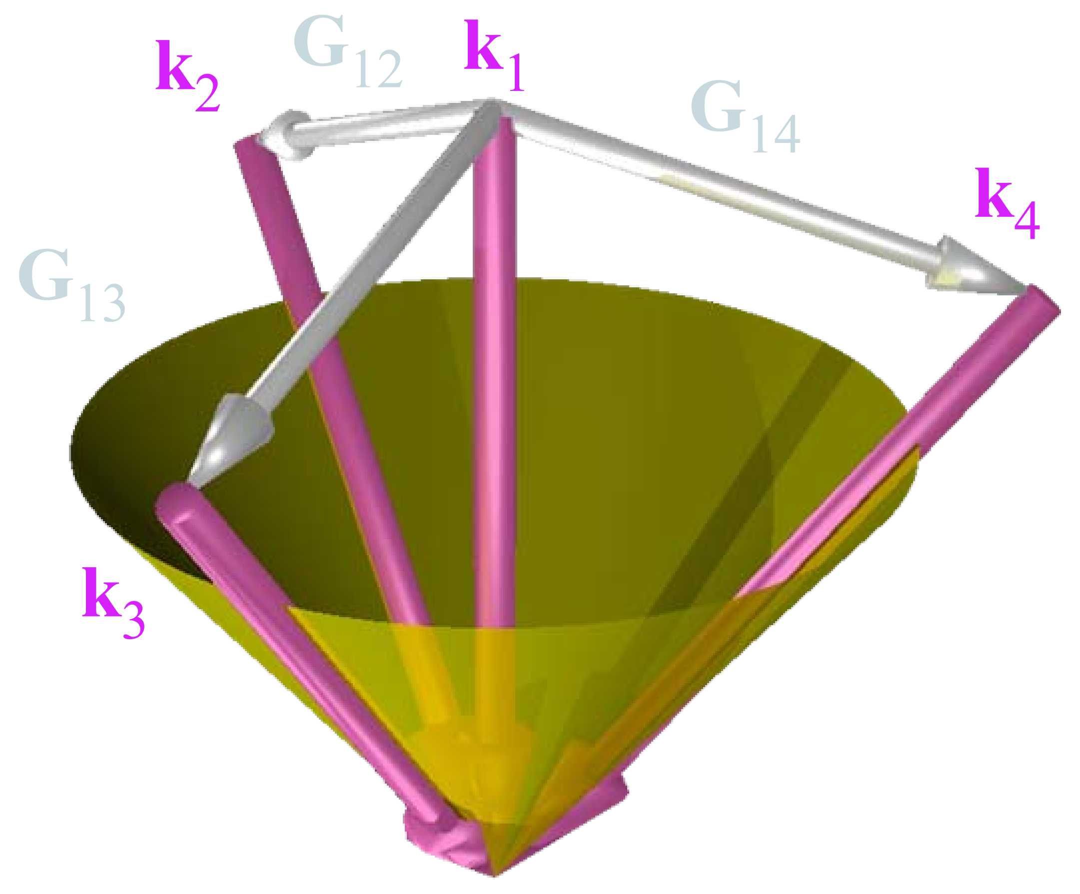
Holographic lithography



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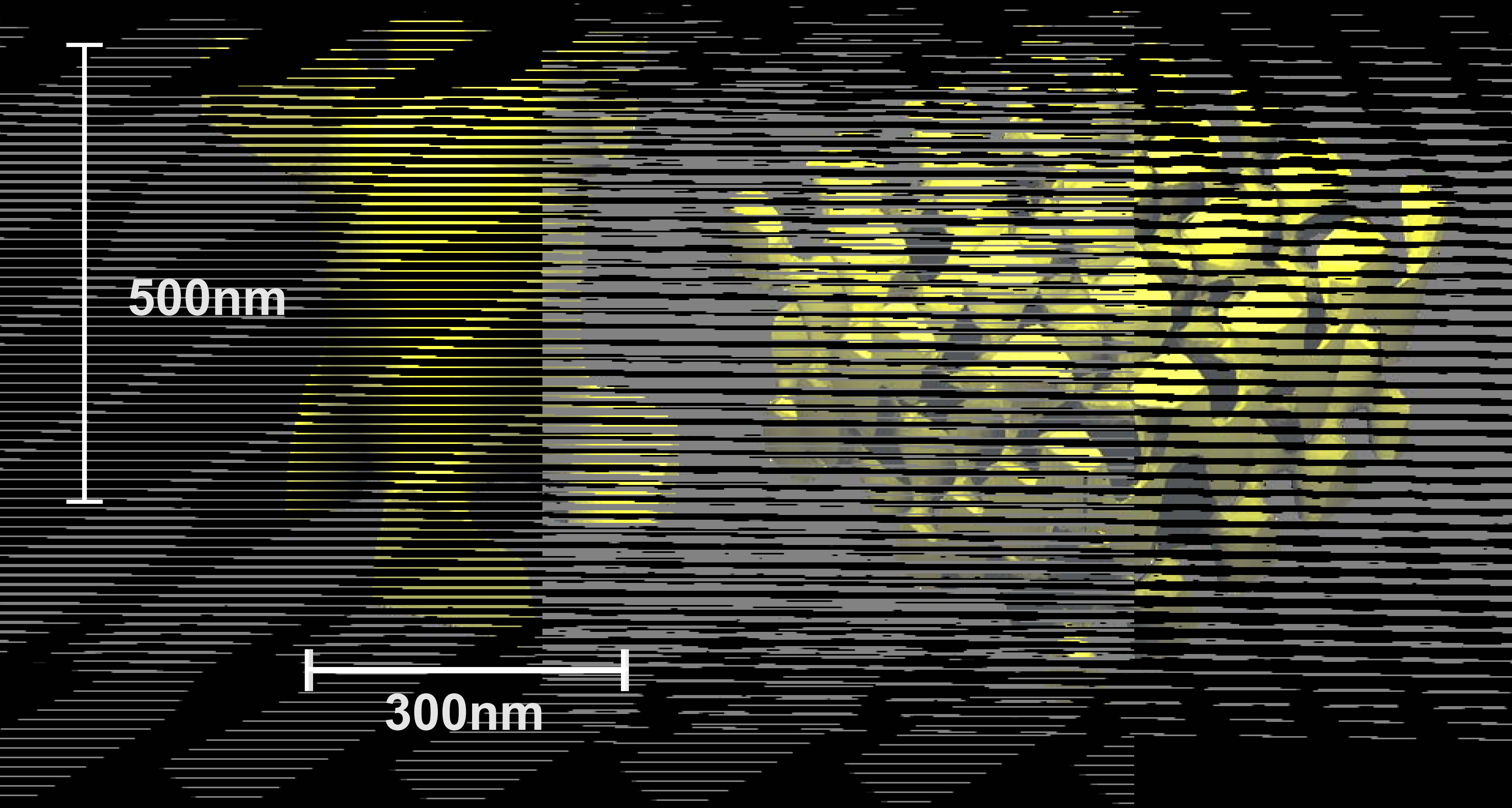
Iso-intensity surfaces



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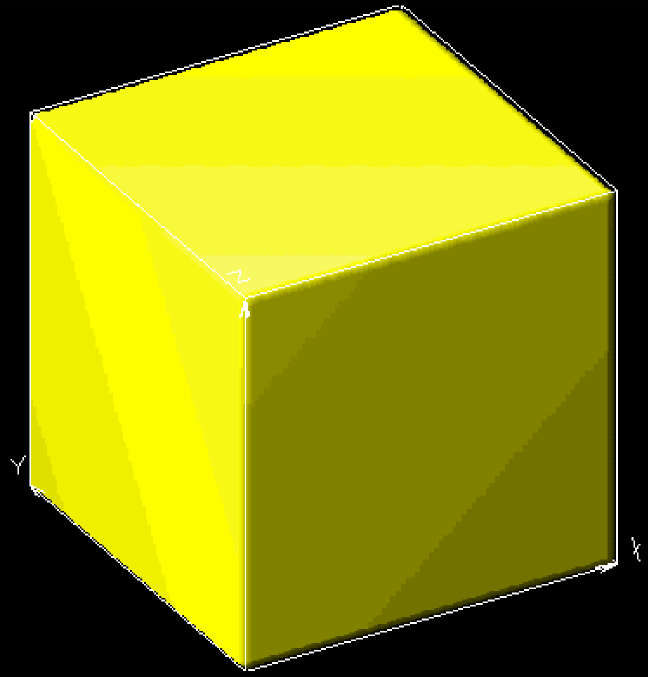
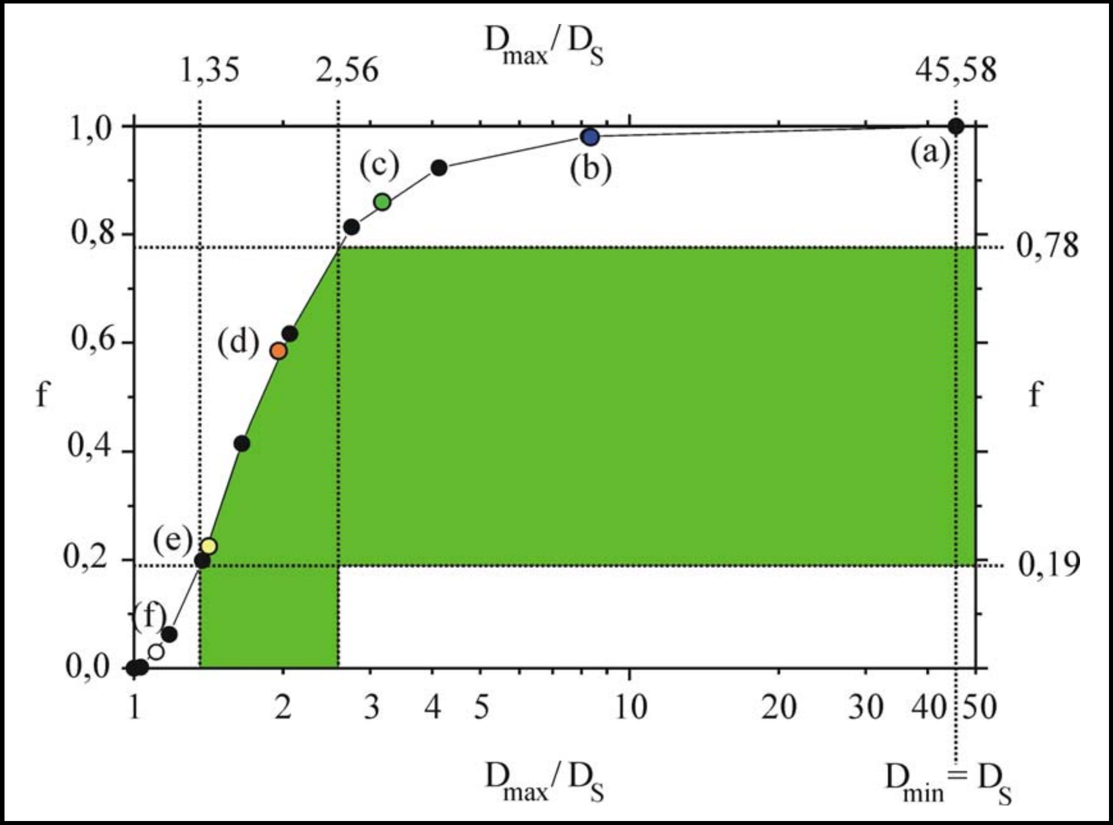
Influence of exposure intensity





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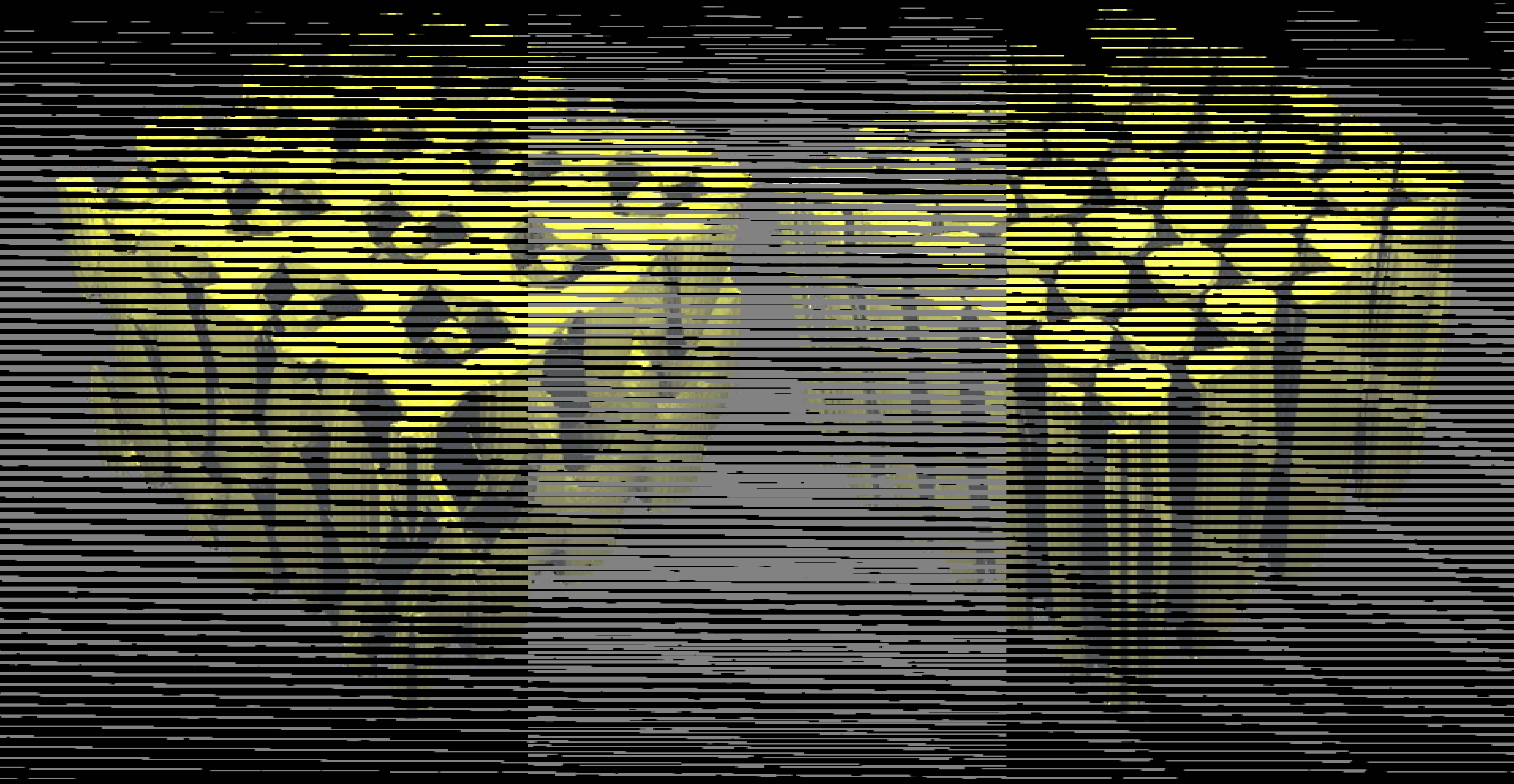
Different polarizations ...



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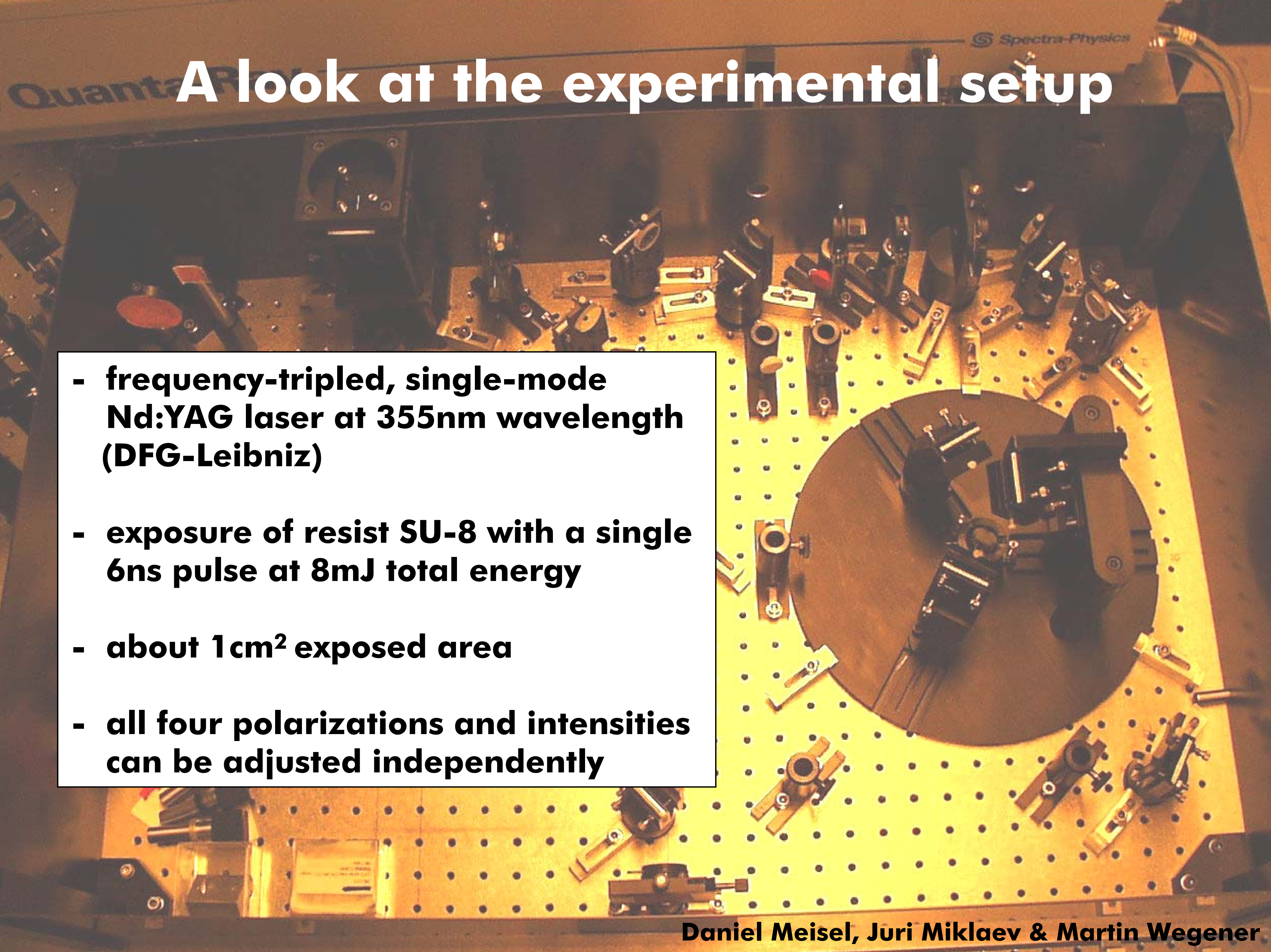
A look at the experimental setup



- frequency-tripled, single-mode Nd:YAG laser at 355nm wavelength (DFG-Leibniz)

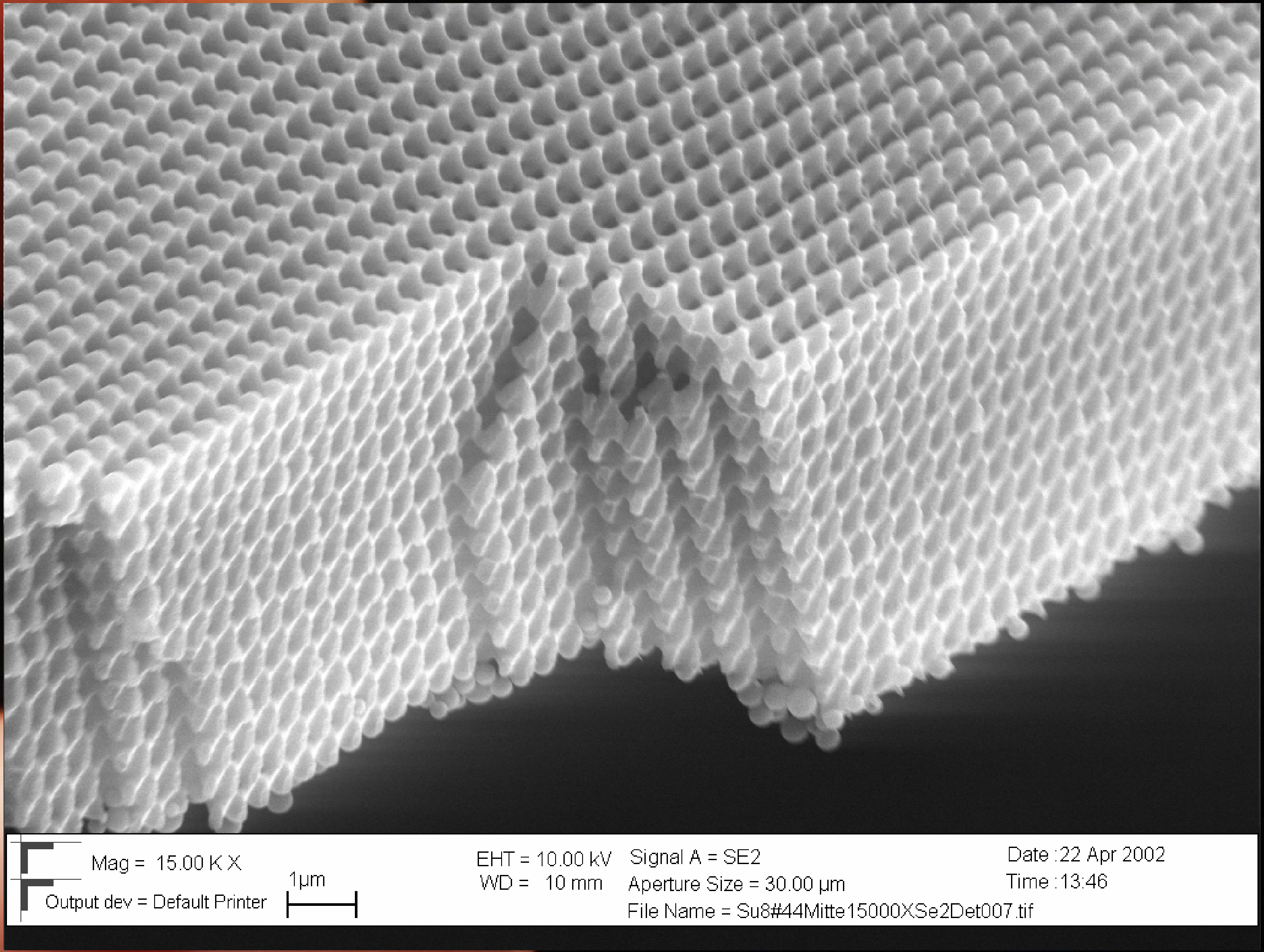
- exposure of resist SU-8 with a single 6ns pulse at 8mJ total energy

- about 1cm^2 exposed area

- all four polarizations and intensities can be adjusted independently

Daniel Meisel, Juri Miklaev & Martin Wegener

Experimental results



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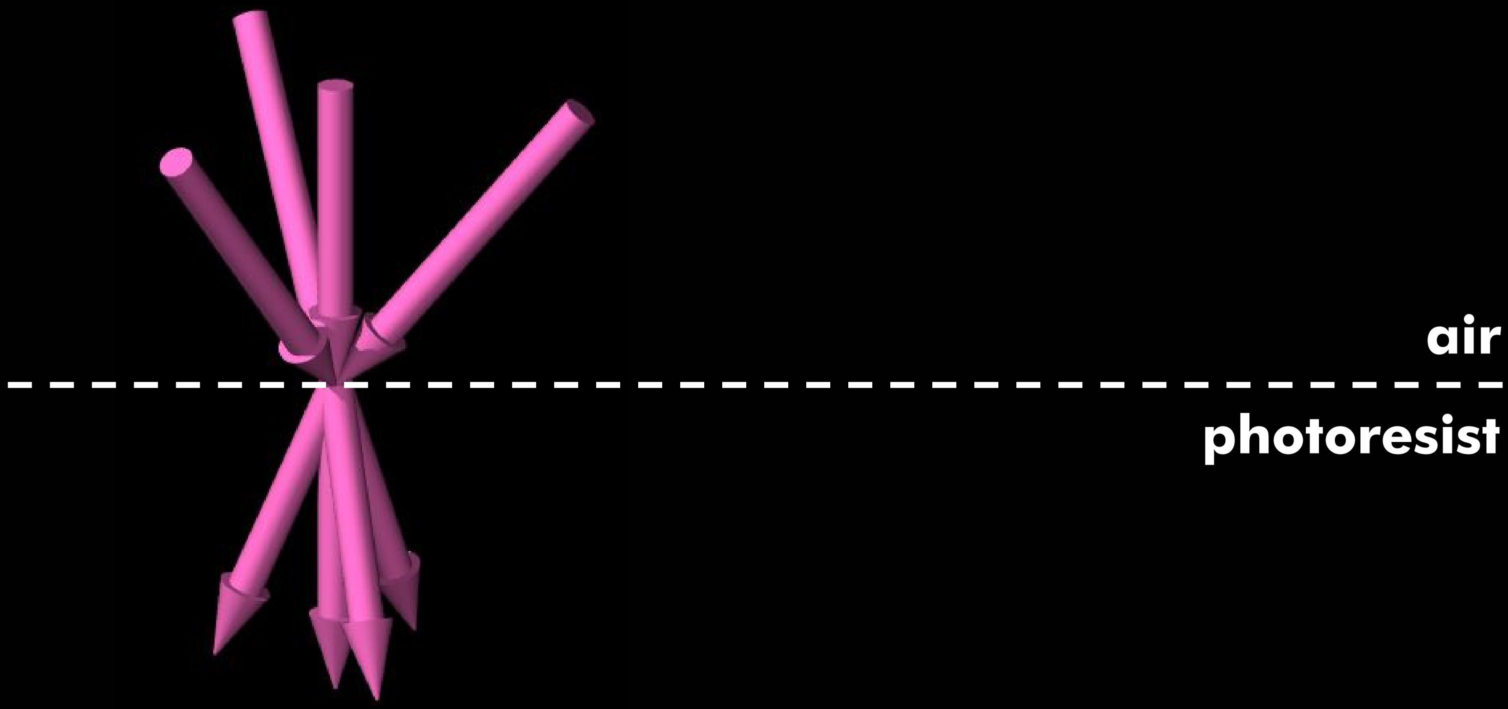
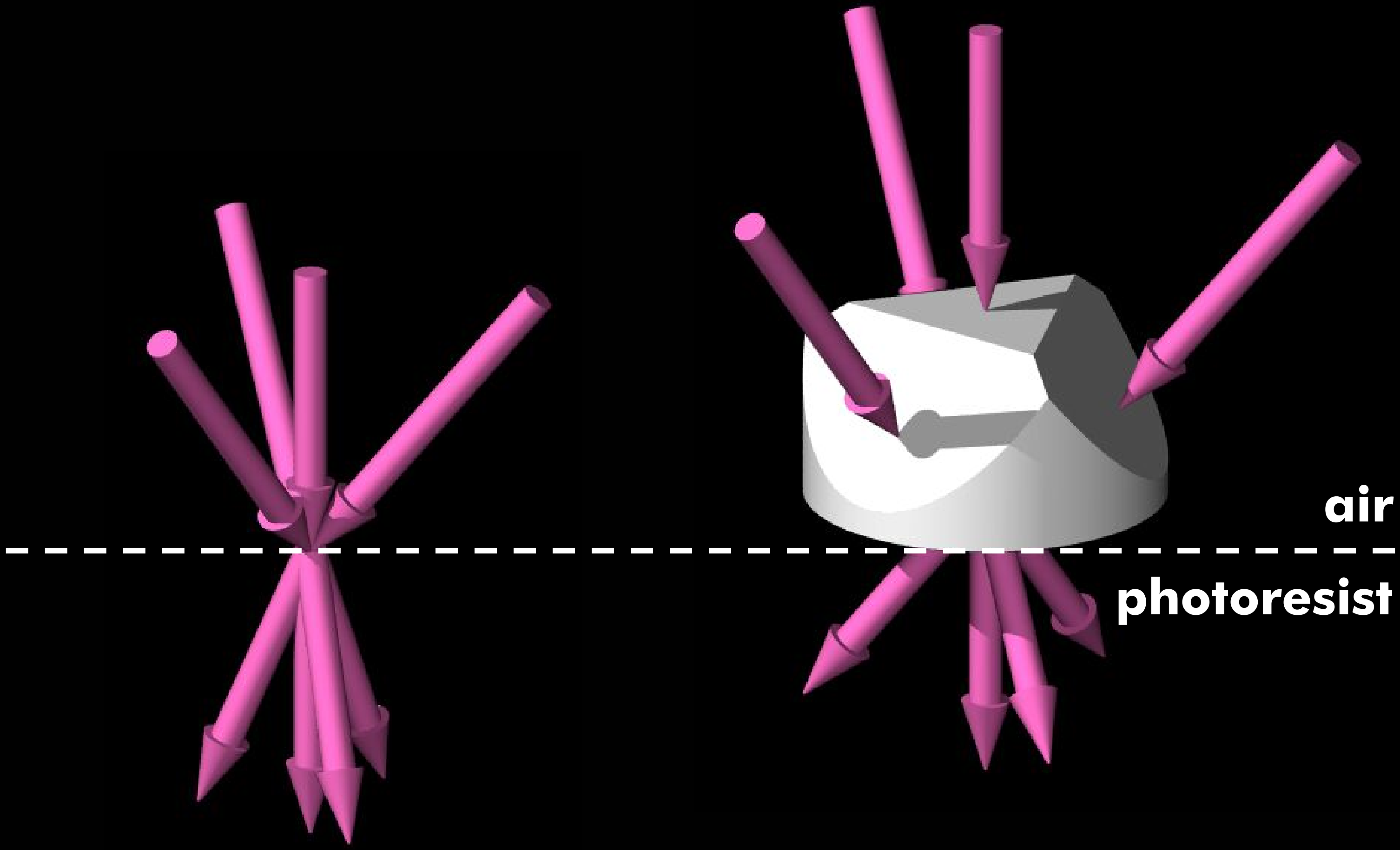
Working against refraction ...



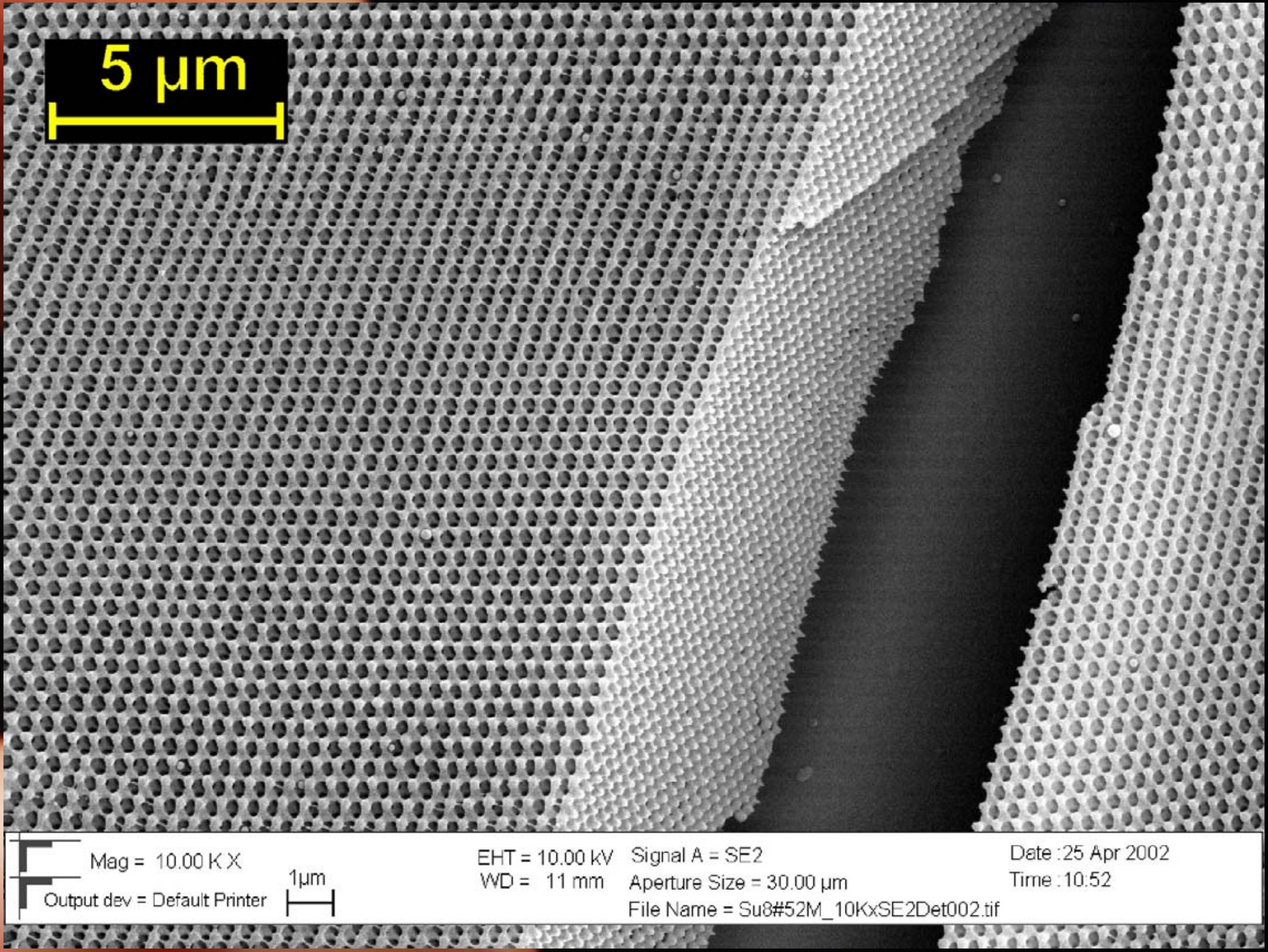
M. Campbell et al., Nature 404, 53 (2000)
X. Wang et al., Appl. Phys. Lett. 82, 2212 (2003)

Working against refraction ...



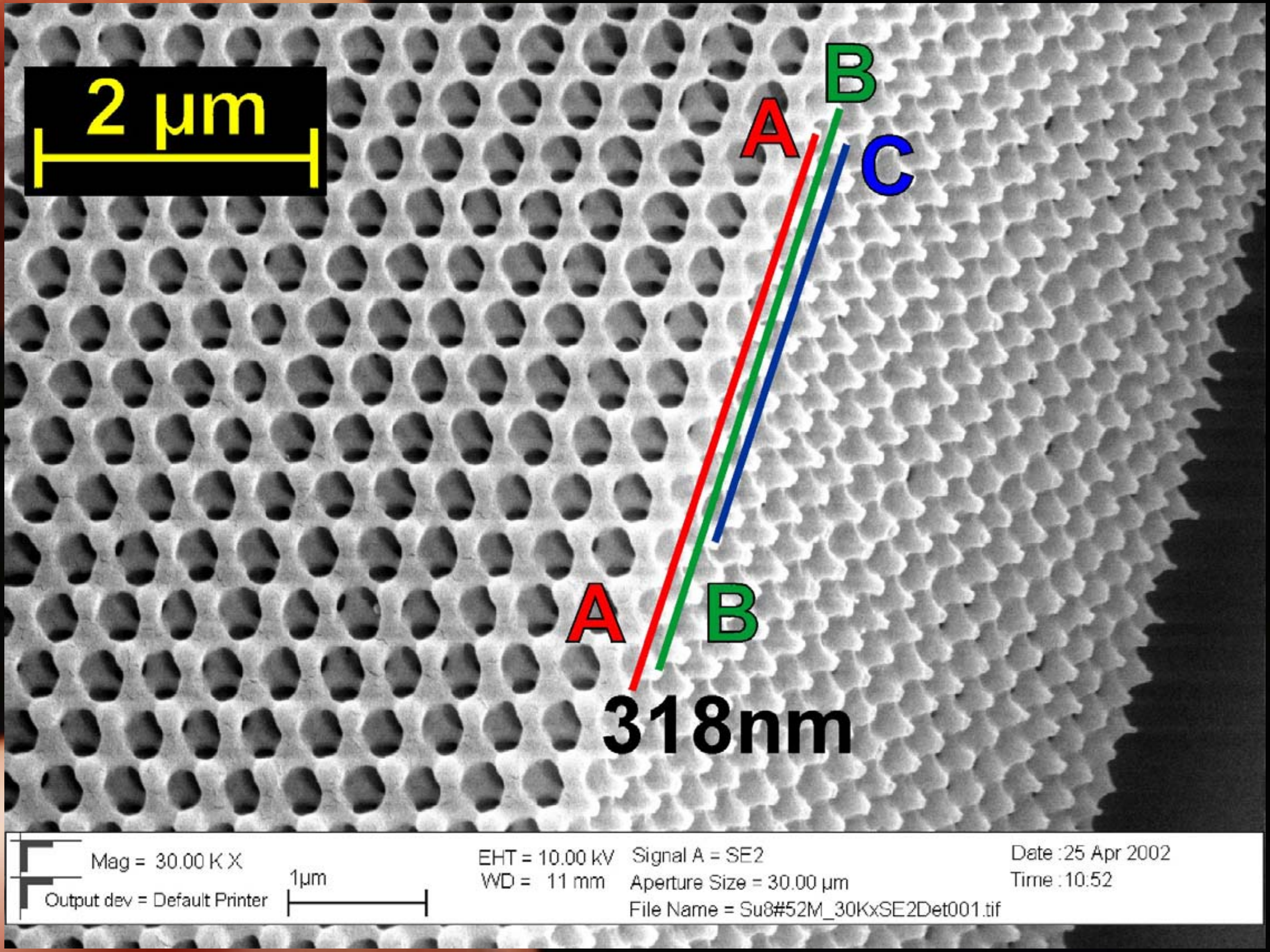
J.M., D.M. & M.W., Patentanmeldung 10233309.2 (INT,FZK)

A fcc Photonic Crystal



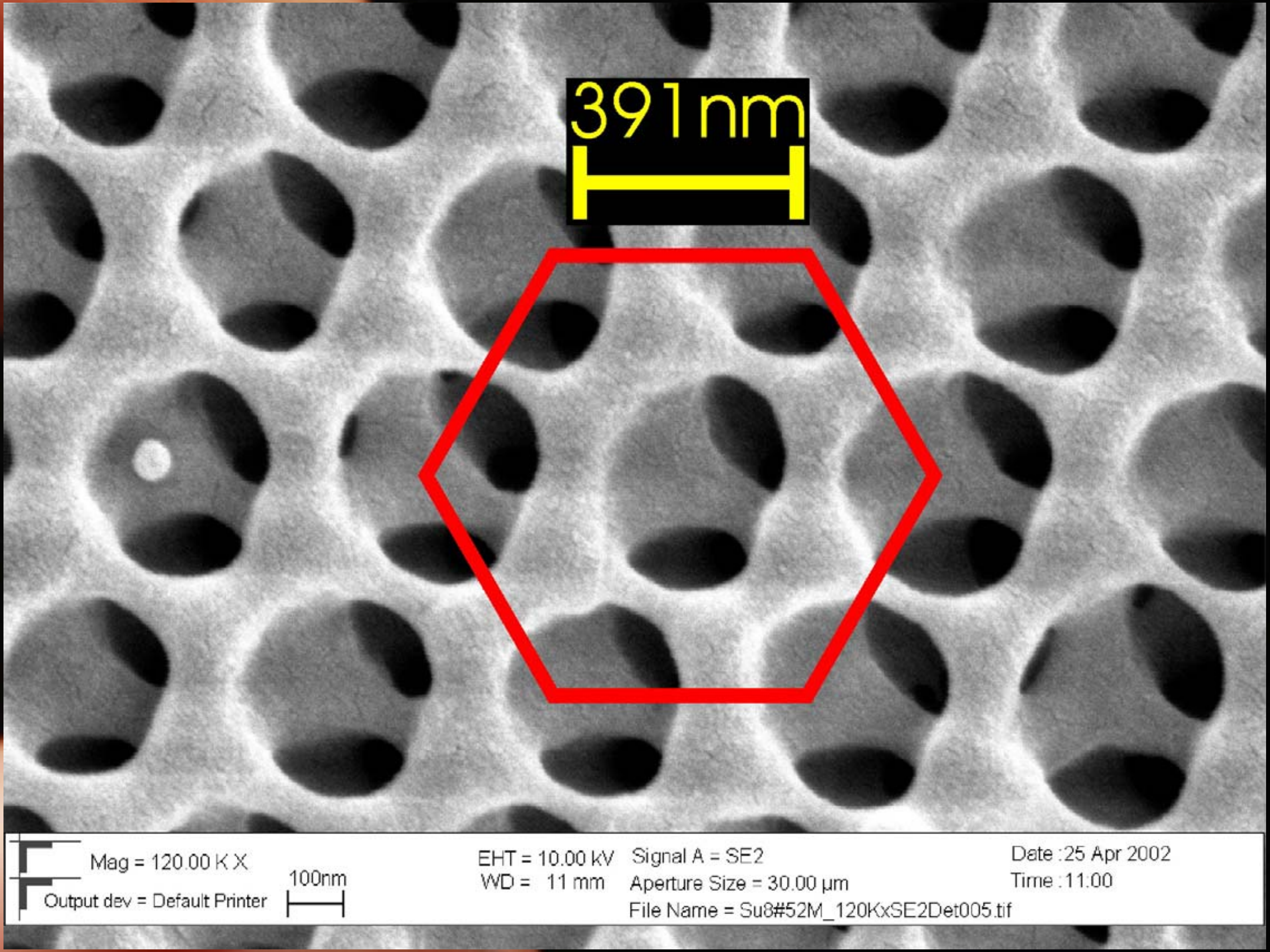
J. Miklaev et al., Appl. Phys. Lett. 82, 1284 (2003)

A fcc Photonic Crystal



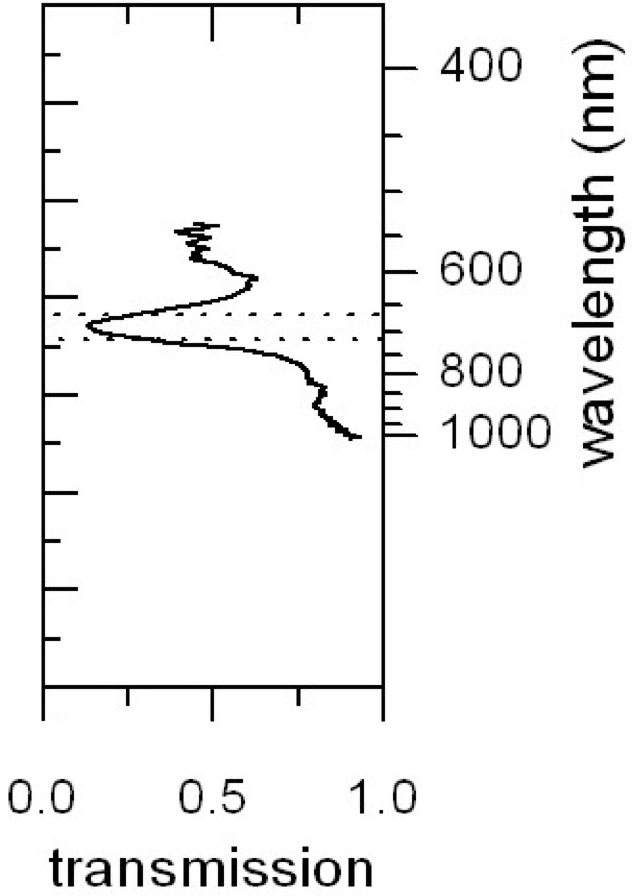
J. Miklaev et al., Appl. Phys. Lett. 82, 1284 (2003)

A fcc Photonic Crystal



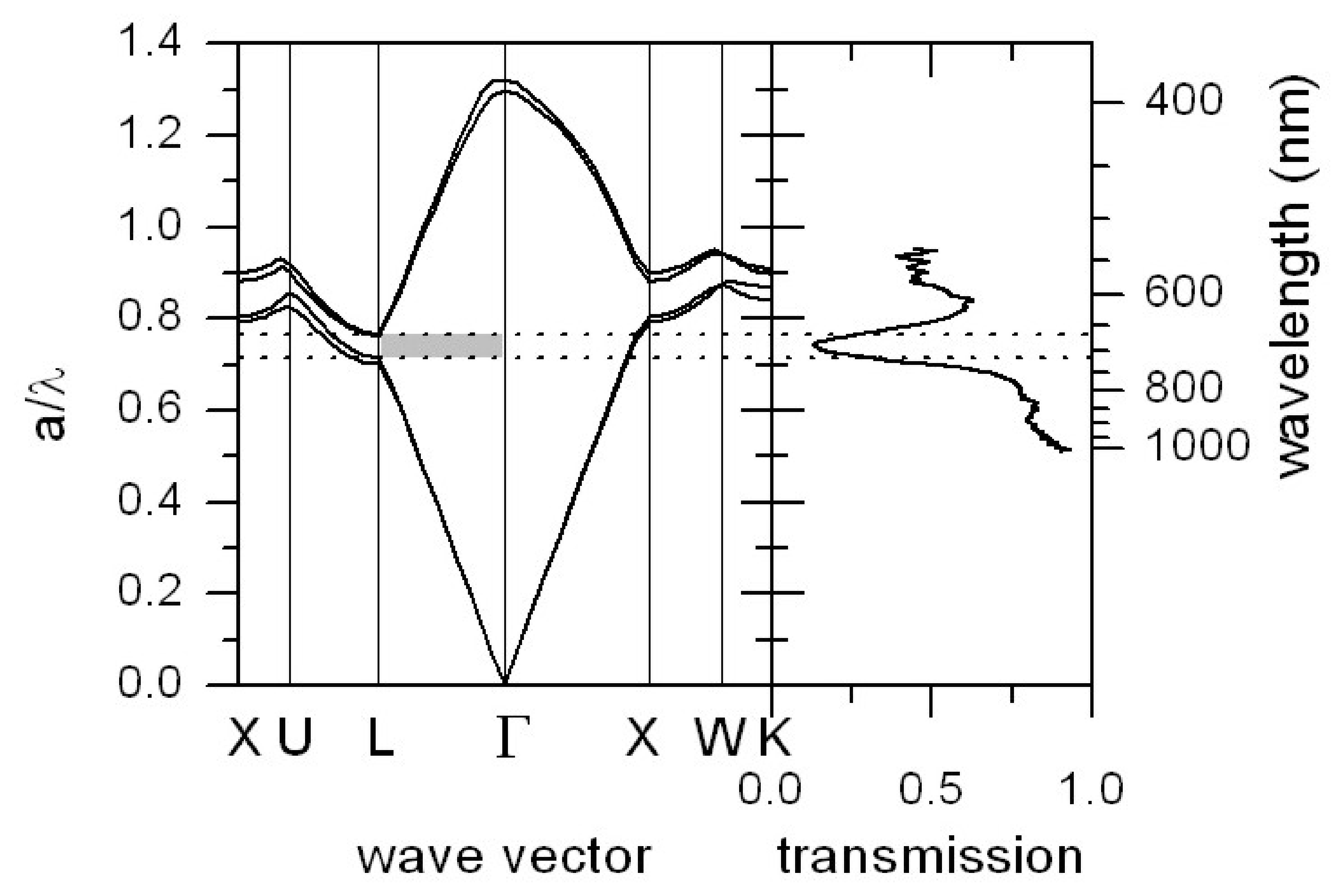
J. Miklaev et al., Appl. Phys. Lett. 82, 1284 (2003)

Optical characterization



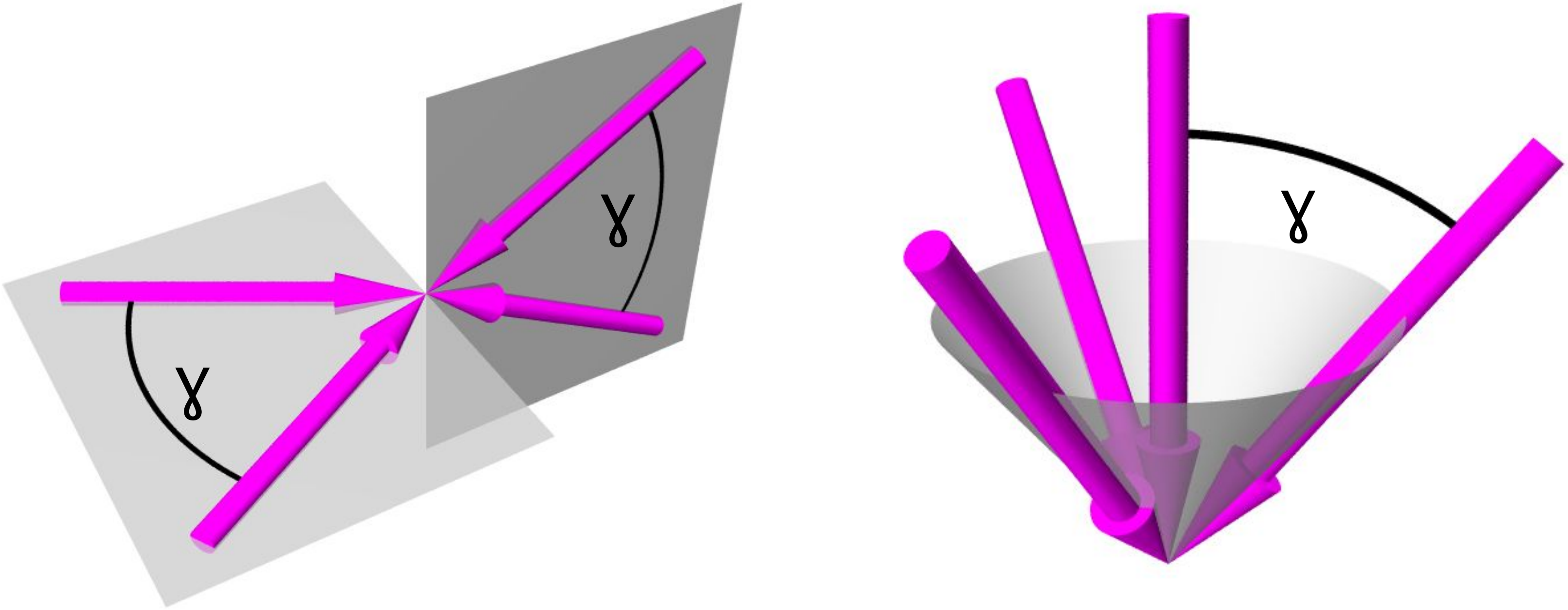
J. Miklaev et al., Appl. Phys. Lett. 82, 1284 (2003)

Comparison with theory



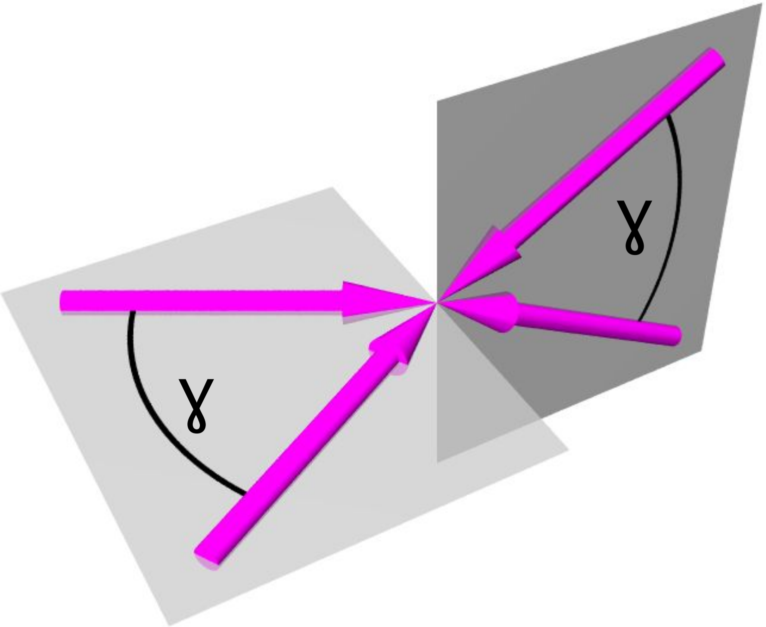
J. Miklaev et al., Appl. Phys. Lett. 82, 1284 (2003)

The 20-parameter problem



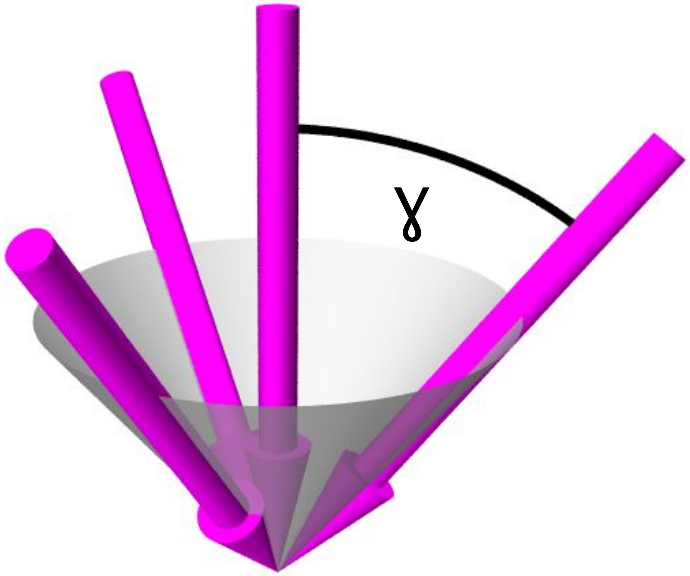
“two-planes” and “umbrella-like“ geometry

The 20-parameter problem



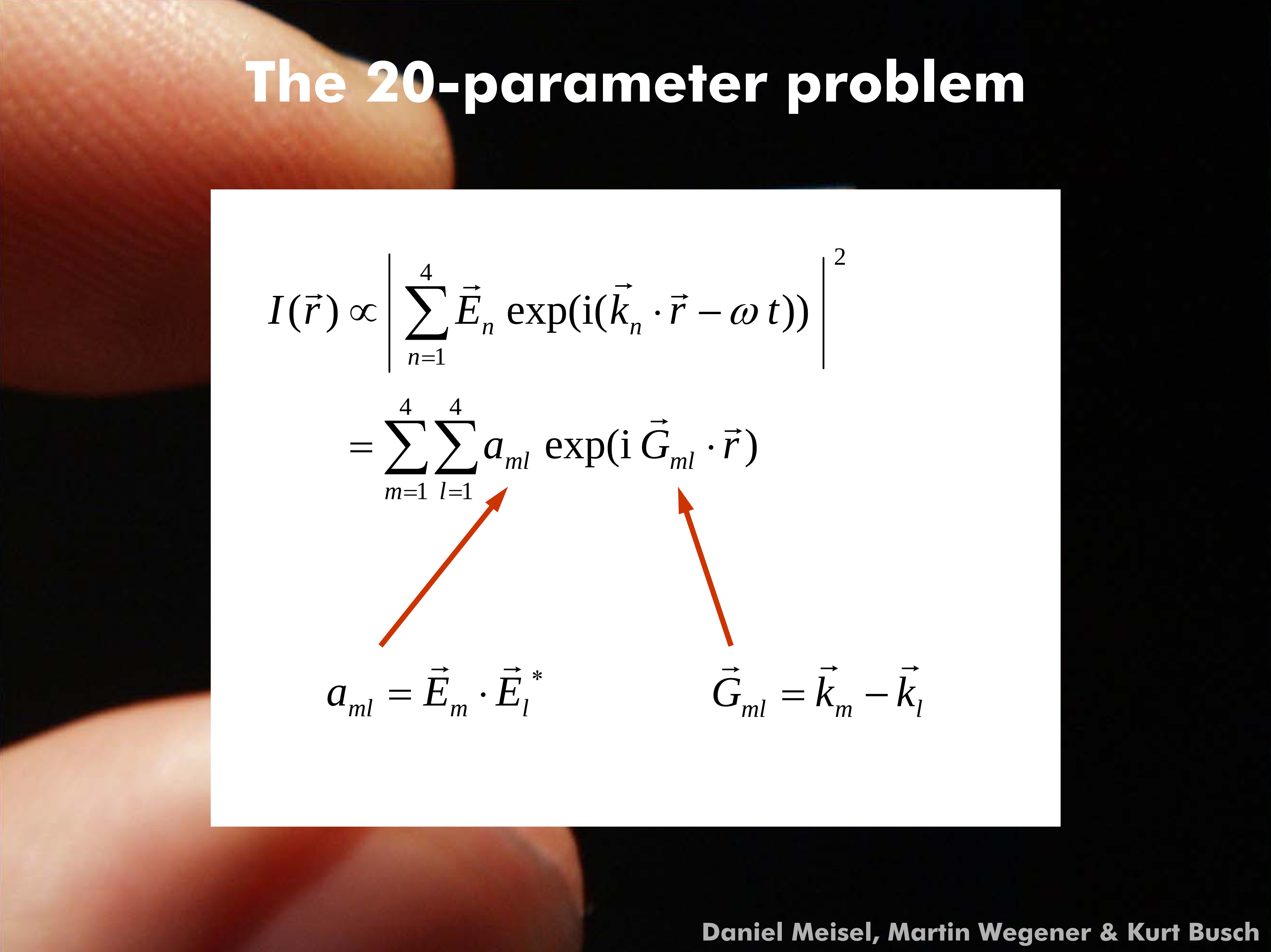
M. Maldovan et al., PRB 65, 165123 (2002) & S. John et al., PRL 92, 043905 (2004)

The 20-parameter problem



D.C. Meisel et al., Phys. Rev. B 70, 165104 (2004)

The 20-parameter problem



$$I(\vec{r}) \propto \left| \sum_{n=1}^{4} \vec{E}_n \exp(\mathrm{i}(\vec{k}_n \cdot \vec{r} - \omega t)) \right|^2$$

$$= \sum_{m=1}^{4} \sum_{l=1}^{4} a_{ml} \exp(\mathrm{i}\, \vec{G}_{ml} \cdot \vec{r})$$

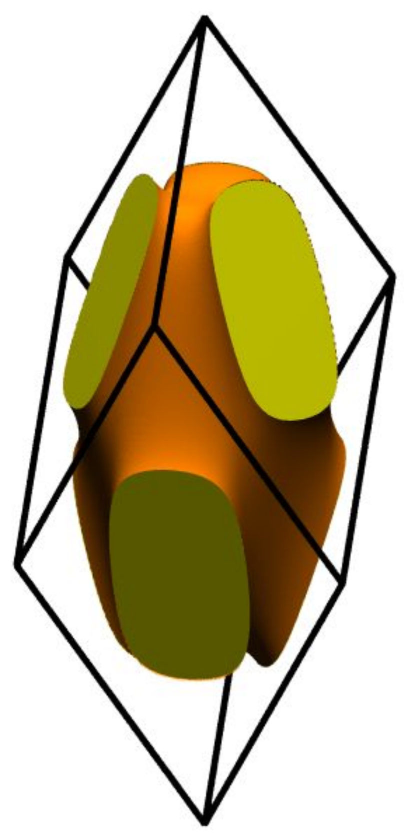
$$a_{ml} = \vec{E}_m \cdot \vec{E}_l^{\,*}$$

$$\vec{G}_{ml} = \vec{k}_m - \vec{k}_l$$

Daniel Meisel, Martin Wegener & Kurt Busch

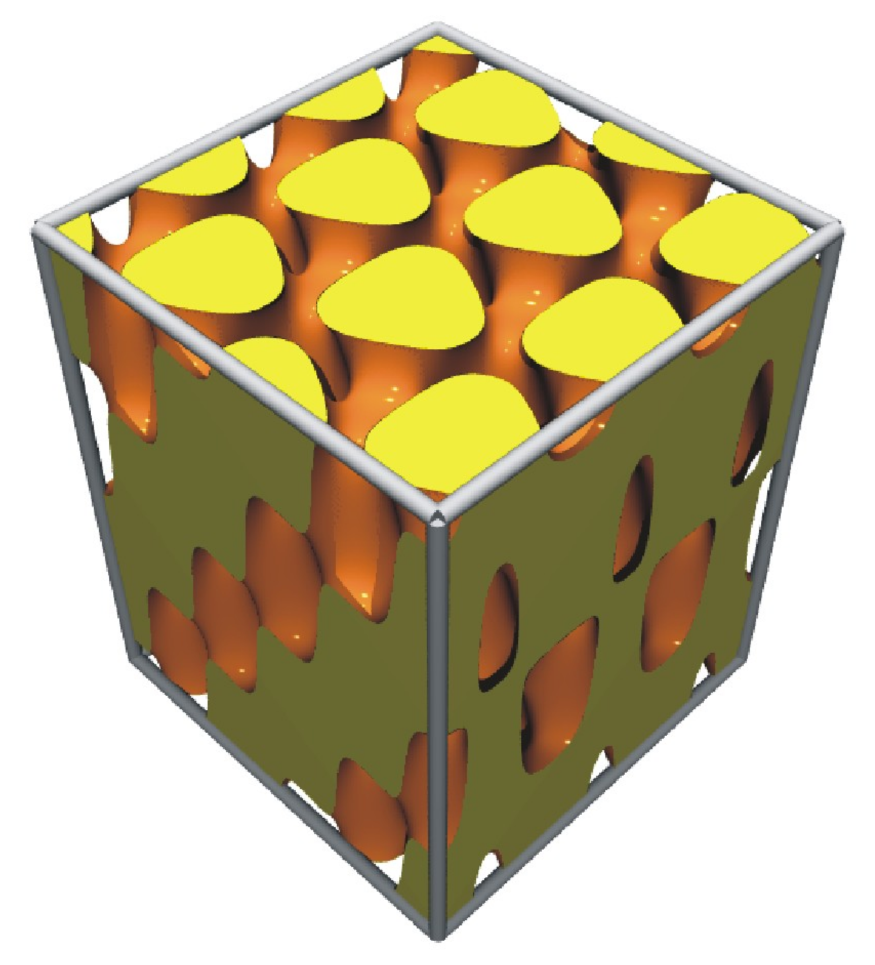
Example I

- fcc translational symmetry
- rhombohedral crystal symmetry
- 5.8% gap/midgap PBG between 2nd and 3rd band for 37% silicon
- resembles celebrated Yablonovite
- 10:1 interference contrast



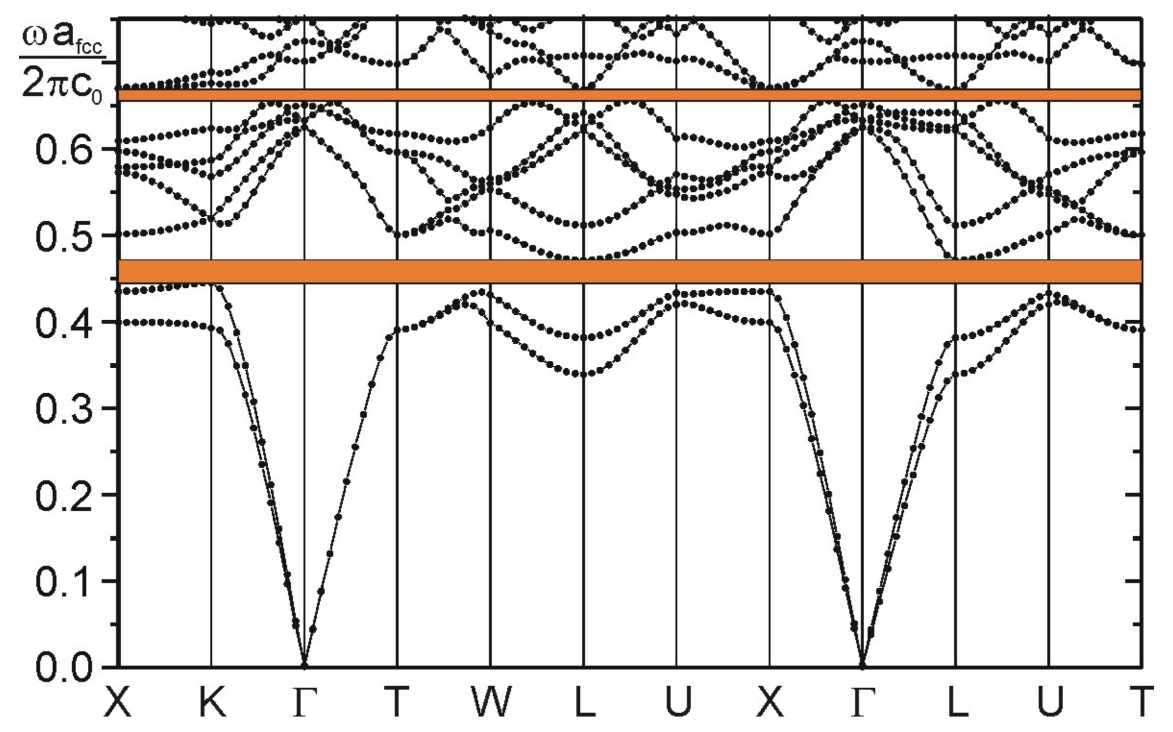
Daniel Meisel, Martin Wegener & Kurt Busch

Example I



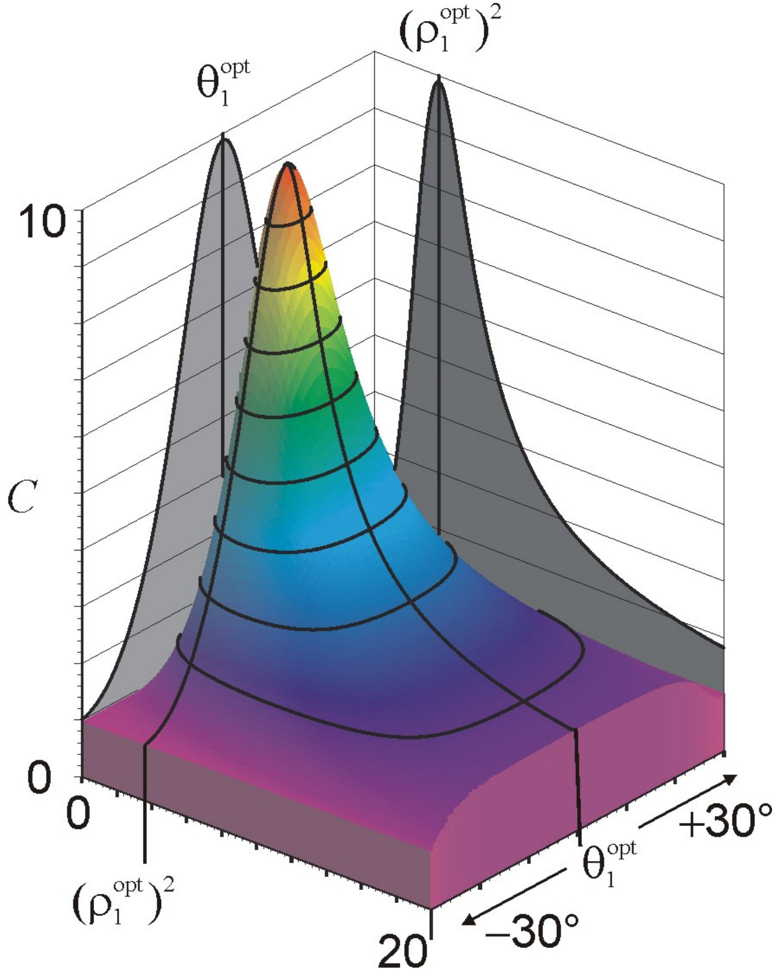
Daniel Meisel, Martin Wegener & Kurt Busch

Example I



Daniel Meisel, Martin Wegener & Kurt Busch

Example I

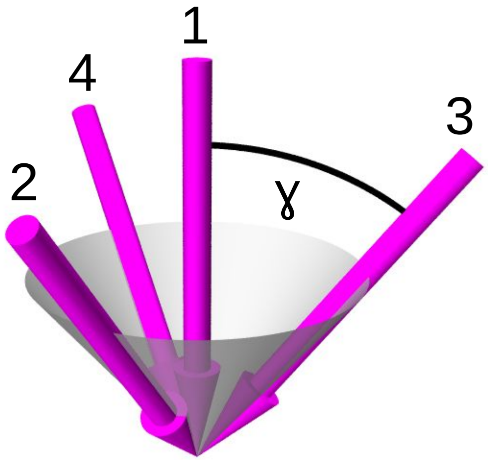


$I_1^{opt} = 1$

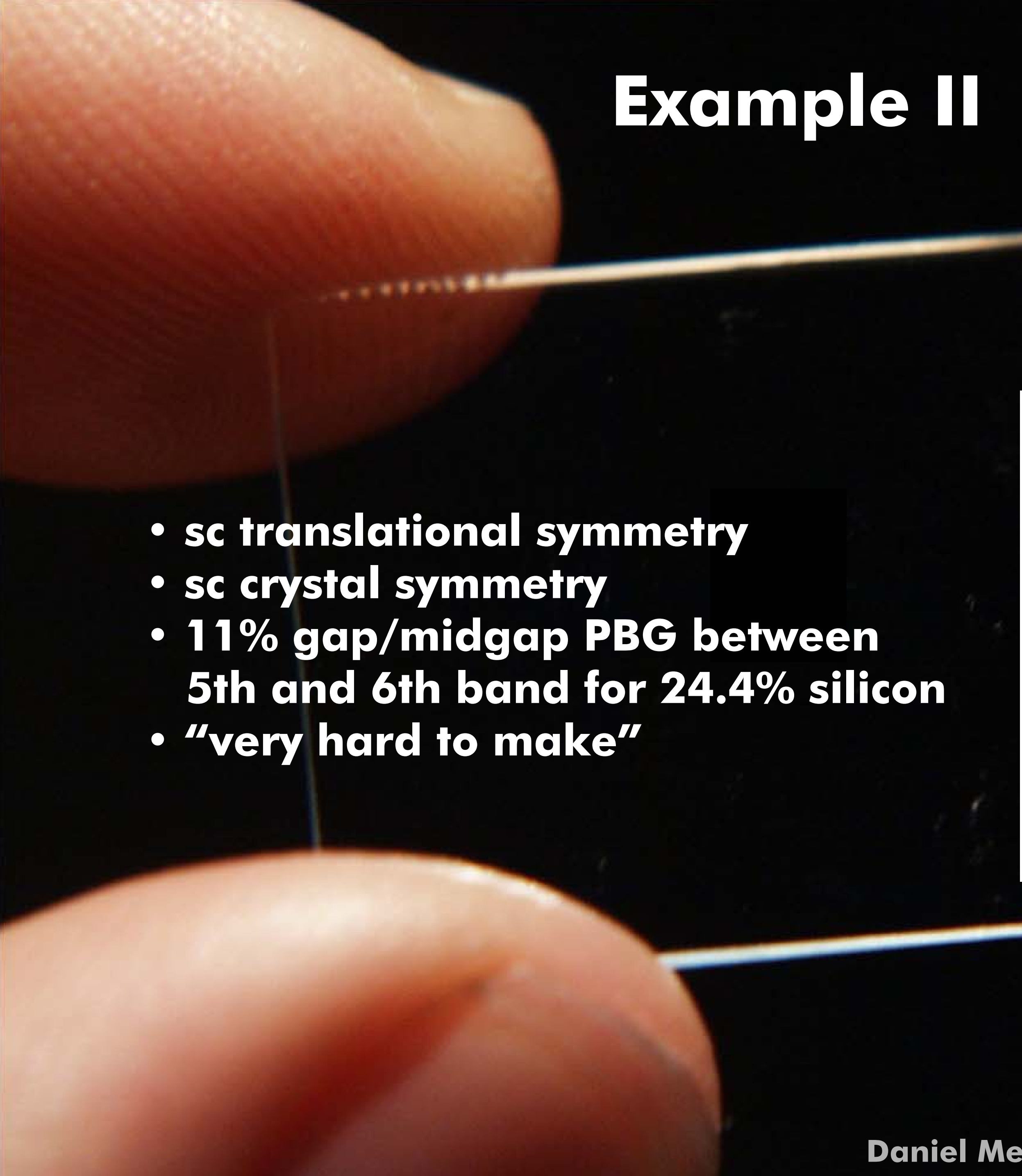
$I_2^{opt} = 0.11$

$I_3^{opt} = 0.44$

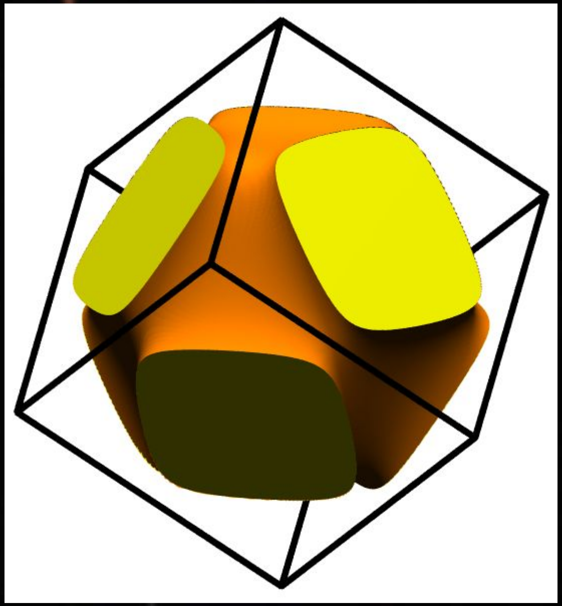
$I_4^{opt} = 0.44$



Example II



- sc translational symmetry
- sc crystal symmetry
- 11% gap/midgap PBG between 5th and 6th band for 24.4% silicon
- “very hard to make”



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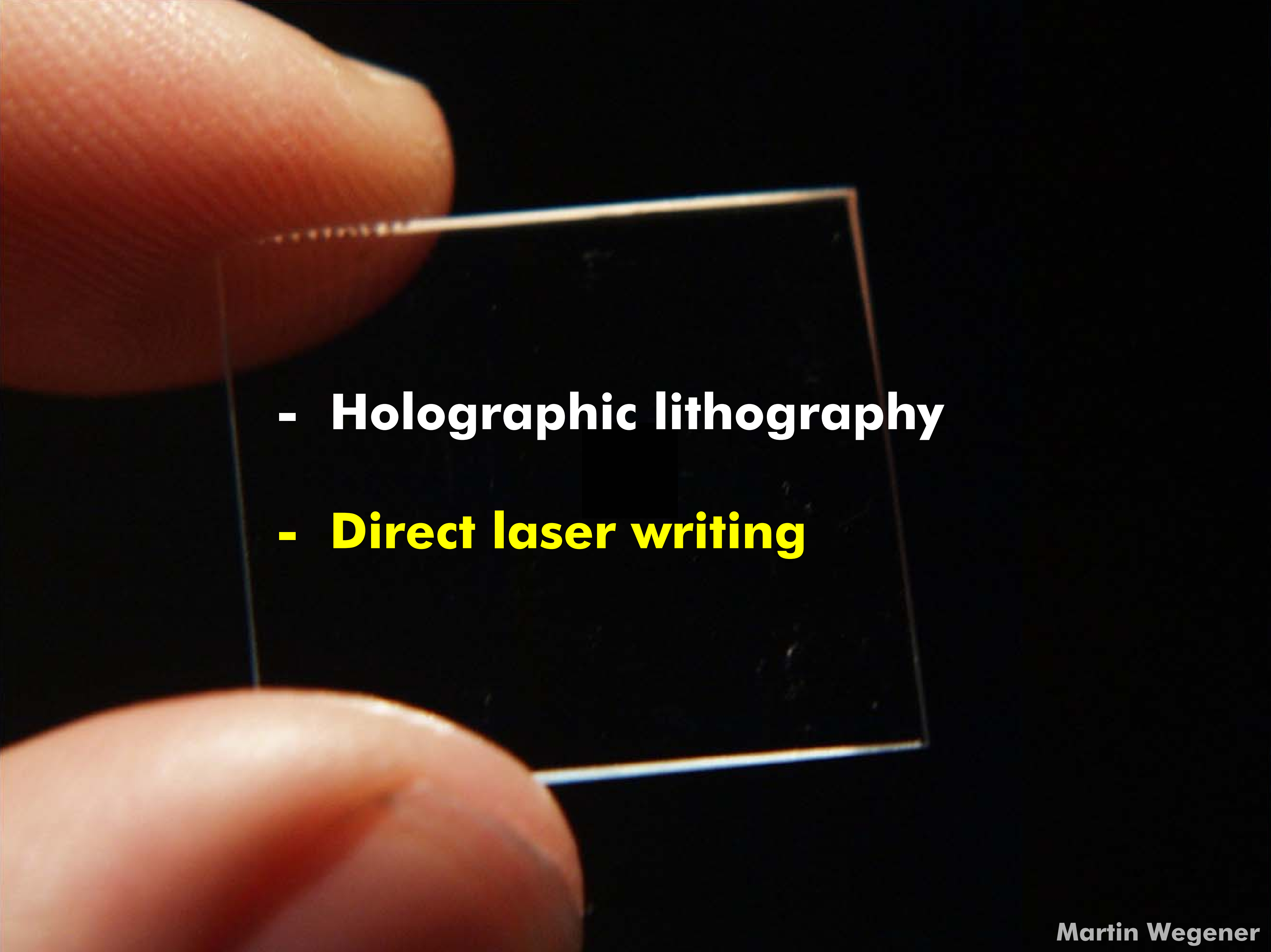
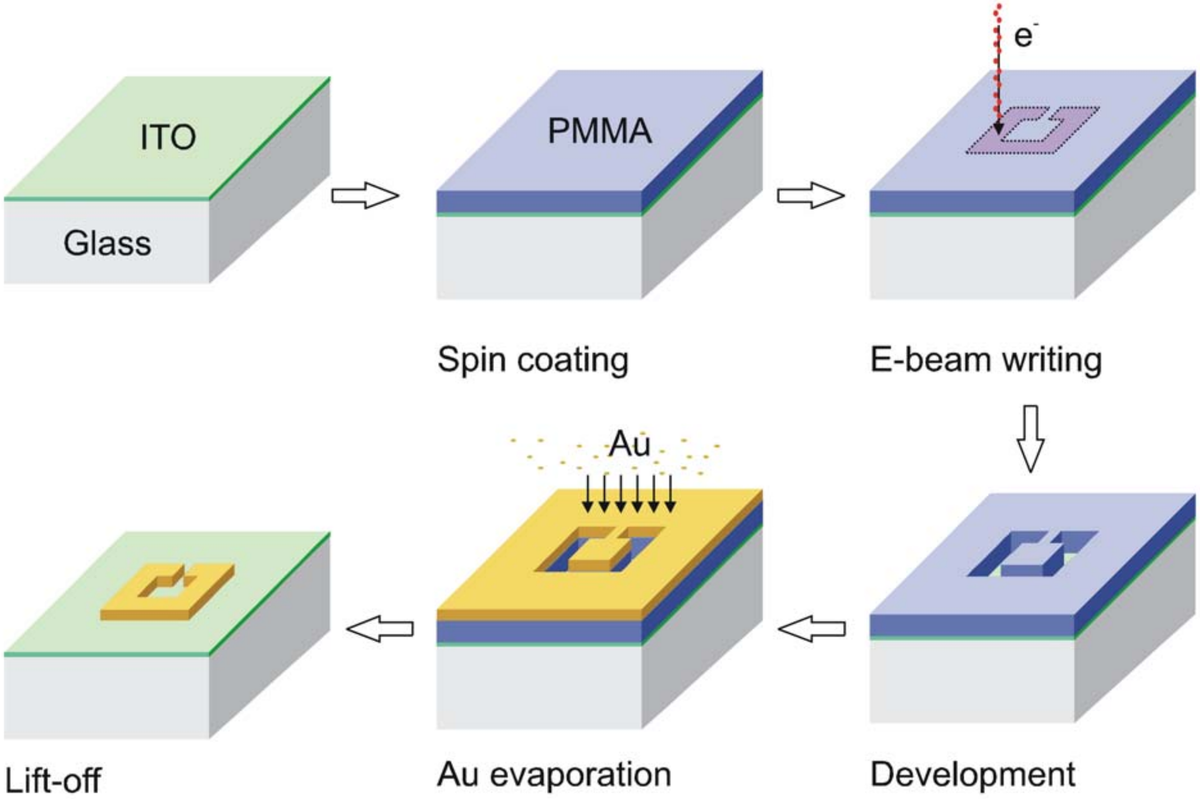


- Holographic lithography
- Direct laser writing

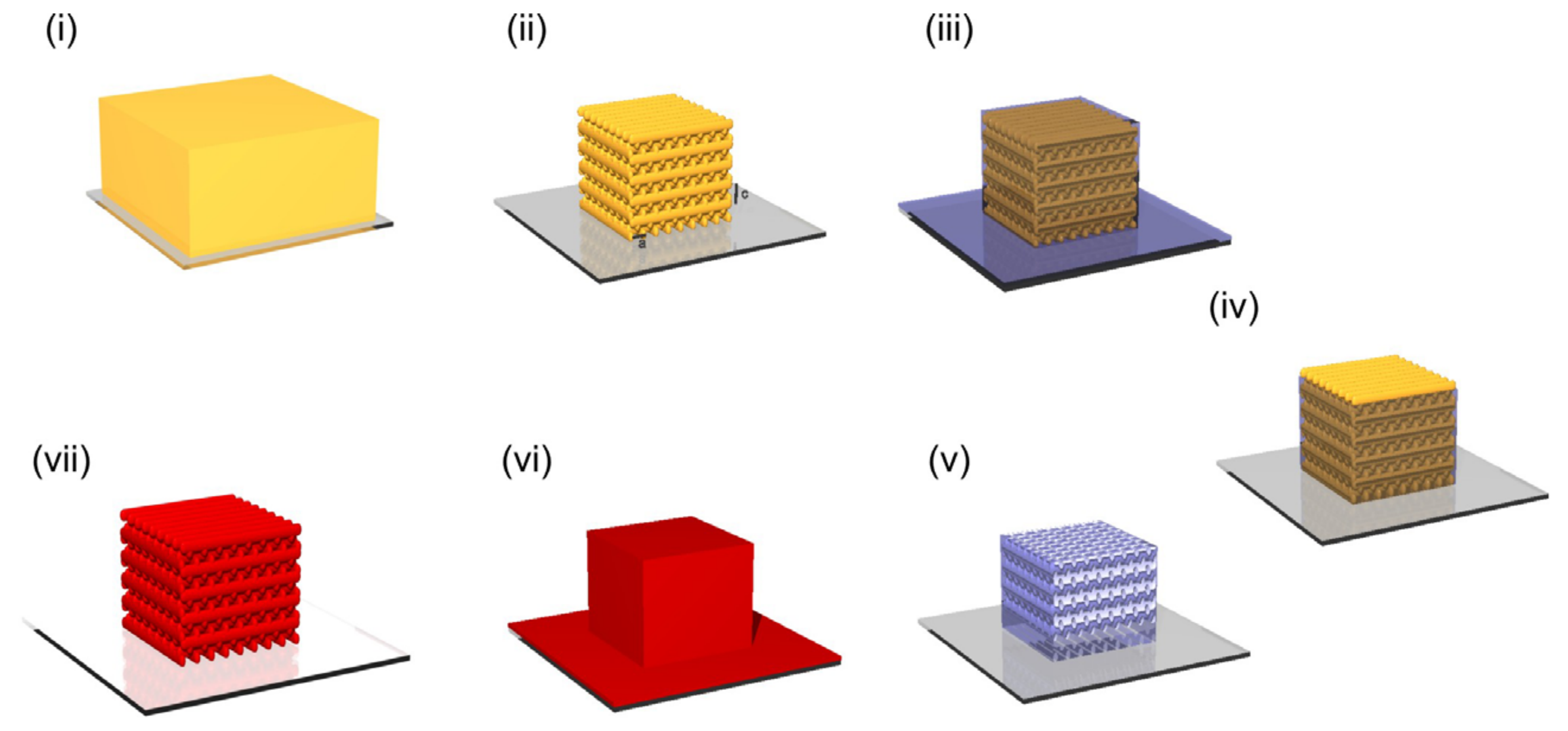
Martin Wegener

2D Electron-beam lithography



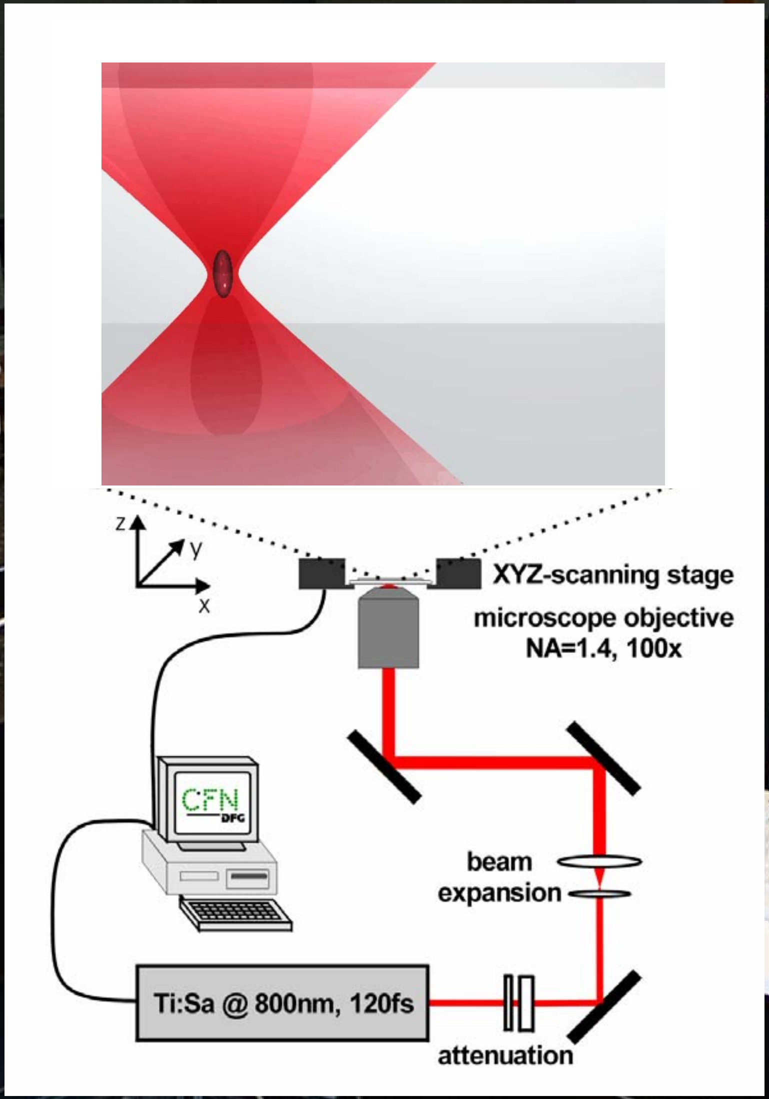
used in, e.g., Science 306, 1351 (2004)

polymer – SiO_2 – Si



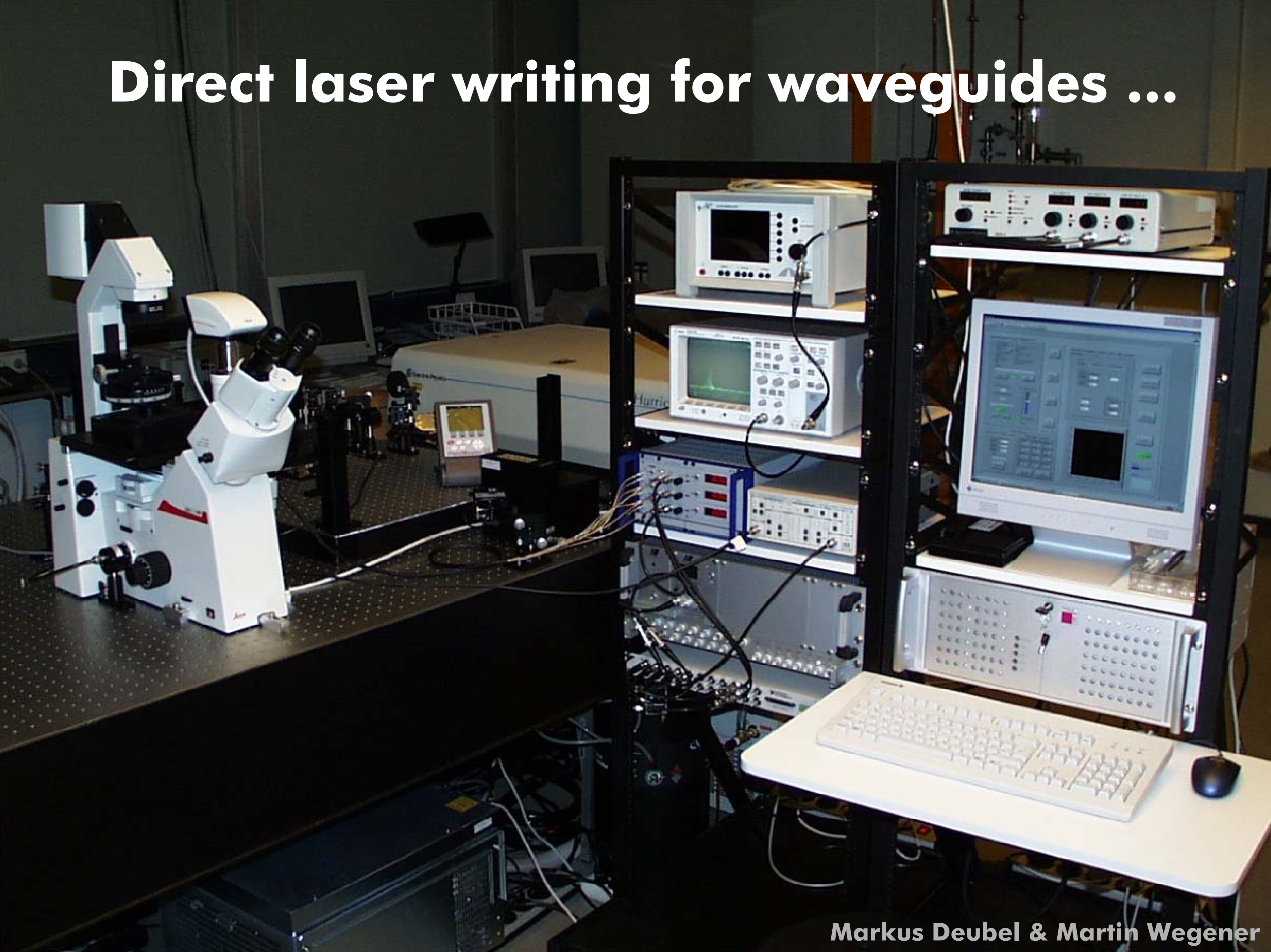
N. Tétreault et al., Adv. Mater. 18, 457 (2006)

Direct laser writing for waveguides …



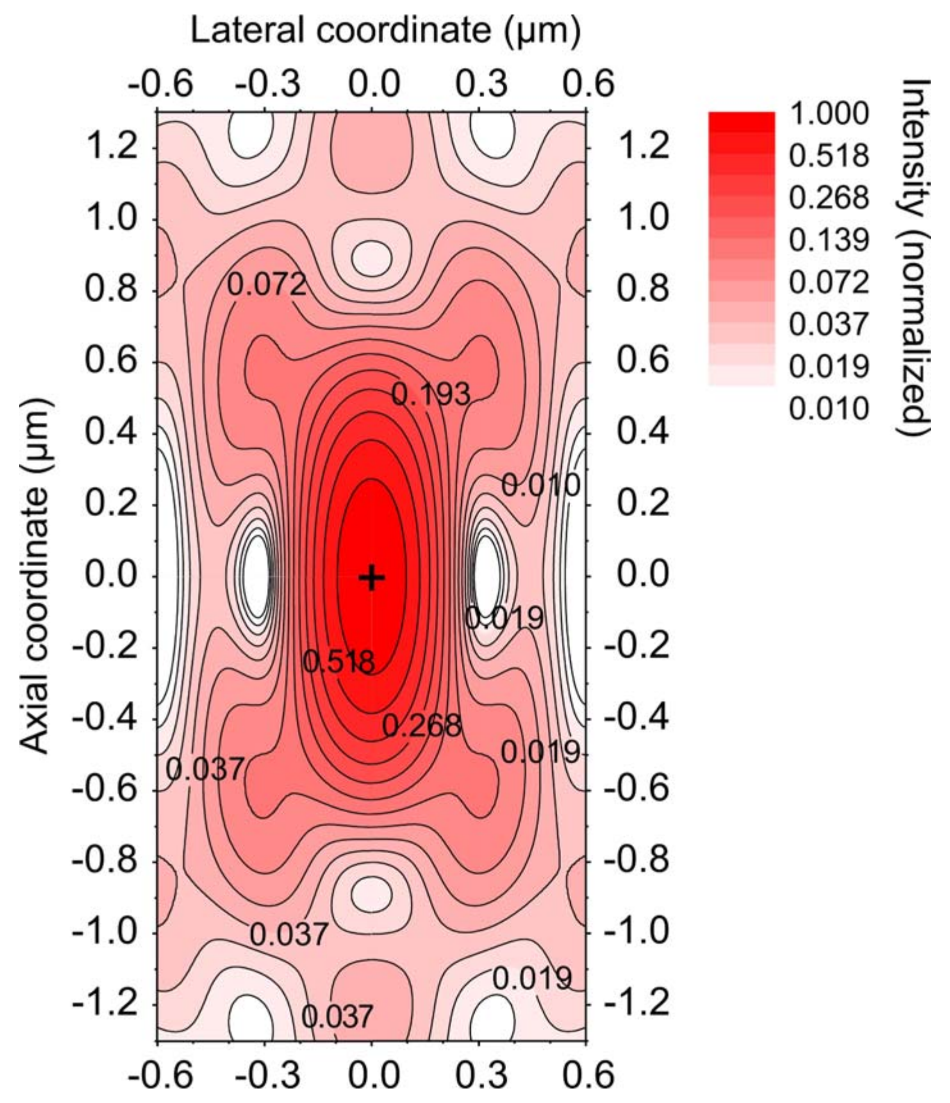
For DLW-technique see: S. Kawata et al., Nature 412, 697 (2001)

Direct laser writing for waveguides ...



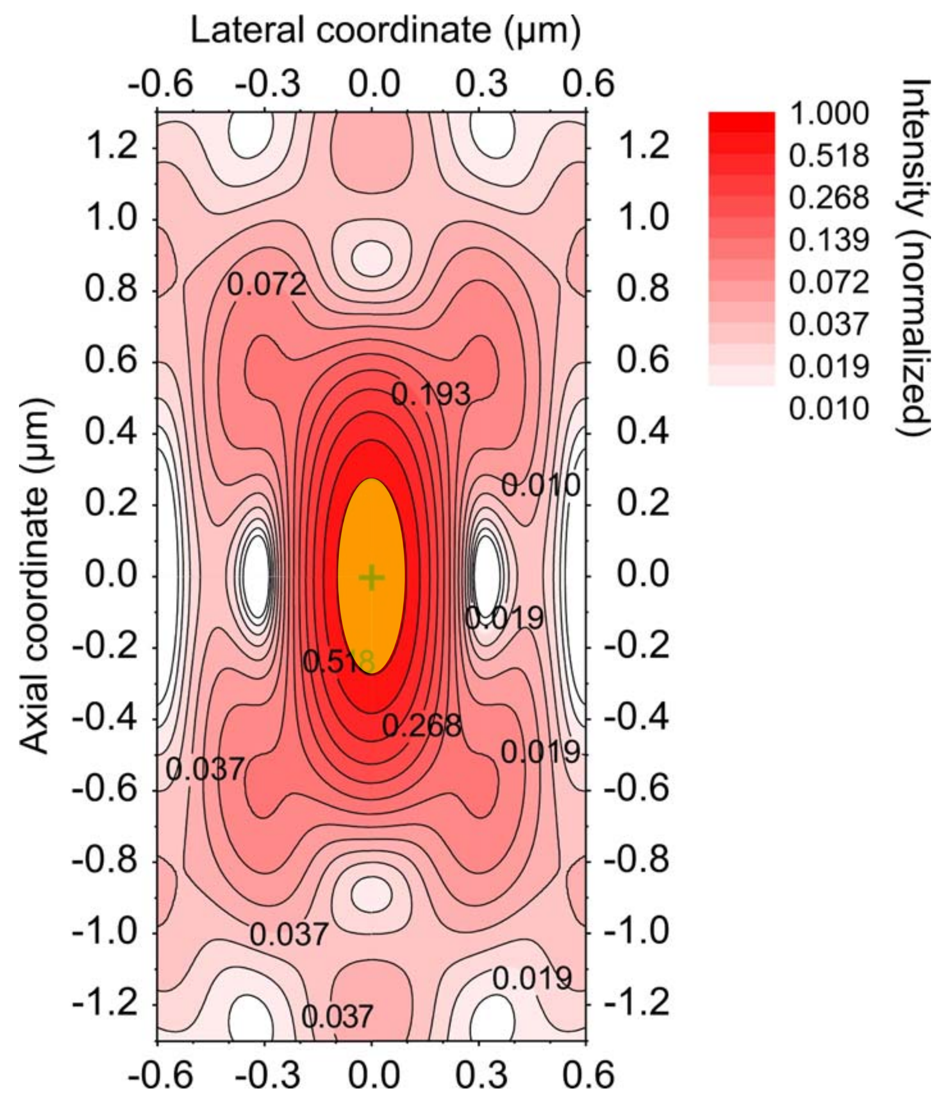
Markus Deubel & Martin Wegener

The building block



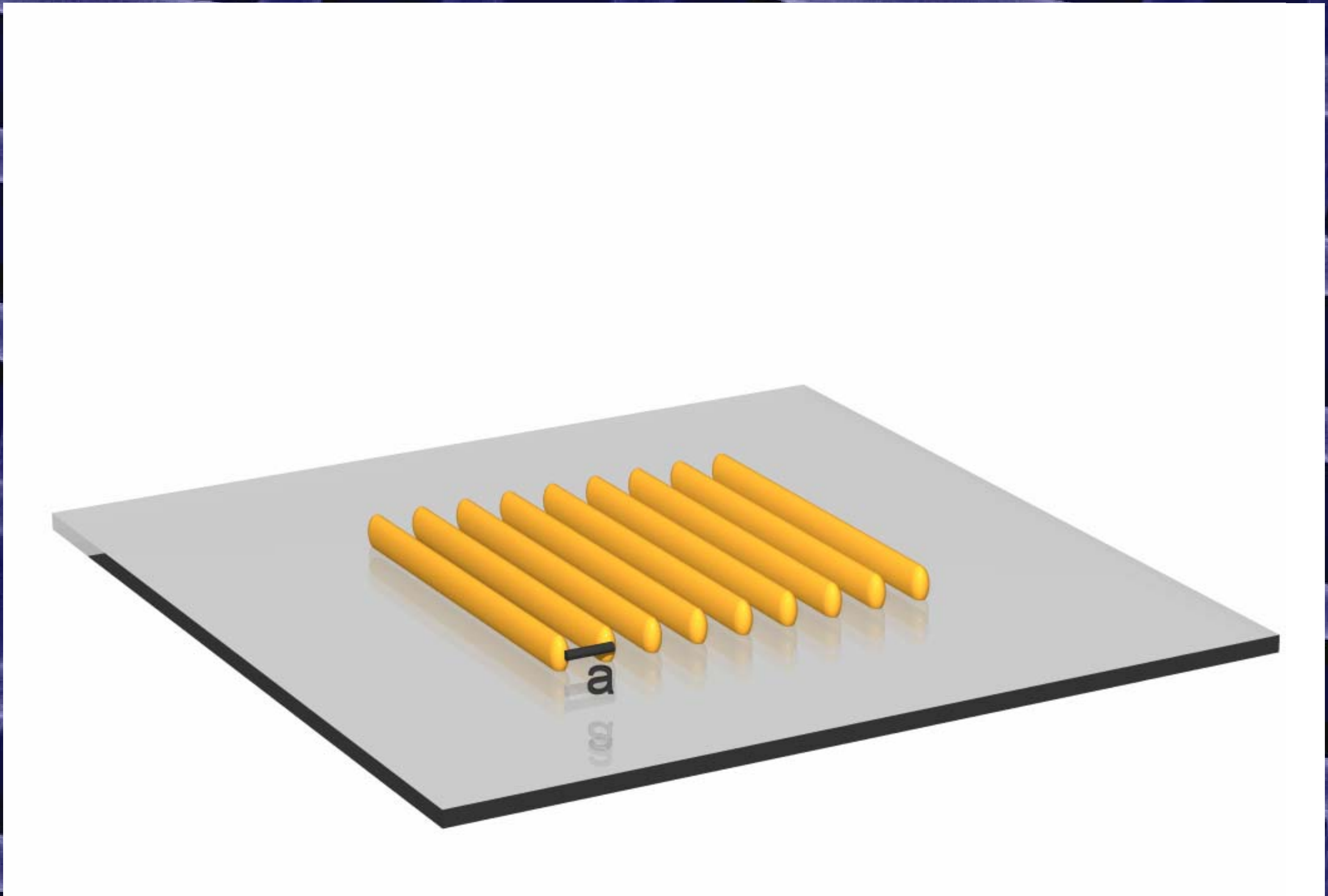
Markus Deubel & Martin Wegener

The building block



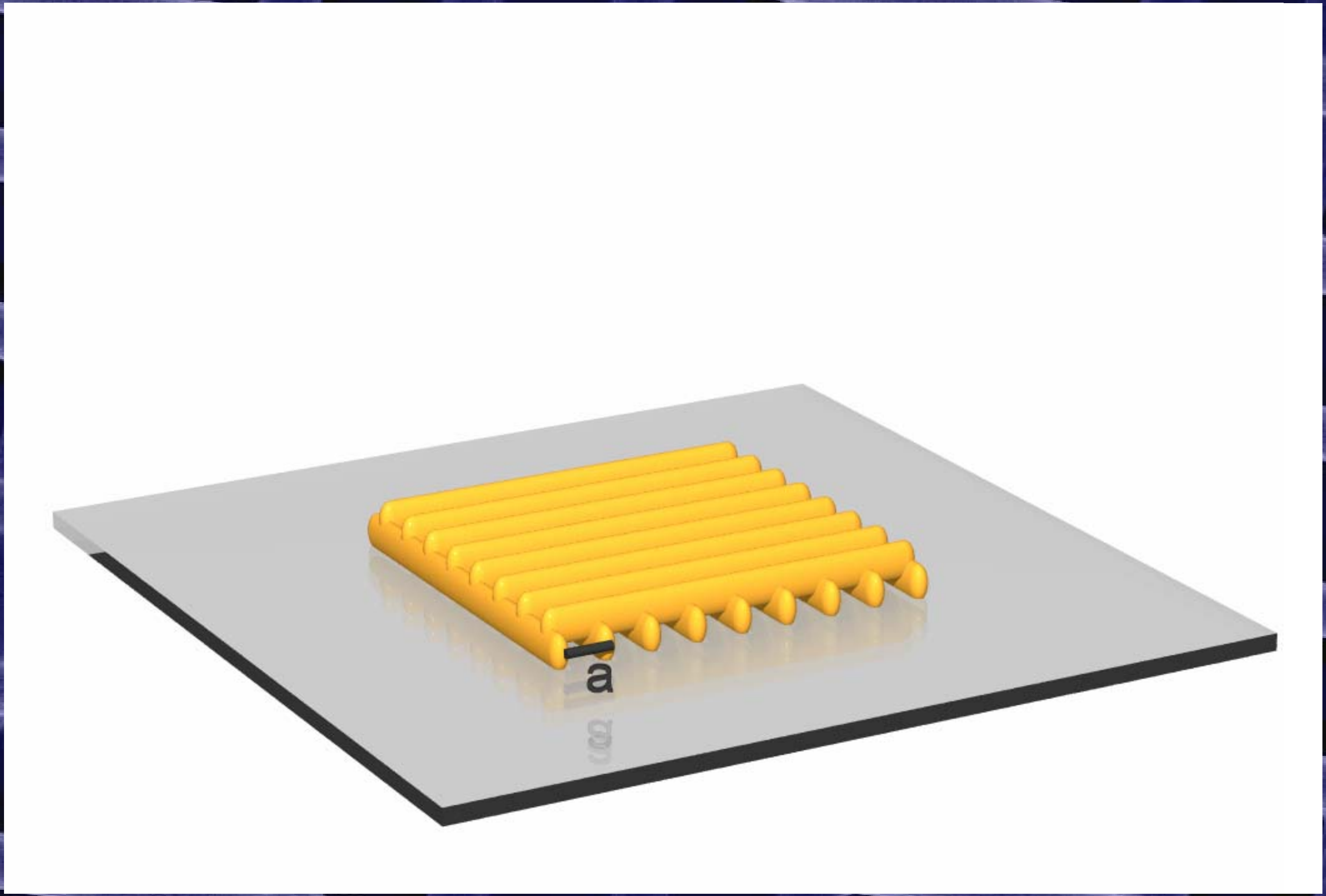
Markus Deubel & Martin Wegener

Woodpile structures



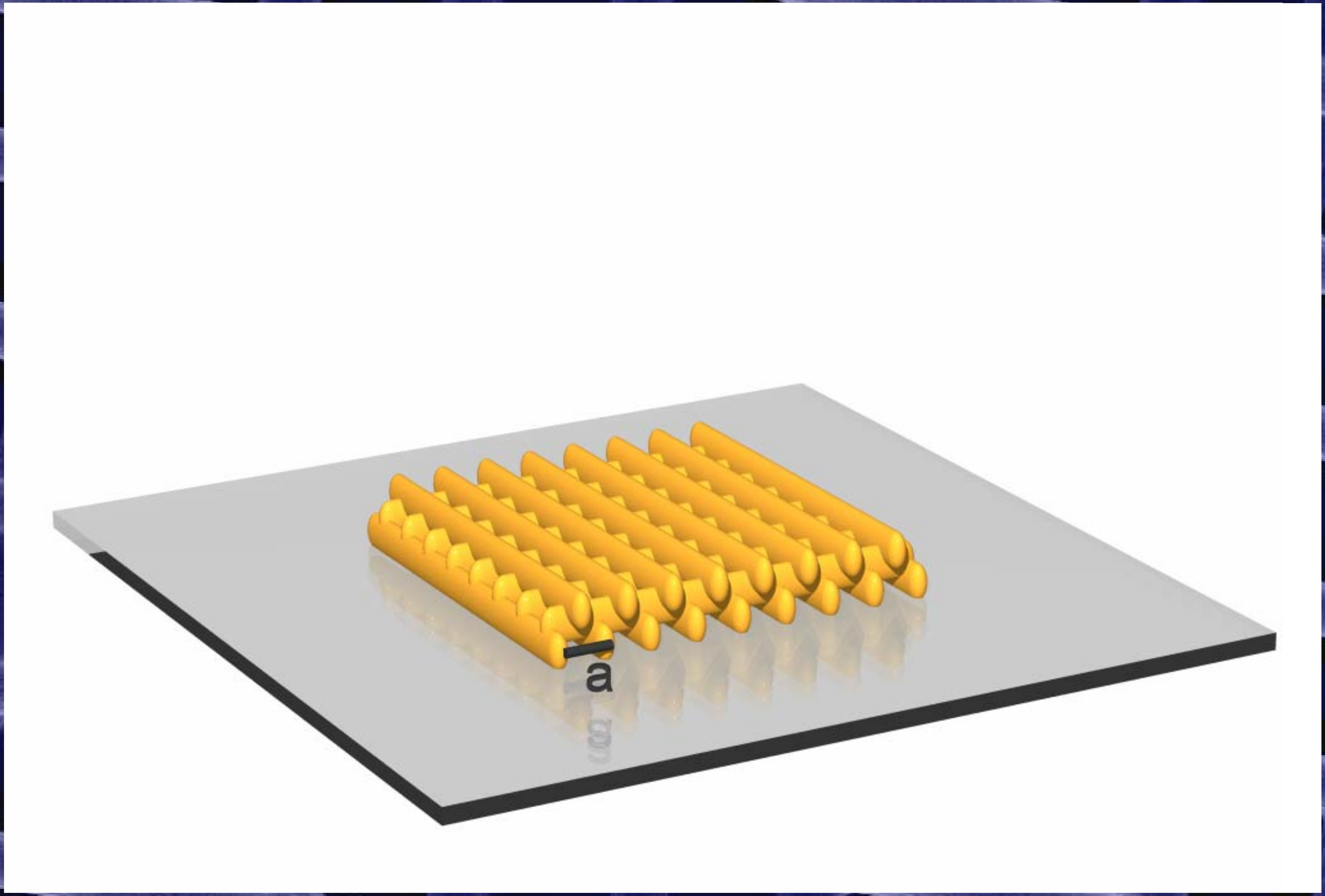
Proposal: C.M. Soukoulis et al., Solid State Commun. 89, 413 (1994)

Woodpile structures



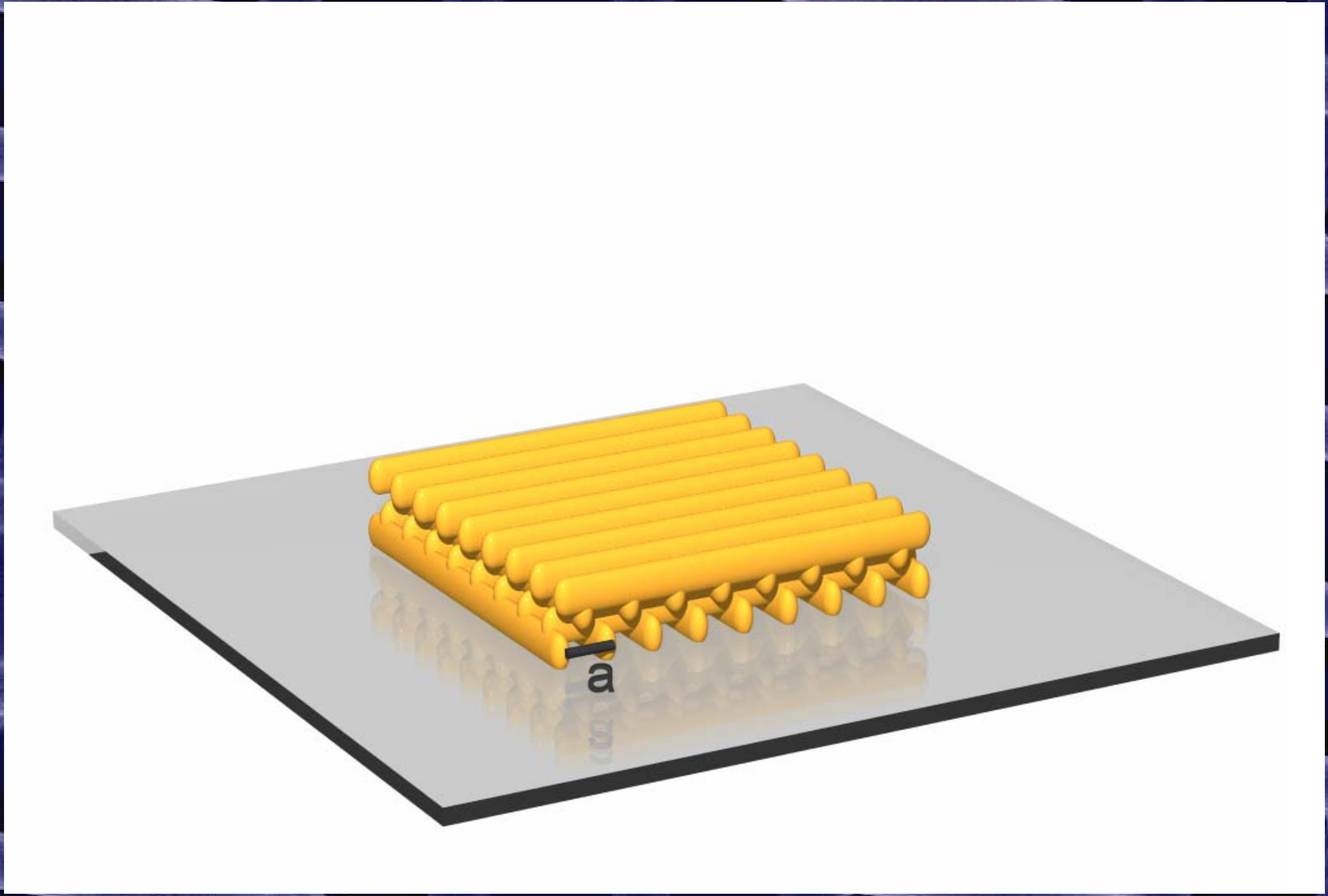
Proposal: C.M. Soukoulis et al., Solid State Commun. 89, 413 (1994)

Woodpile structures



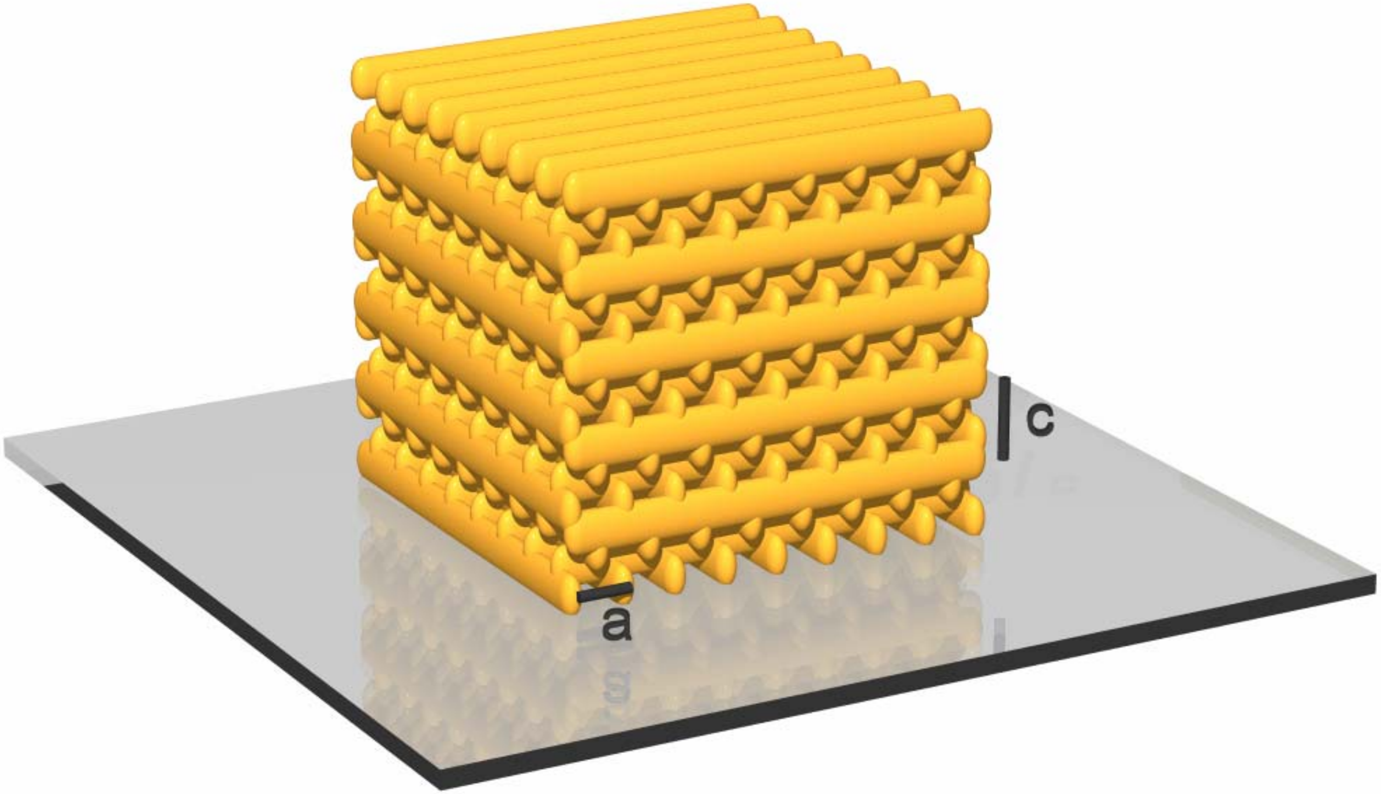
Proposal: C.M. Soukoulis et al., Solid State Commun. 89, 413 (1994)

Woodpile structures



Proposal: C.M. Soukoulis et al., Solid State Commun. 89, 413 (1994)

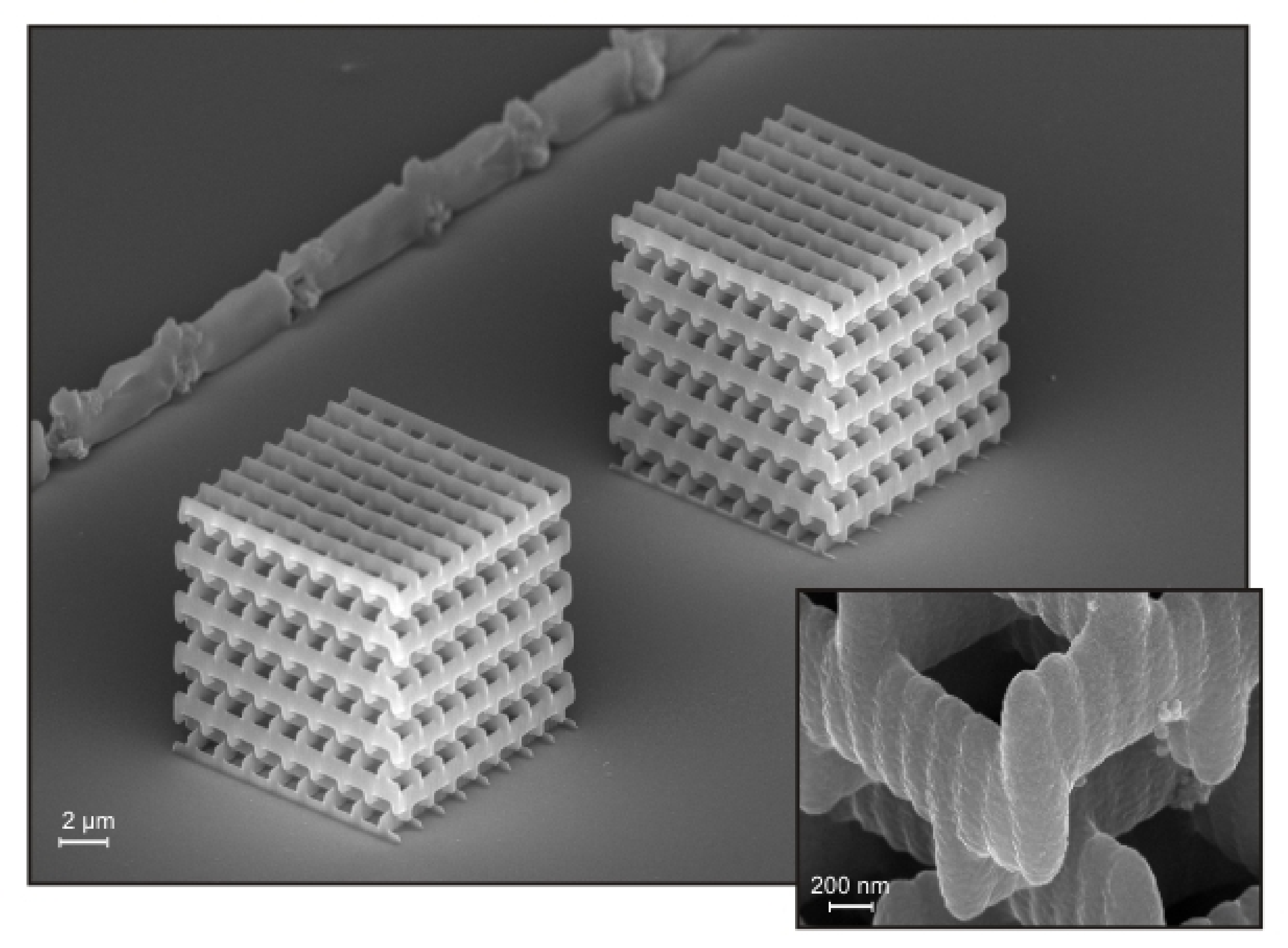
Woodpile structures



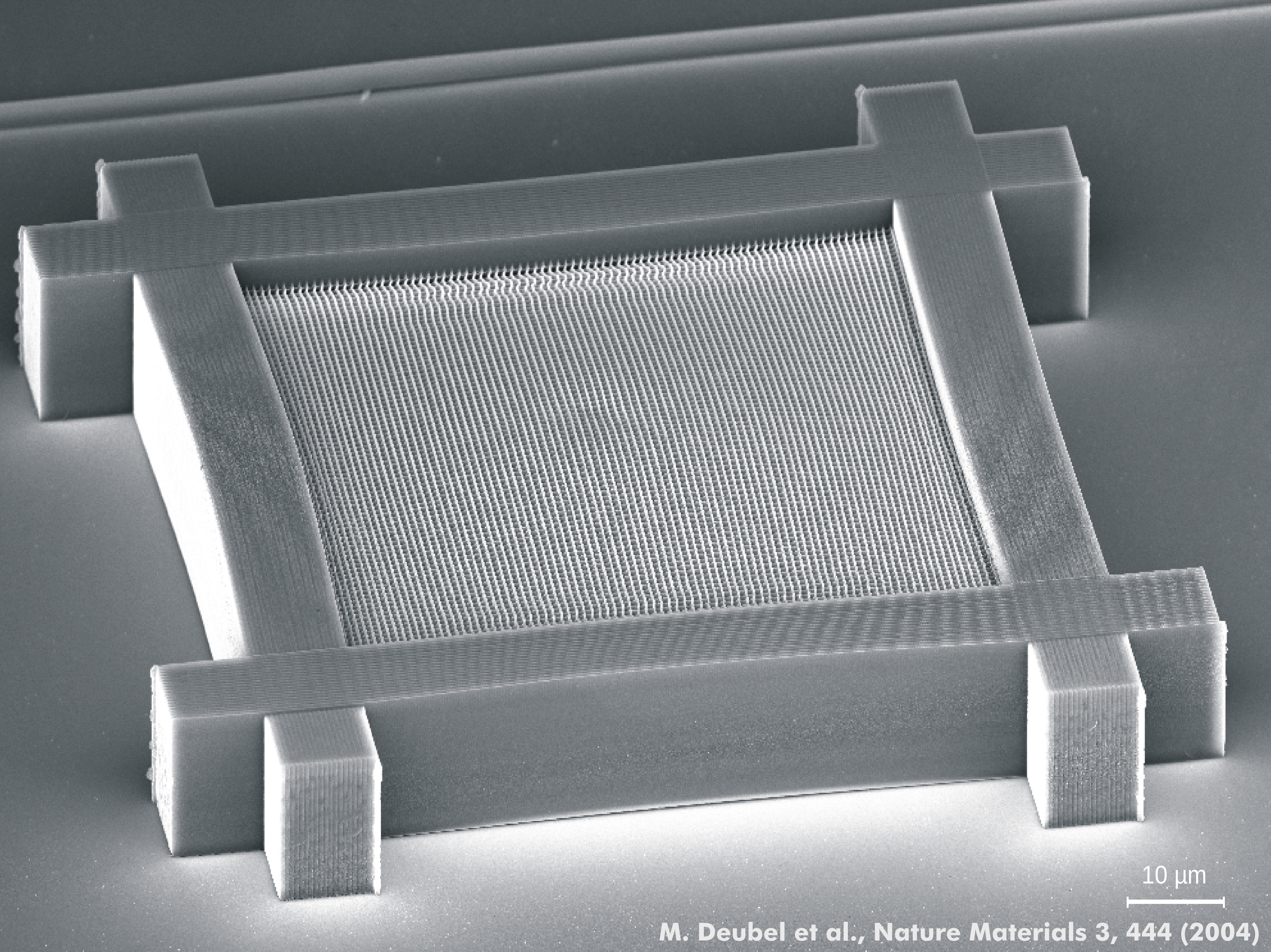
fcc for $(c/a)^2=2$, full gap for index contrast > 1.9, 25% gap for holes in Si

Proposal: C.M. Soukoulis et al., Solid State Commun. 89, 413 (1994)

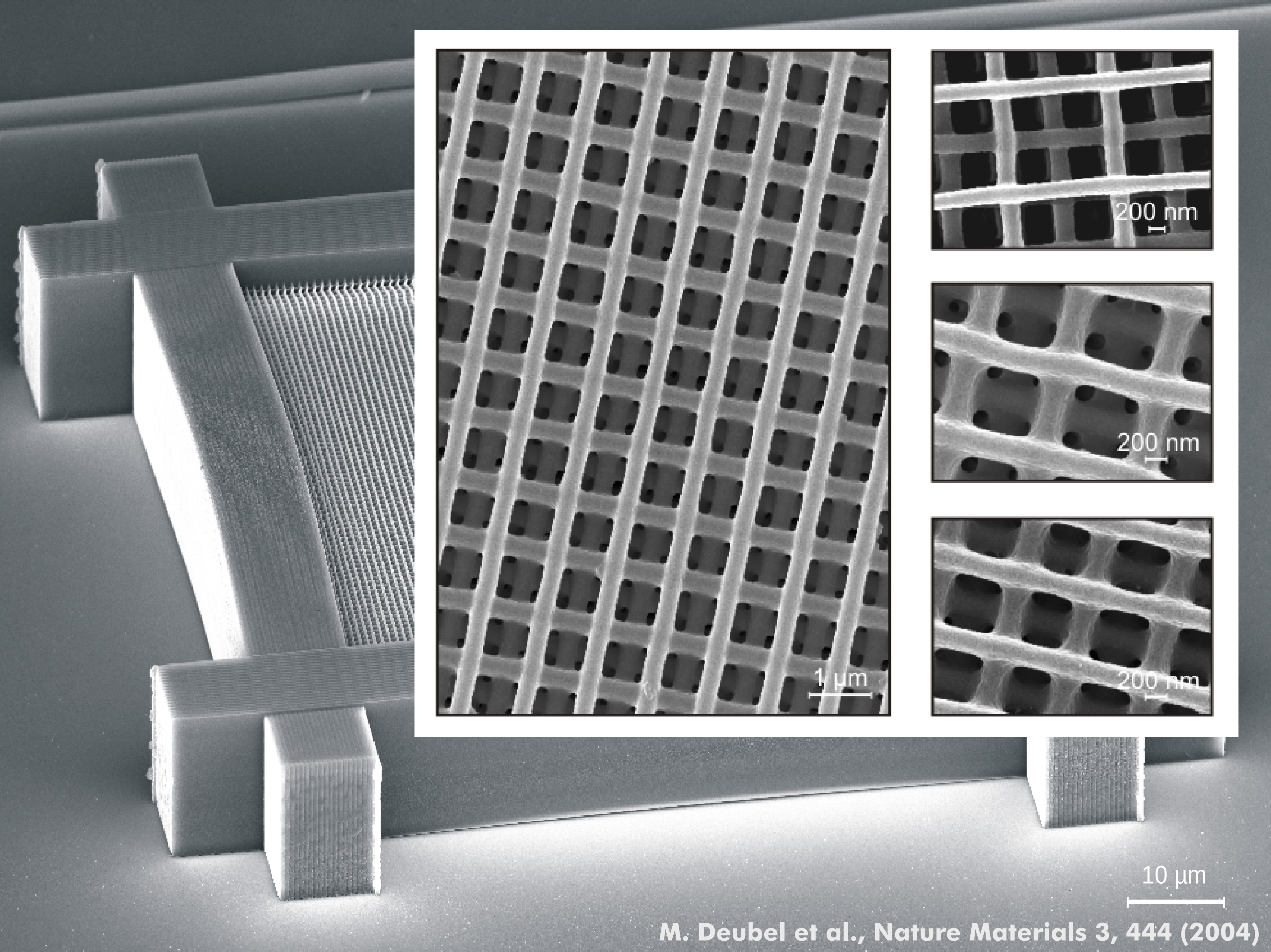
Experimental results …



Markus Deubel & Martin Wegener

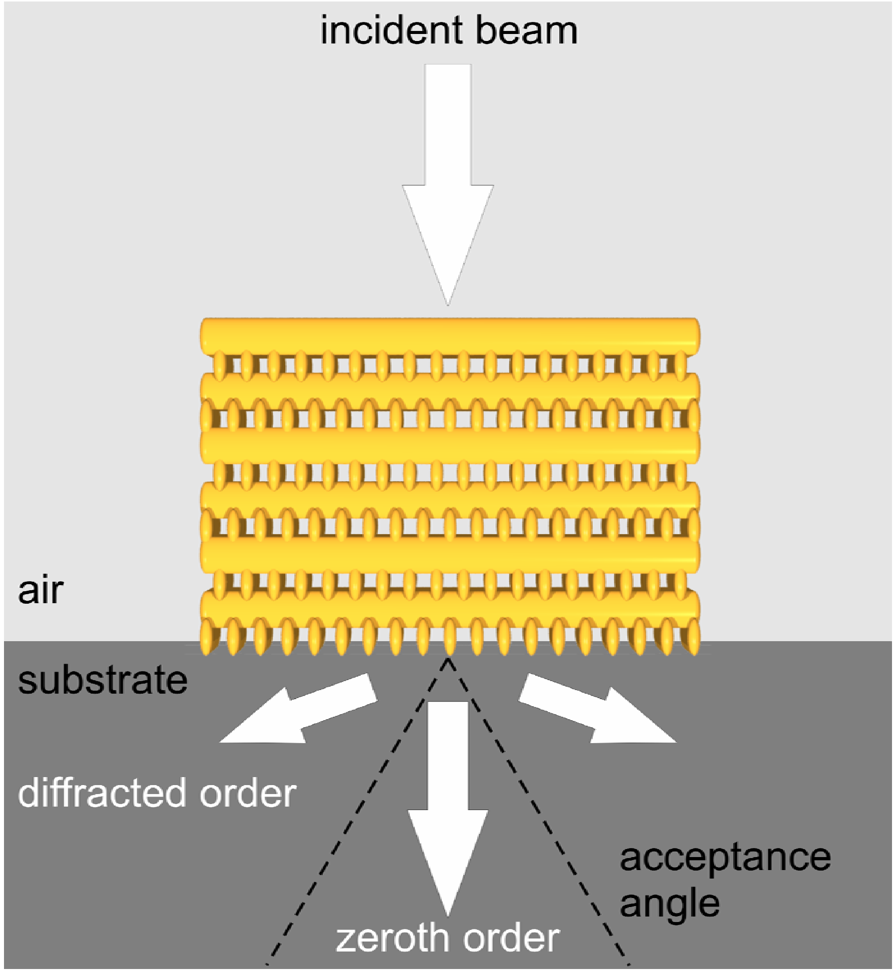


M. Deubel et al., Nature Materials 3, 444 (2004)

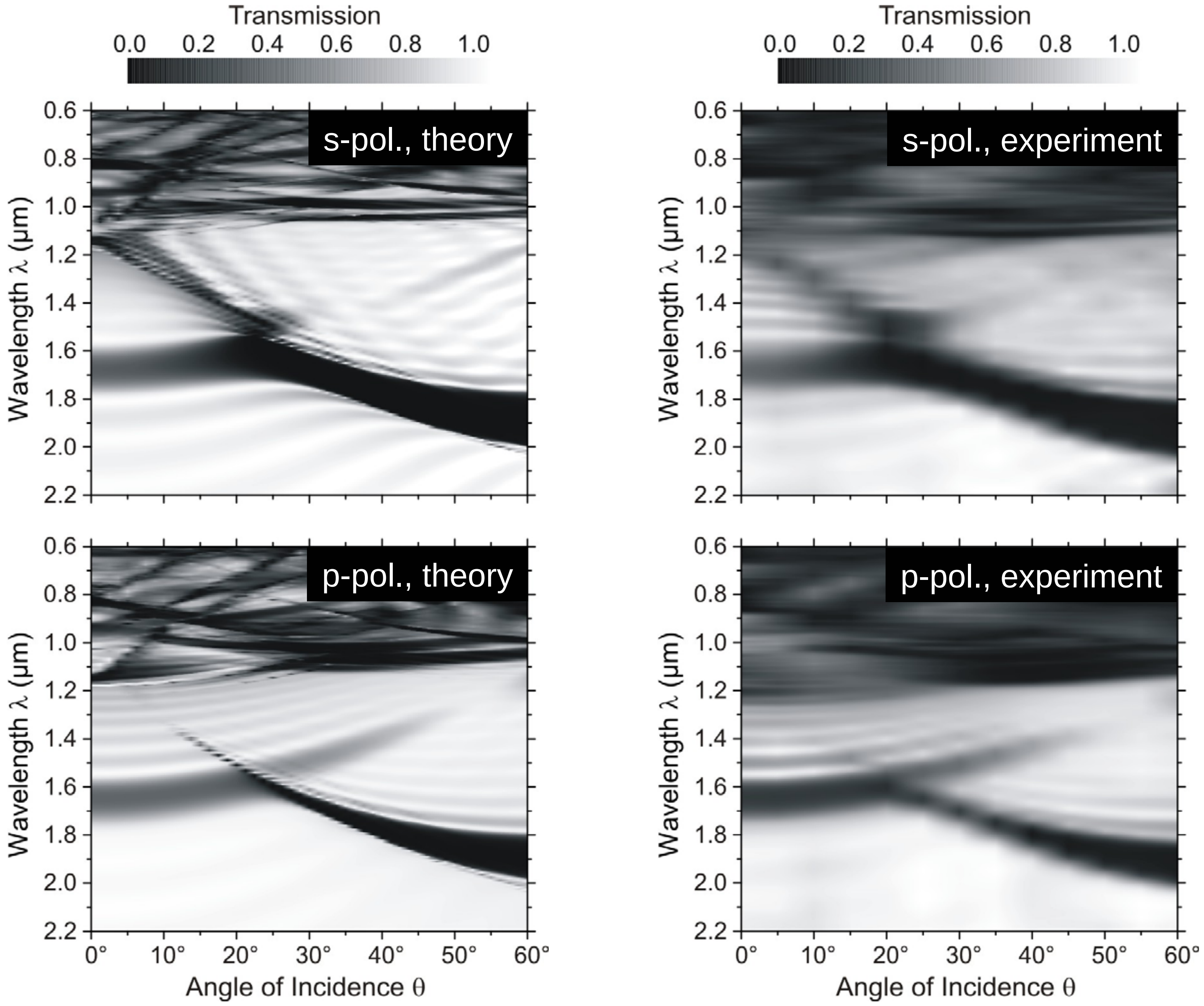


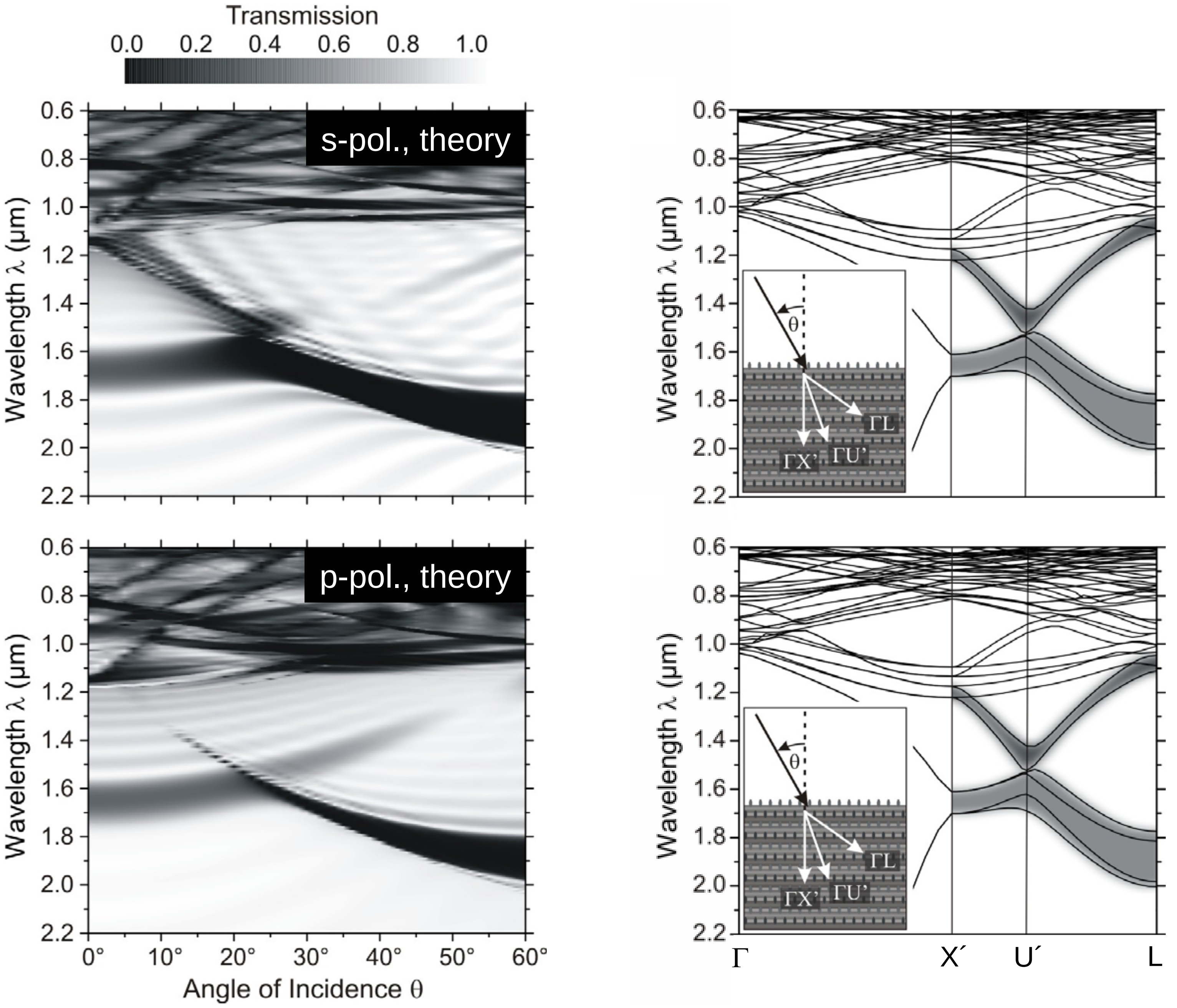
M. Deubel et al., Nature Materials 3, 444 (2004)

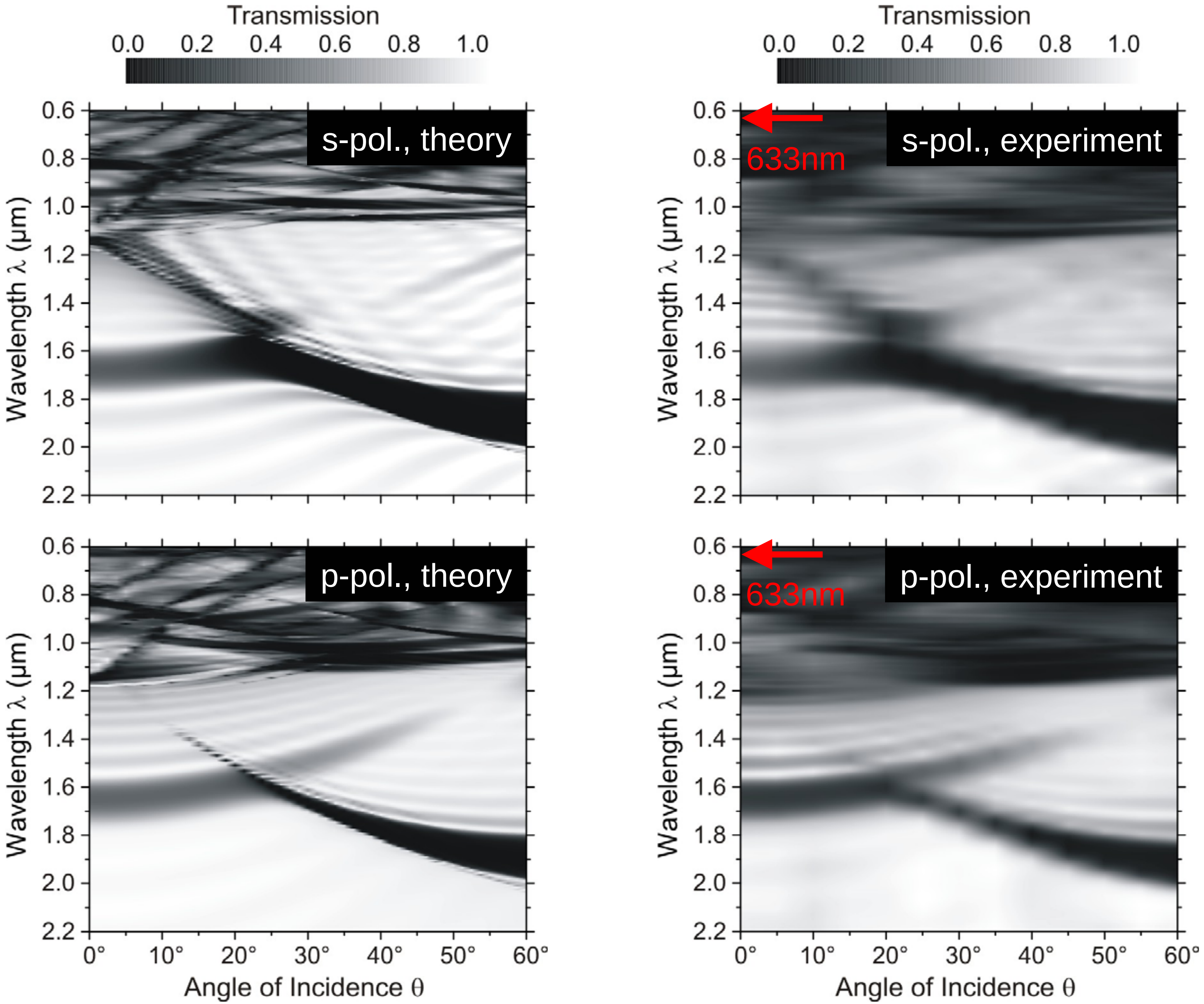
Analysis of transmission spectra



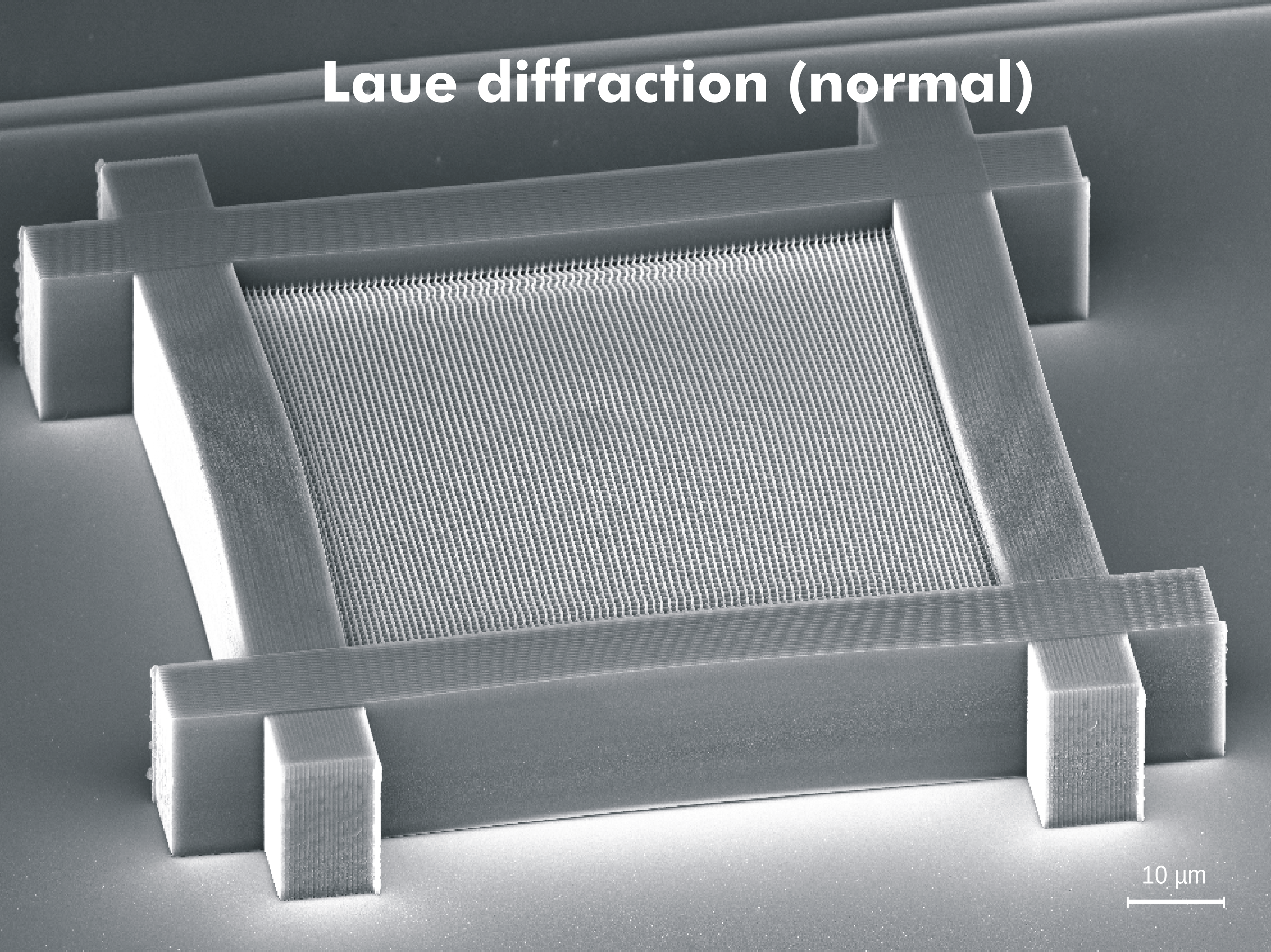
M. Deubel et al., Appl. Phys. Lett. 87, 221104 (2005)


Transmission
0.0
0.2
0.4
0.6
0.8
1.0
s-pol., theory
s-pol., experiment
p-pol., theory
p-pol., experiment
Wavelength λ (µm)
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
0°
10°
20°
30°
40°
50°
60°
Angle of Incidence θ

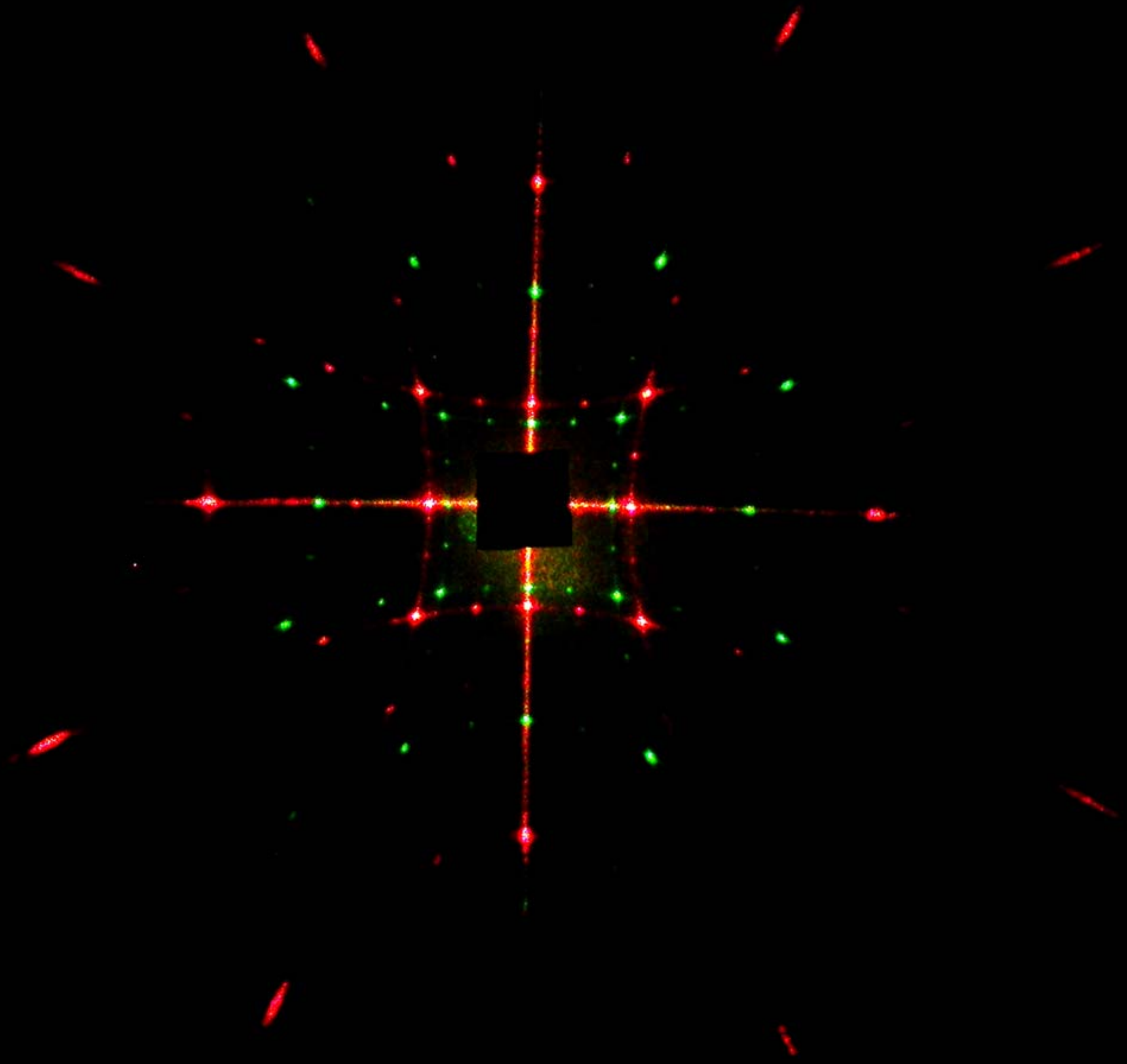

Transmission
0.0
0.2
0.4
0.6
0.8
1.0
s-pol., theory
p-pol., theory
Wavelength λ (µm)
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
0°
10°
20°
30°
40°
50°
60°
Angle of Incidence θ
θ
ΓL
ΓU'
ΓX'
Γ
X´
U´
L

Transmission
0.0
0.2
0.4
0.6
0.8
1.0
s-pol., theory
s-pol., experiment
p-pol., theory
p-pol., experiment
633nm
633nm
Wavelength λ (µm)
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
0°
10°
20°
30°
40°
50°
60°
Angle of Incidence θ

Laue diffraction (normal)

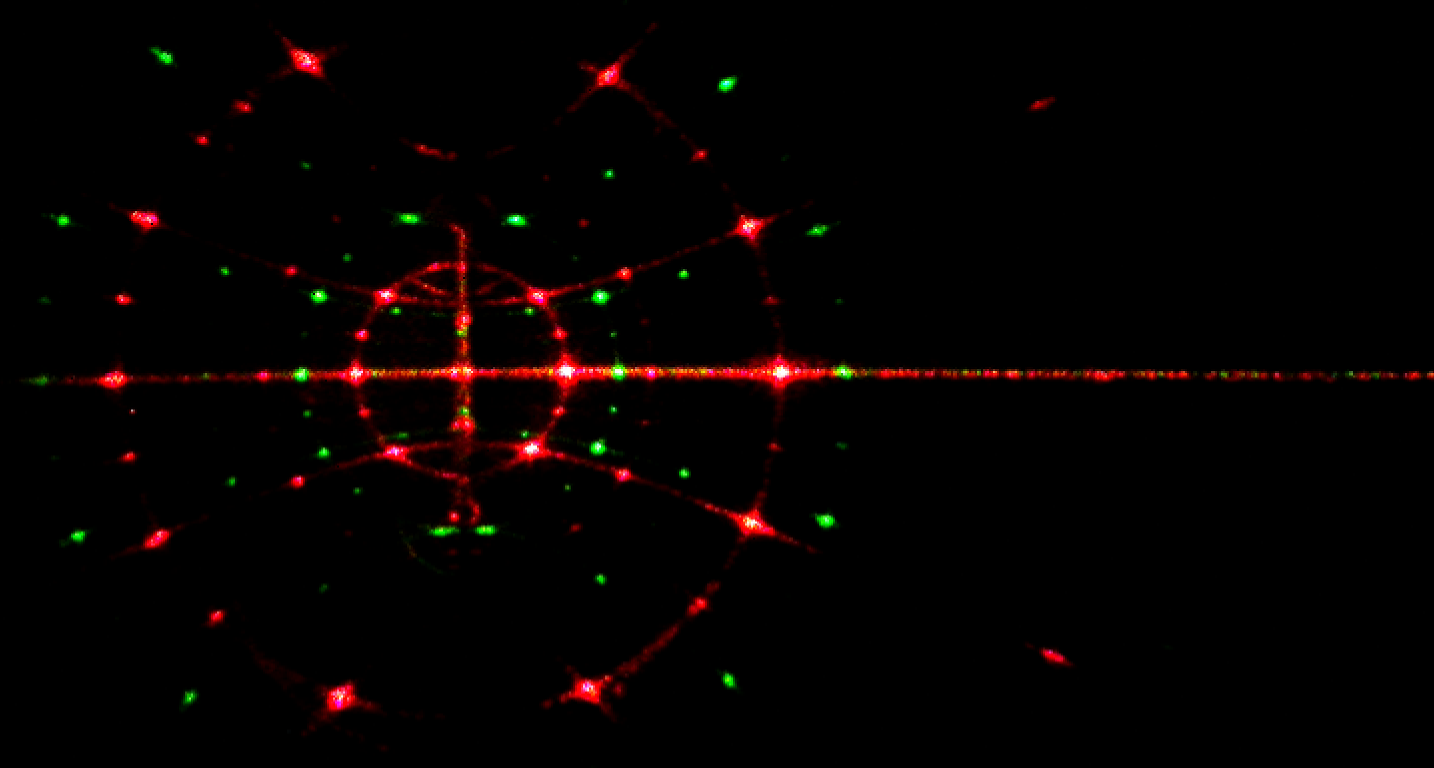


Laue diffraction (normal)

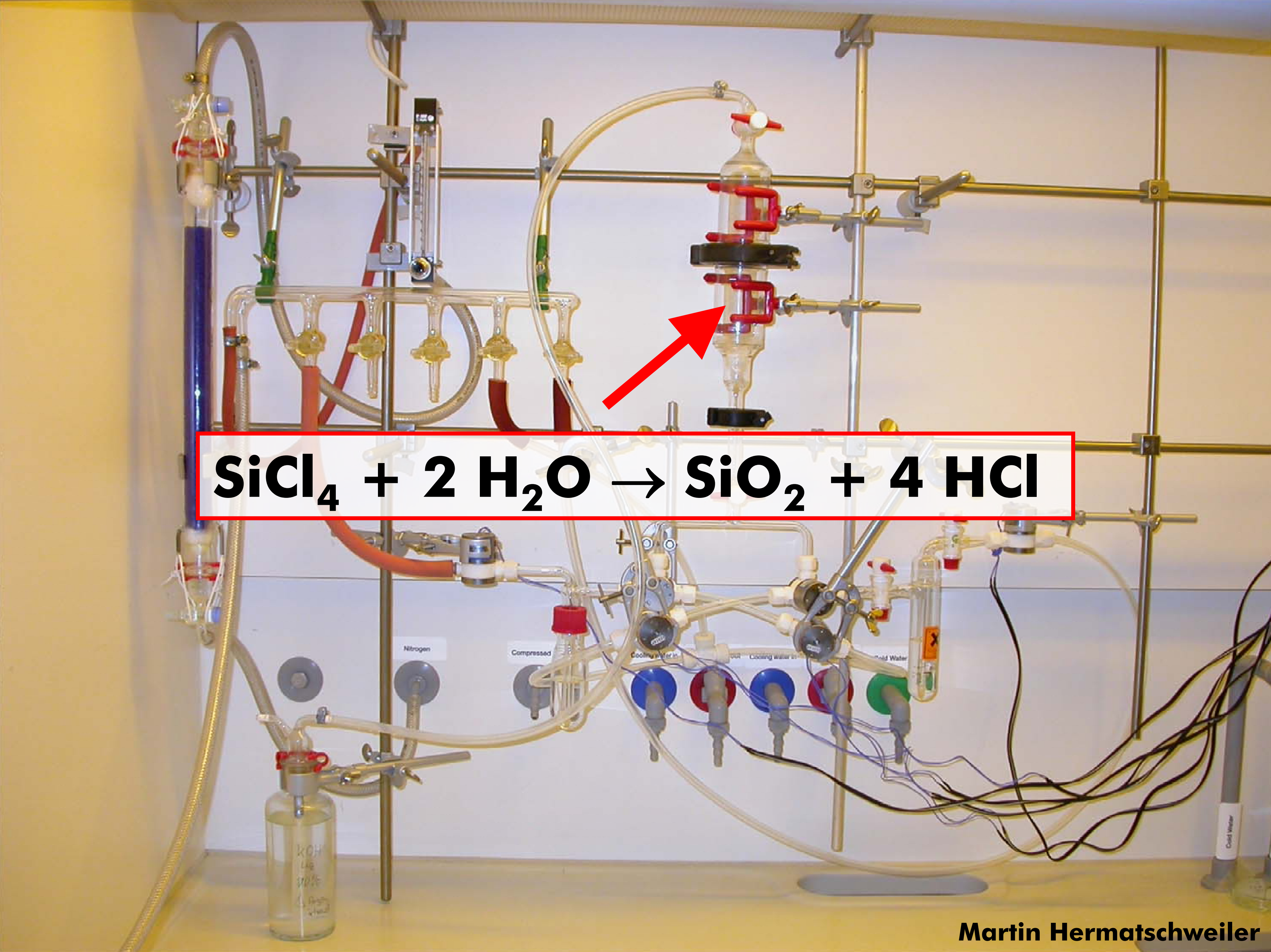


532nm and 633nm wavelength

Laue diffraction (side)

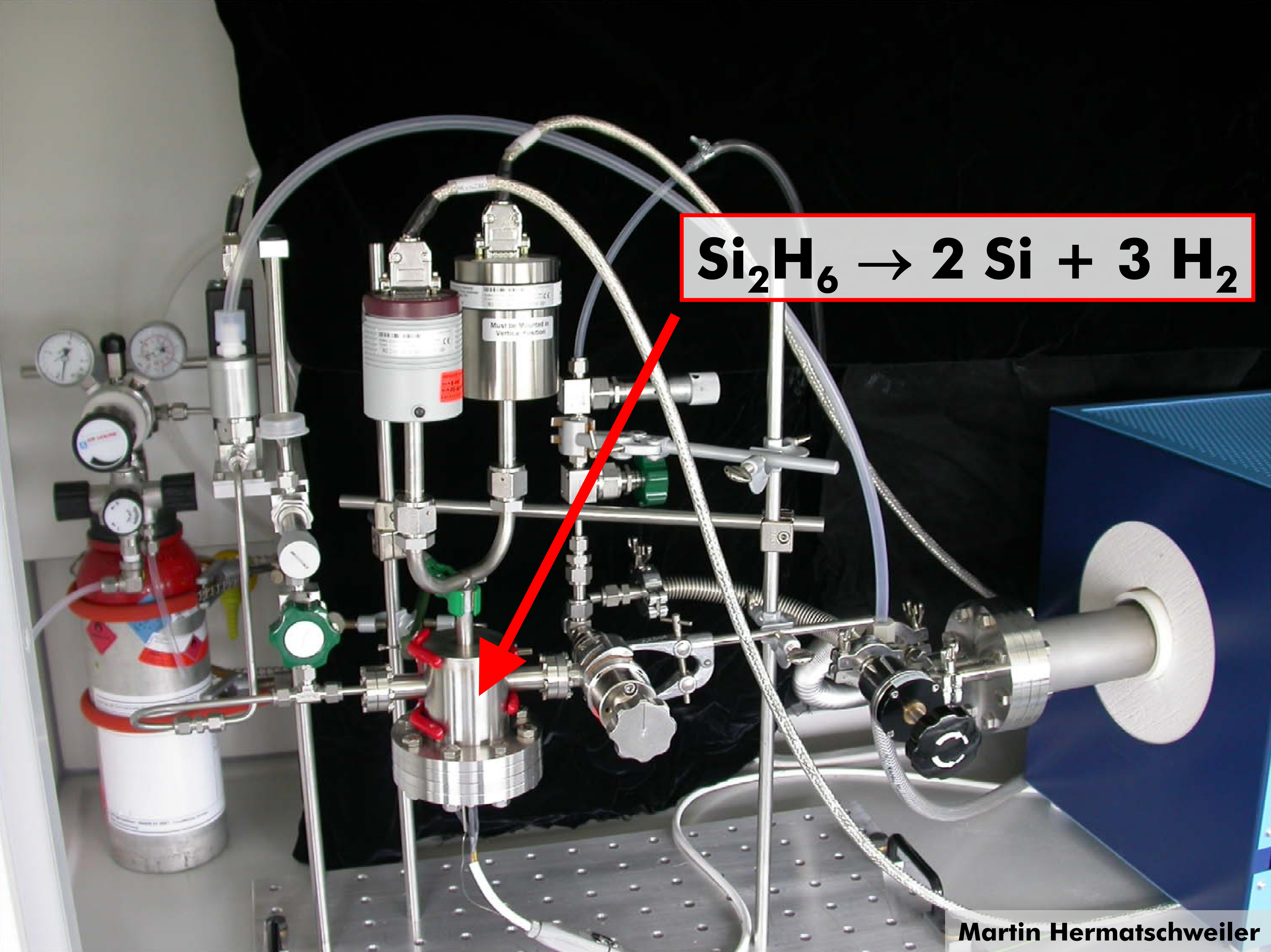


532nm and 633nm wavelength



$SiCl_4 + 2\ H_2O \rightarrow SiO_2 + 4\ HCl$

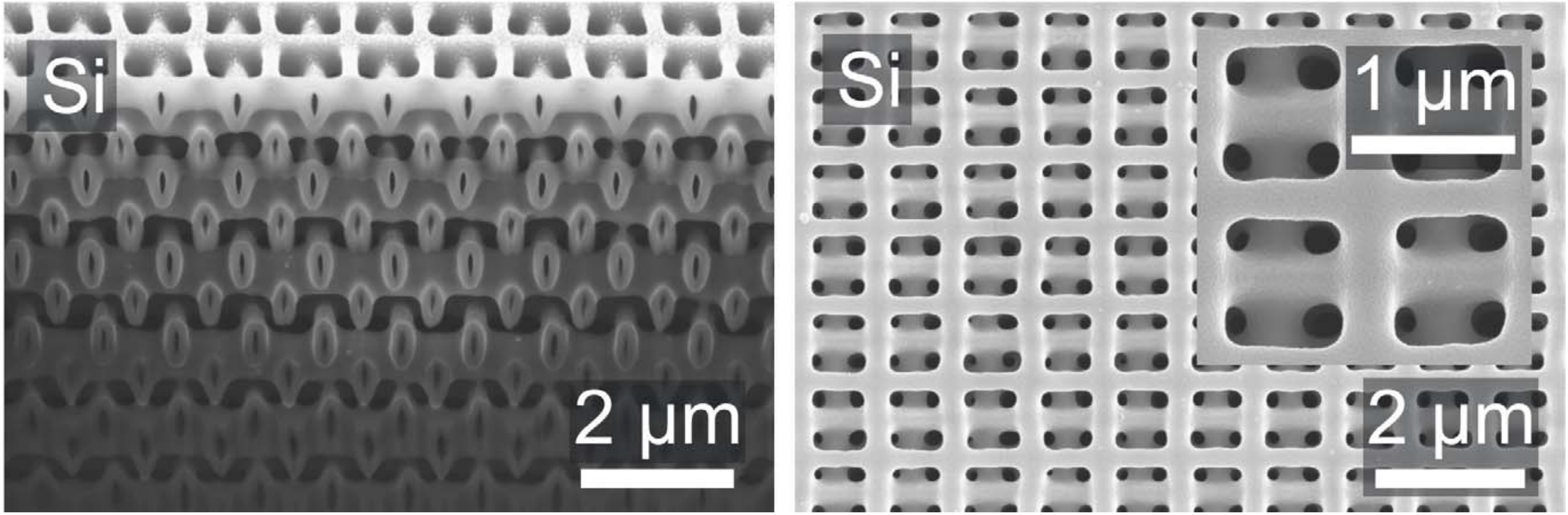
Martin Hermatschweiler



$Si_2H_6 \rightarrow 2\ Si + 3\ H_2$

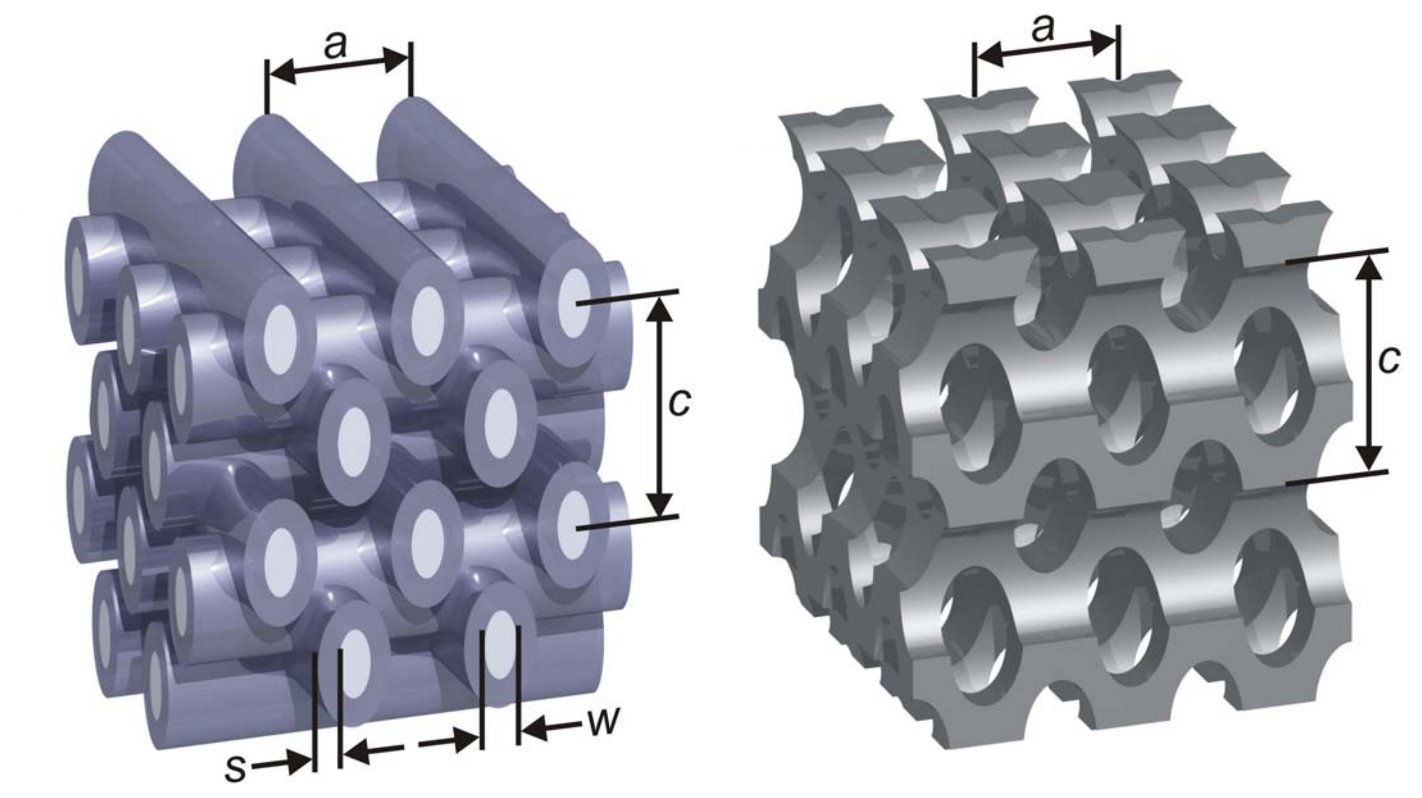
Martin Hermatschweiler

Resulting silicon woodpiles …



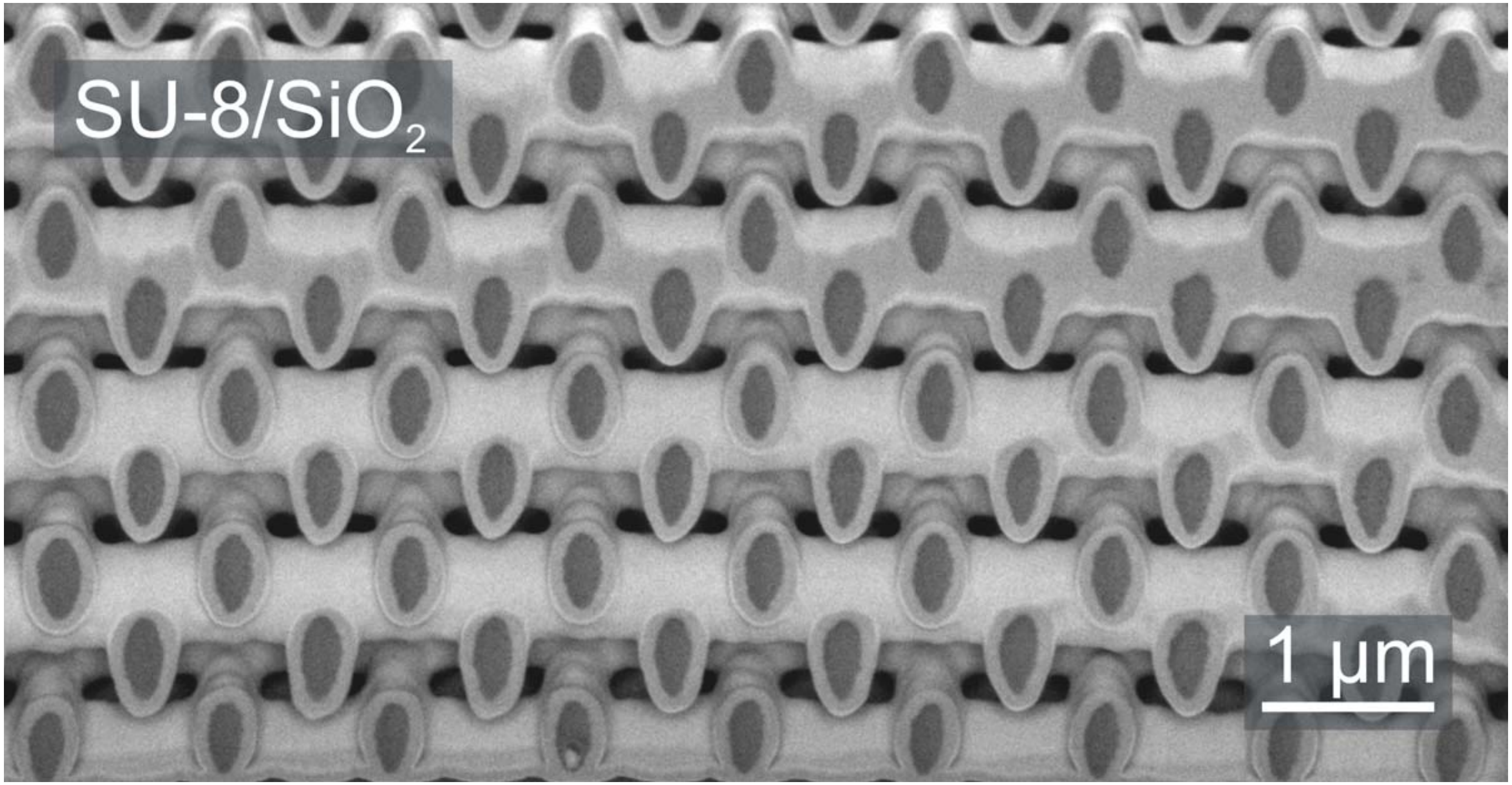
N. Tétreault et al., Adv. Mater. 18, 457 (2006)

… or inverse silicon woodpiles



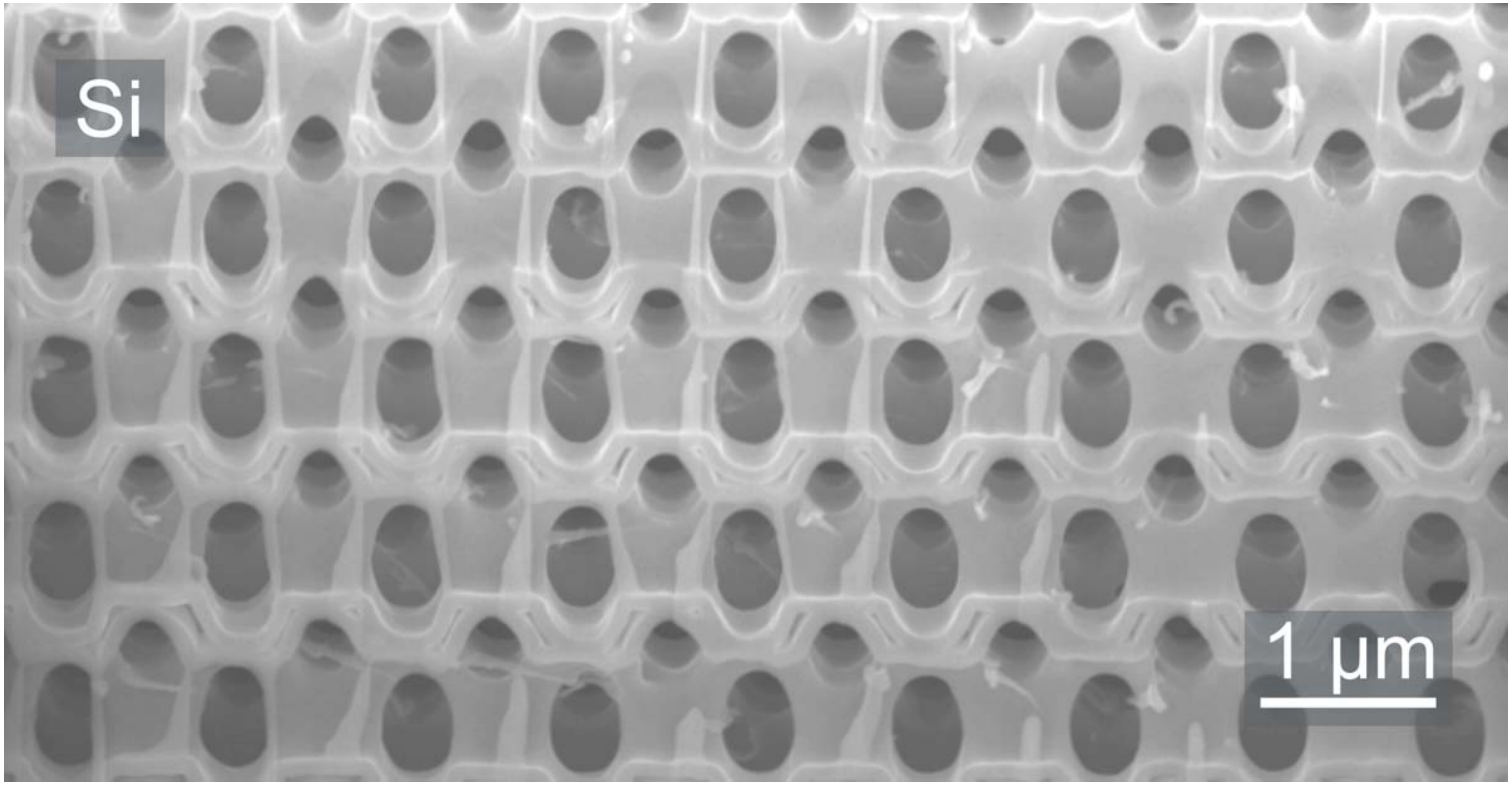
M. Hermatschweiler et al., Adv. Funct. Mater. 17, 2273 (2007)

… or inverse silicon woodpiles



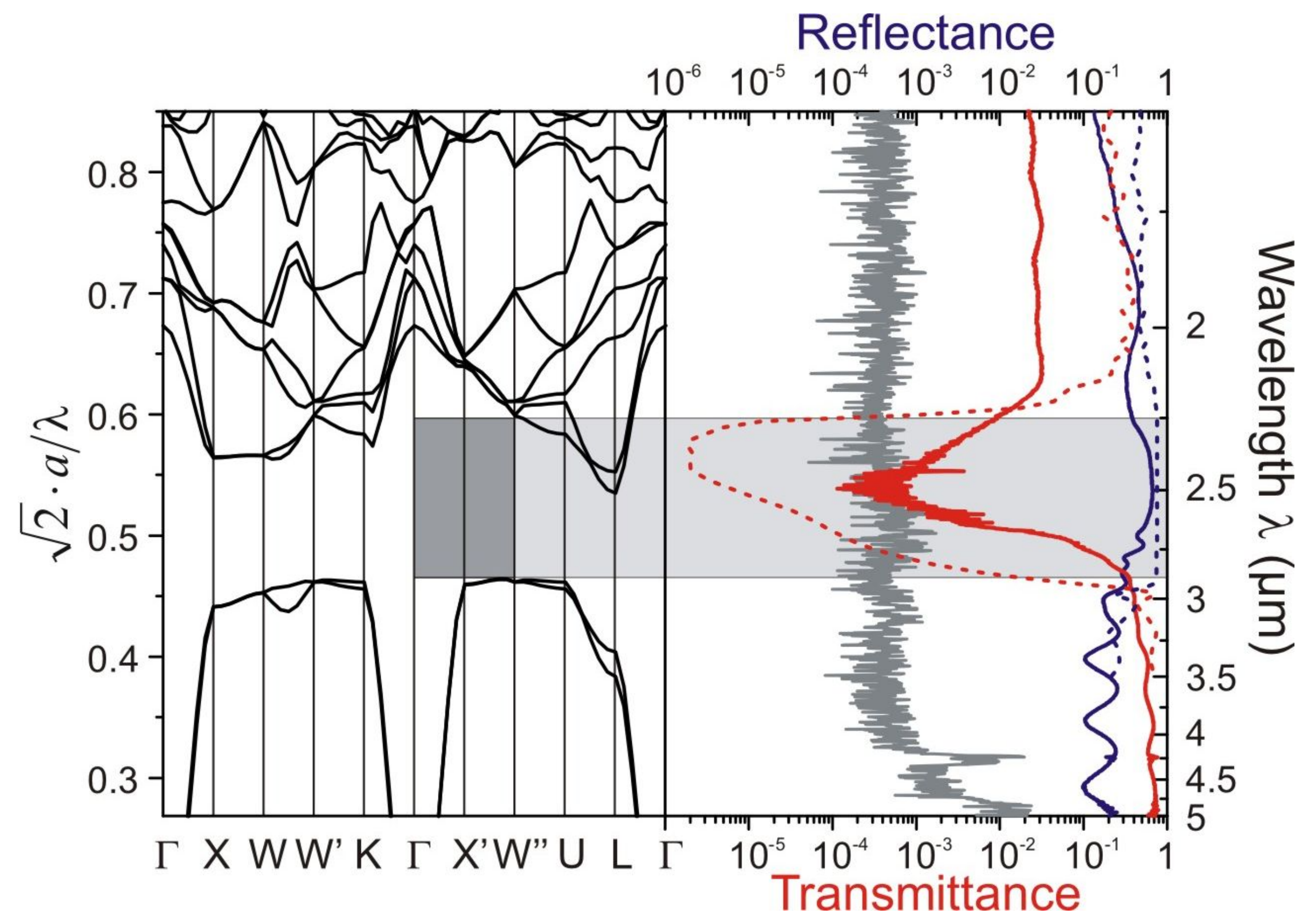
M. Hermatschweiler et al., Adv. Funct. Mater. 17, 2273 (2007)

… or inverse silicon woodpiles



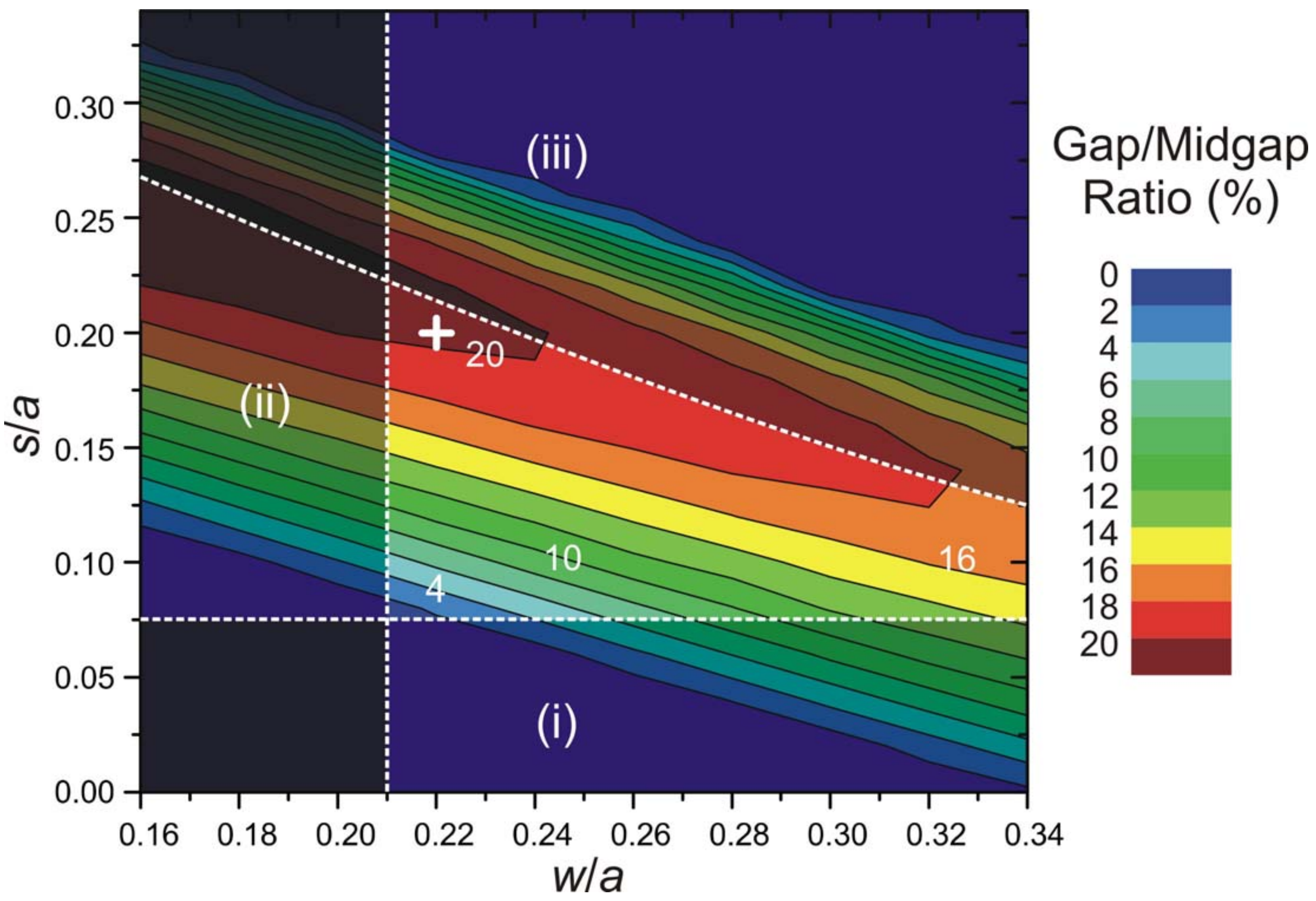
M. Hermatschweiler et al., Adv. Funct. Mater. 17, 2273 (2007)

14% PBG @ 2.5 µm achieved



M. Hermatschweiler et al., Adv. Funct. Mater. 17, 2273 (2007)

20% PBG @ 1.5 μm in reach



M. Hermatschweiler et al., Adv. Funct. Mater. 17, 2273 (2007)

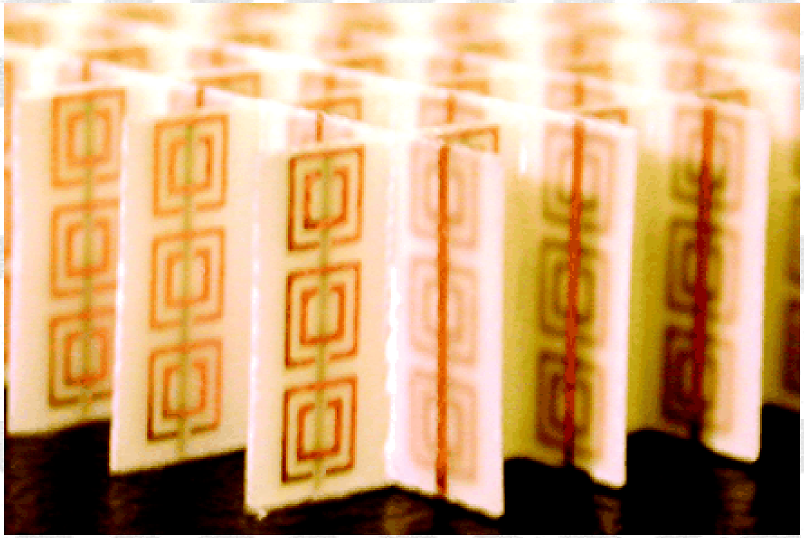
"Normal" crystals ...

... have lattice constants
much smaller than
the wavelength of light

... are common optical
materials; they have
a refractive index n>0

Natur-Lexikon.com

Metamaterials ...

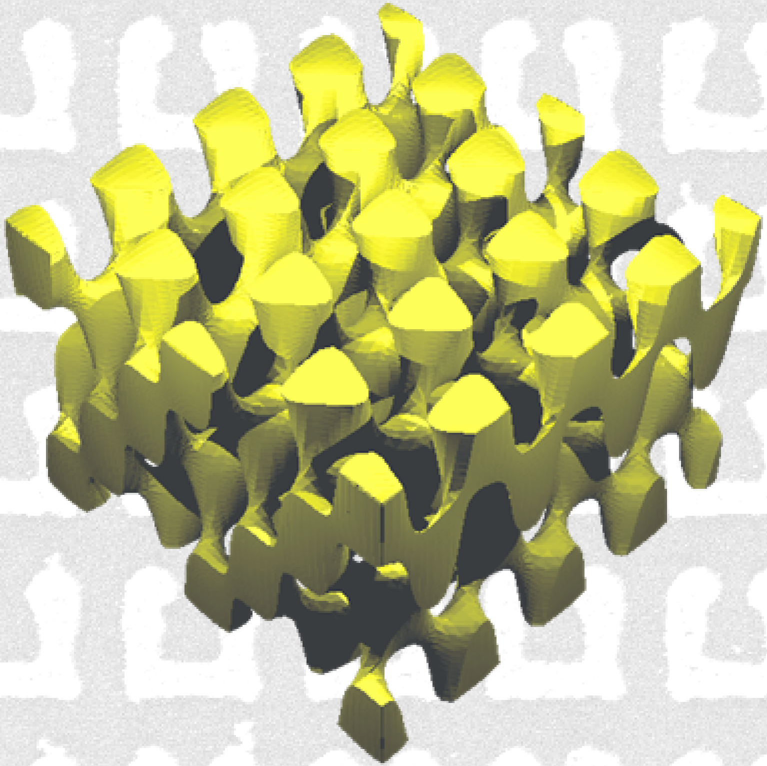


... have lattice constants
smaller than
the wavelength of light

... can be left-handed, i.e.,
n<0, which is the basis
for, e.g., "perfect lenses"

R.A. Shelby et al., Science 292, 77 (2001)

Photonic Crystals ...



... have lattice constants
comparable to
the wavelength of light

... can be
"semiconductors for light"
(see section 2.)

S. John, Phys. Rev. Lett. 58, 2586 (1987)
E. Yablonovitch, Phys. Rev. Lett. 58, 2059 (1987)

1. Basics
    - 1.1. Analogy between photonics and electronics
    - 1.2. Paradigms of nanophotonics
        - 1.2.1. Evanescent waves & the limit of electrostatics
        - 1.2.2. Mie-scattering of nanoparticles
        - 1.2.3. Nanoscale metal particles: Particle plasmons
        - 1.2.4. A nano-aperture in an ideal metal film: Bethe-Bouwkamp theory

2. Photonic band gap (PBG) materials
    - 2.1. One-dimensional Photonic Crystals (i.e., dielectric mirrors)
    - 2.2. Two-dimensional Photonic Crystals
        - 2.2.1. Negative refraction and the superprism effect
        - 2.2.2. "Semiconductors for light": Waveguides and defect cavities
    - 2.3. Three-dimensional Photonic Crystals
        - 2.3.1. Quantum optics: Modified Planck's law
        - 2.3.2. Fabrication: Overview and state-of-the-art
        - 2.3.3. Fabrication: Current efforts within the CFN

3. Metamaterials
    - **3.1. Left-handed or Veselago materials (LHM)**
    - 3.2. "Perfect lenses" made from LHM
    - 3.3. Towards metamaterials @ optical frequencies (CFN activities)

4. Plasmonics
    - 4.1. Field-enhancement & surface-enhanced Raman scattering (SERS)
    - 4.2. Surface-plasmon amplification by stimulated emission of radiation (SPASER)
    - 4.3. Transmission through sub-wavelength hole arrays

Plane waves ...

... are solutions of the Maxwell equations

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

For linear materials ...

$$\vec{D} = \varepsilon_0 \varepsilon \, \vec{E}$$

$$\vec{B} = \mu_0 \mu \, \vec{H}$$

Plane waves …

… are characterized by their phase velocity c with

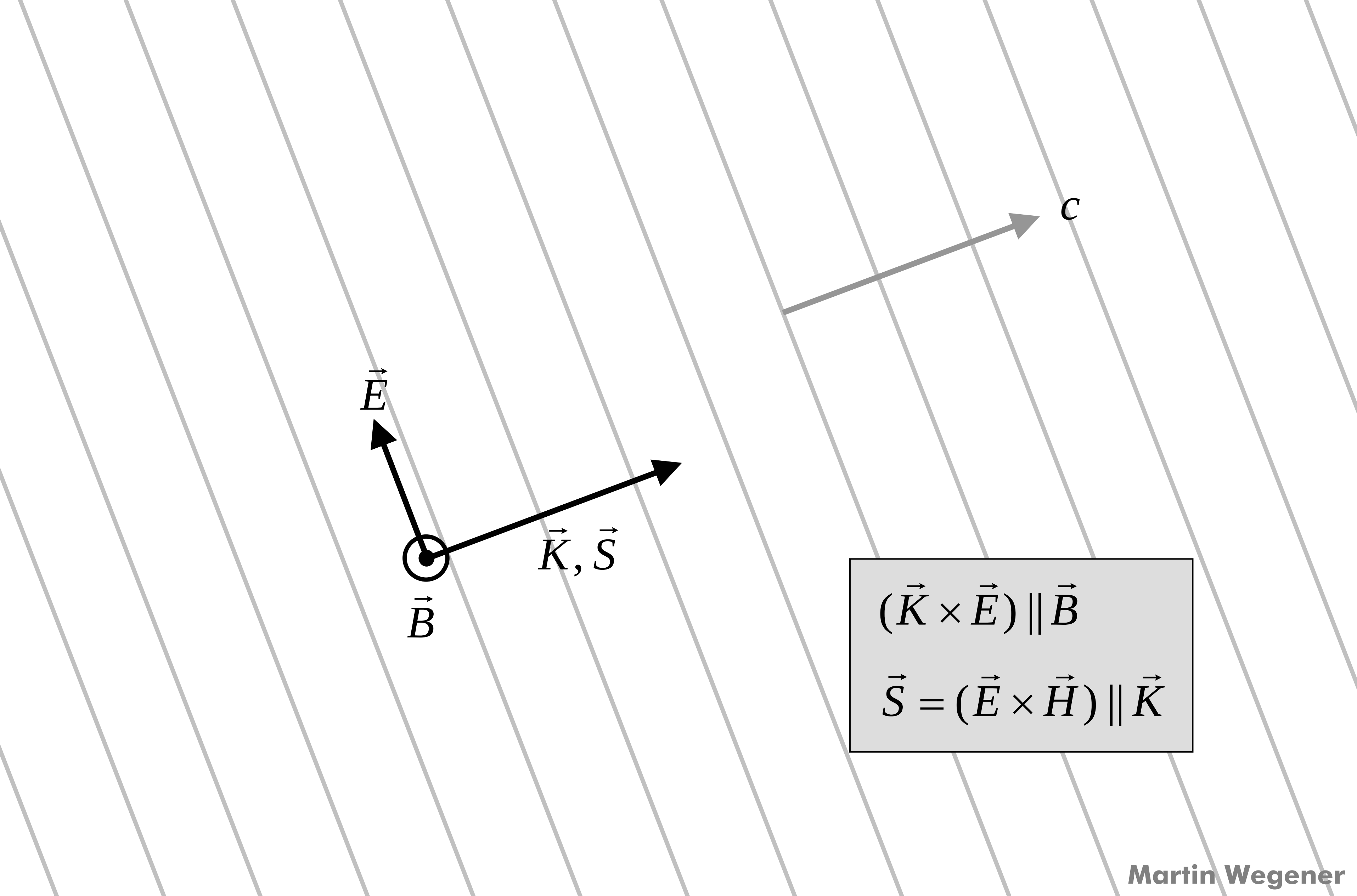
$$c^2 = \frac{1}{\varepsilon_0 \varepsilon \mu_0 \mu} = \frac{c_0^2}{\varepsilon \mu} = \frac{c_0^2}{n^2}$$

$$n^2 = \varepsilon \mu \quad \Rightarrow n = \pm\sqrt{\varepsilon \mu}$$

… and by their impedance

$$Z = \sqrt{\frac{\mu_0 \mu}{\varepsilon_0 \varepsilon}} \qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 376.7\,\Omega$$

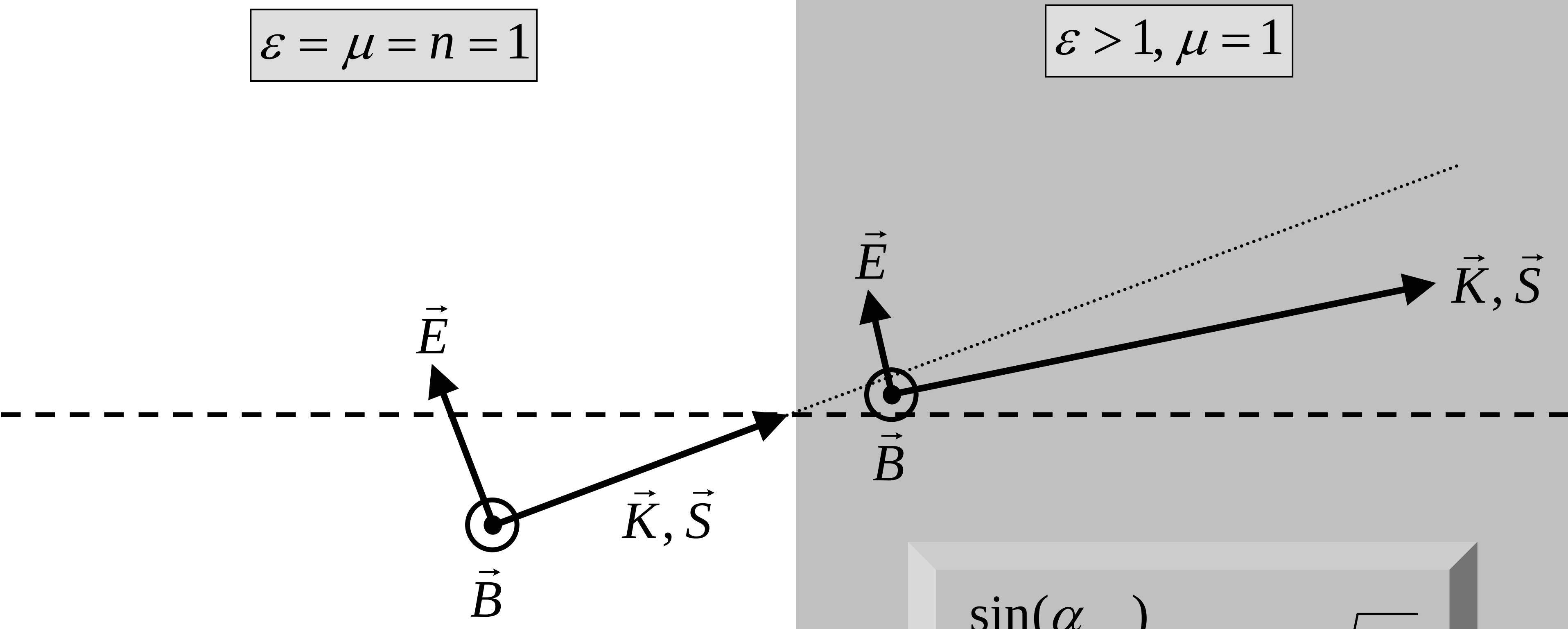
Planes waves in normal media



$$(\vec{K} \times \vec{E}) \parallel \vec{B}$$

$$\vec{S} = (\vec{E} \times \vec{H}) \parallel \vec{K}$$

Refraction at an interface



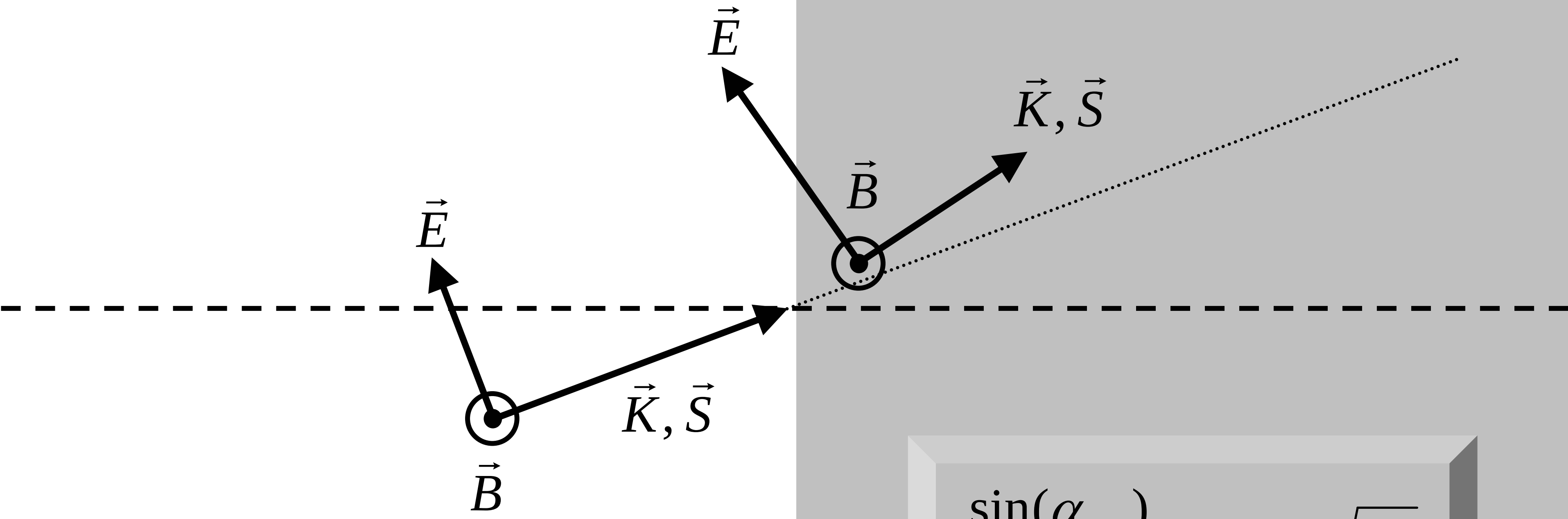
$$\frac{\sin(\alpha_{\text{vac}})}{\sin(\alpha_{\text{med}})} = n = +\sqrt{\varepsilon\mu}$$

Snell's law of refraction

Refraction at an interface

$\varepsilon = \mu = n = 1$

$0 < \varepsilon < 1,\ \mu = 1$



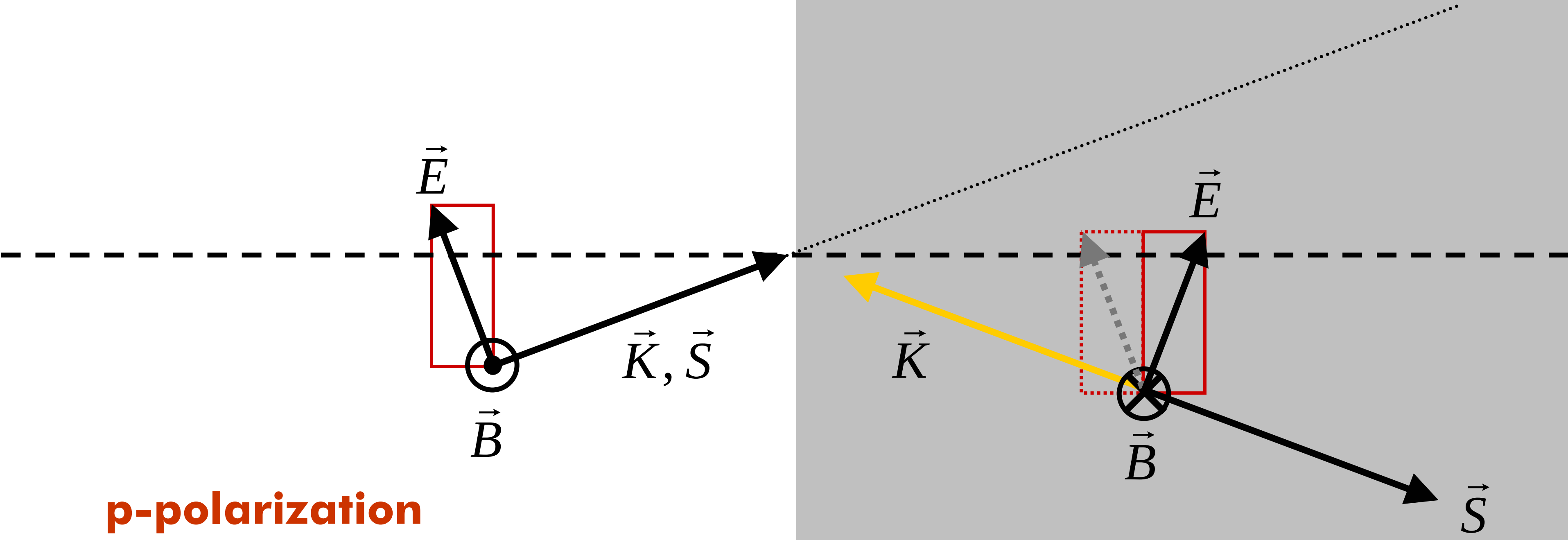
$$\frac{\sin(\alpha_{\text{vac}})}{\sin(\alpha_{\text{med}})} = n = +\sqrt{\varepsilon\mu}$$

Snell's law of refraction

Refraction at an interface

$\varepsilon = \mu = n = 1$

$$\varepsilon = \mu = -1 \Rightarrow Z = \sqrt{\frac{\mu_0 \mu}{\varepsilon_0 \varepsilon}} = Z_0$$



p-polarization

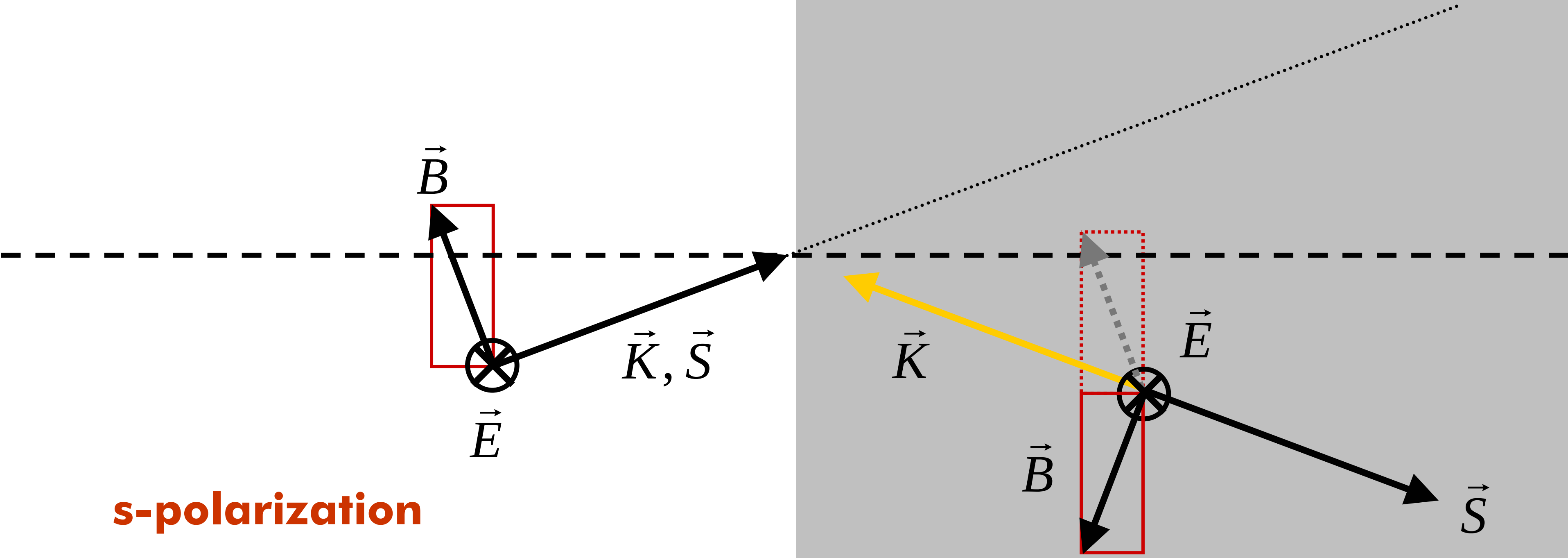
$$n = -\sqrt{\varepsilon\mu} = -1$$

Martin Wegener

Refraction at an interface

$$\varepsilon = \mu = n = 1$$

$$\varepsilon = \mu = -1 \Rightarrow Z = \sqrt{\frac{\mu_0 \mu}{\varepsilon_0 \varepsilon}} = Z_0$$



s-polarization

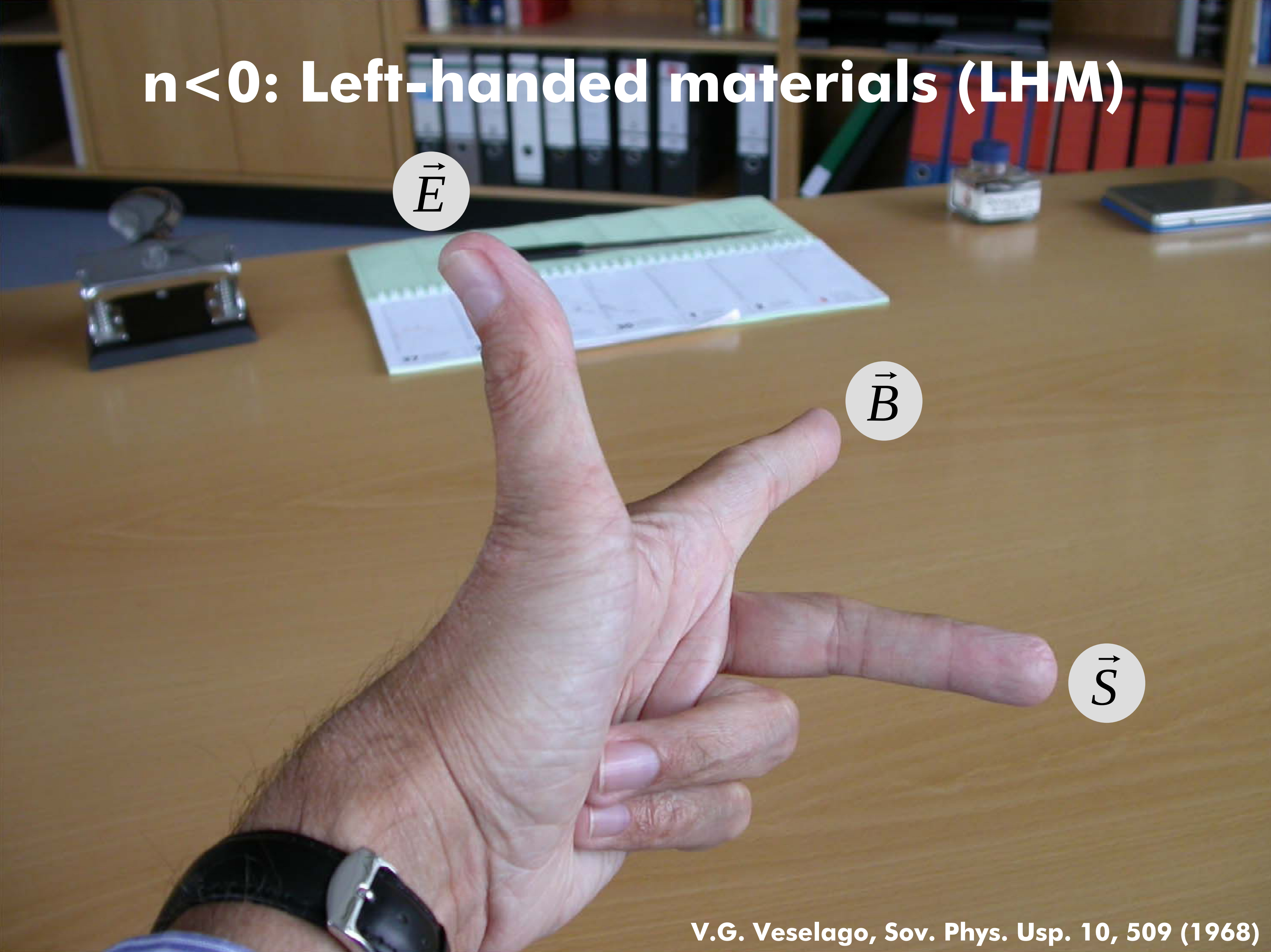
$$n = -\sqrt{\varepsilon \mu} = -1$$

Martin Wegener

Materials with n>0 are right-handed

Martin Wegener

$n<0$: Left-handed materials (LHM)



V.G. Veselago, Sov. Phys. Usp. 10, 509 (1968)

The phenomenon of negative refraction …

… can occur in **Photonic Crystals** with
positive permittivity and unity permeability.
It is a result of Bragg reflection (section 2.2.1.).

… can occur in **left-handed metamaterials** with
negative permittivity and negative permeability.
Bragg reflection plays no role (this section).

1. Basics
   - 1.1. Analogy between photonics and electronics
   - 1.2. Paradigms of nanophotonics
     - 1.2.1. Evanescent waves & the limit of electrostatics
     - 1.2.2. Mie-scattering of nanoparticles
     - 1.2.3. Nanoscale metal particles: Particle plasmons
     - 1.2.4. A nano-aperture in an ideal metal film: Bethe-Bouwkamp theory

2. Photonic band gap (PBG) materials
   - 2.1. One-dimensional Photonic Crystals (i.e., dielectric mirrors)
   - 2.2. Two-dimensional Photonic Crystals
     - 2.2.1. Negative refraction and the superprism effect
     - 2.2.2. "Semiconductors for light": Waveguides and defect cavities
   - 2.3. Three-dimensional Photonic Crystals
     - 2.3.1. Quantum optics: Modified Planck's law
     - 2.3.2. Fabrication: Overview and state-of-the-art
     - 2.3.3. Fabrication: Current efforts within the CFN

3. Metamaterials
   - 3.1. Left-handed or Veselago materials (LHM)
   - **3.2. "Perfect lenses" made from LHM**
   - 3.3. Towards metamaterials @ optical frequencies (CFN activities)

4. Plasmonics
   - 4.1. Field-enhancement & surface-enhanced Raman scattering (SERS)
   - 4.2. Surface-plasmon amplification by stimulated emission of radiation (SPASER)
   - 4.3. Transmission through sub-wavelength hole arrays

A “perfect lens“ from a LHM

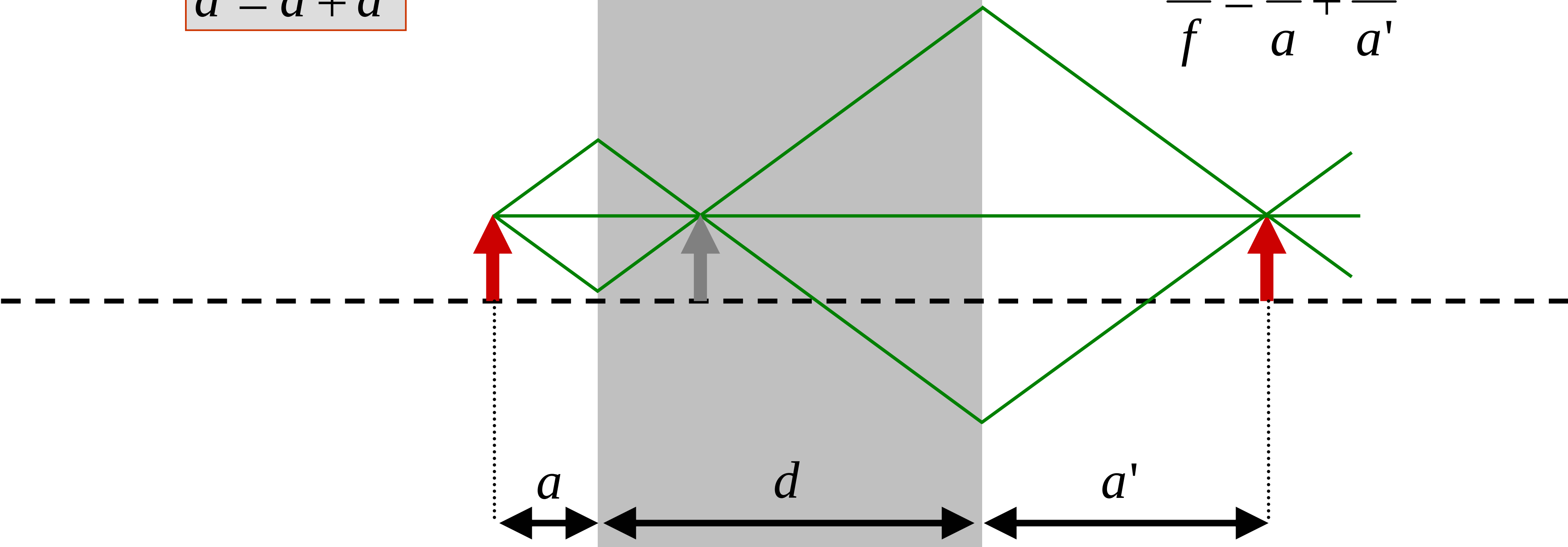
$$\boxed{\varepsilon = \mu = n = -1}$$

perfect lens

$$\boxed{d = a + a'}$$

usual lens

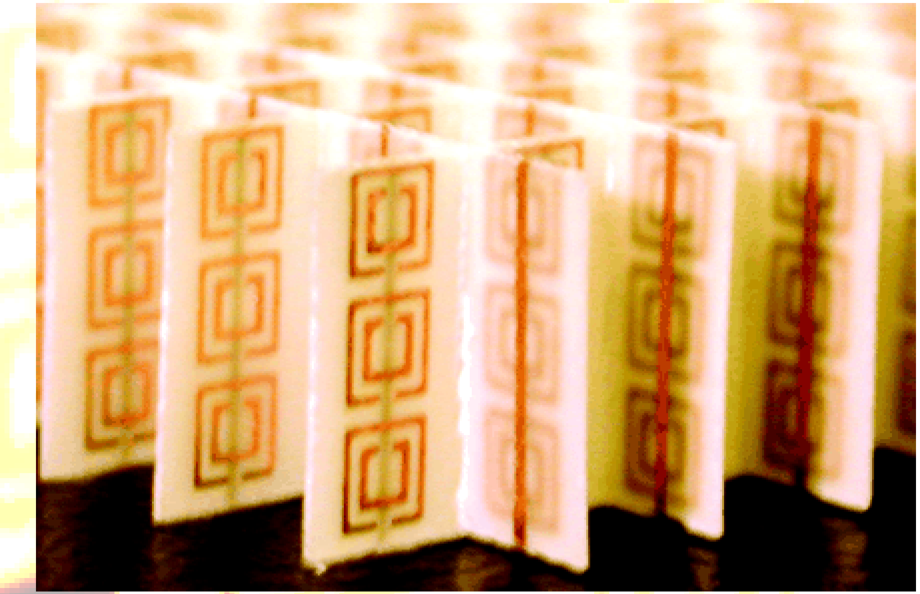
$$\frac{1}{f} = \frac{1}{a} + \frac{1}{a'}$$



J.B. Pendry, Phys. Rev. Lett. 85, 3966 (2000)

First left-handed materials

10 GHz (3 cm)



R.A. Shelby et al., Science 292, 77 (2001)

Science magazine:

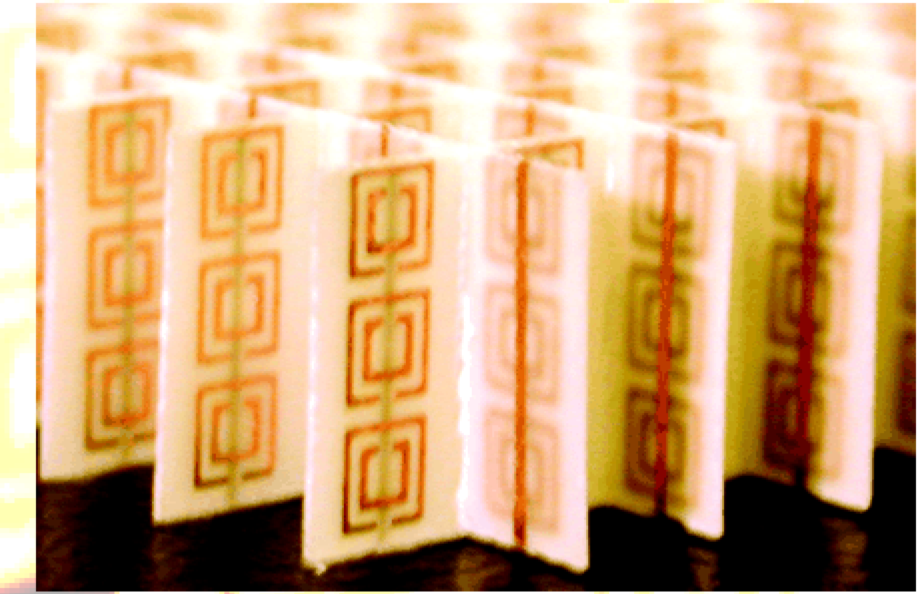
"LHM one of the top ten scientific breakthroughs in 2003"

Editorial staff, Science 302, 2039 (2003)

Confirmation: A.A. Houck et al., Phys. Rev. Lett. 90, 137 (2003)

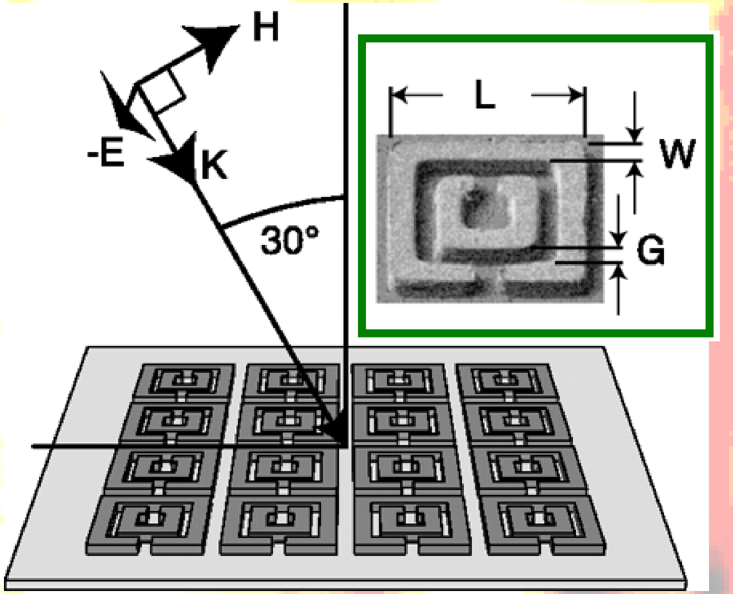
First left-handed materials

10 GHz (3 cm)



R.A. Shelby et al., Science 292, 77 (2001)

1 THz (300 μm)



T.J. Yen et al., Science 303, 1494 (2004)

Review: D.R. Smith et al., Science 305, 788 (2004)

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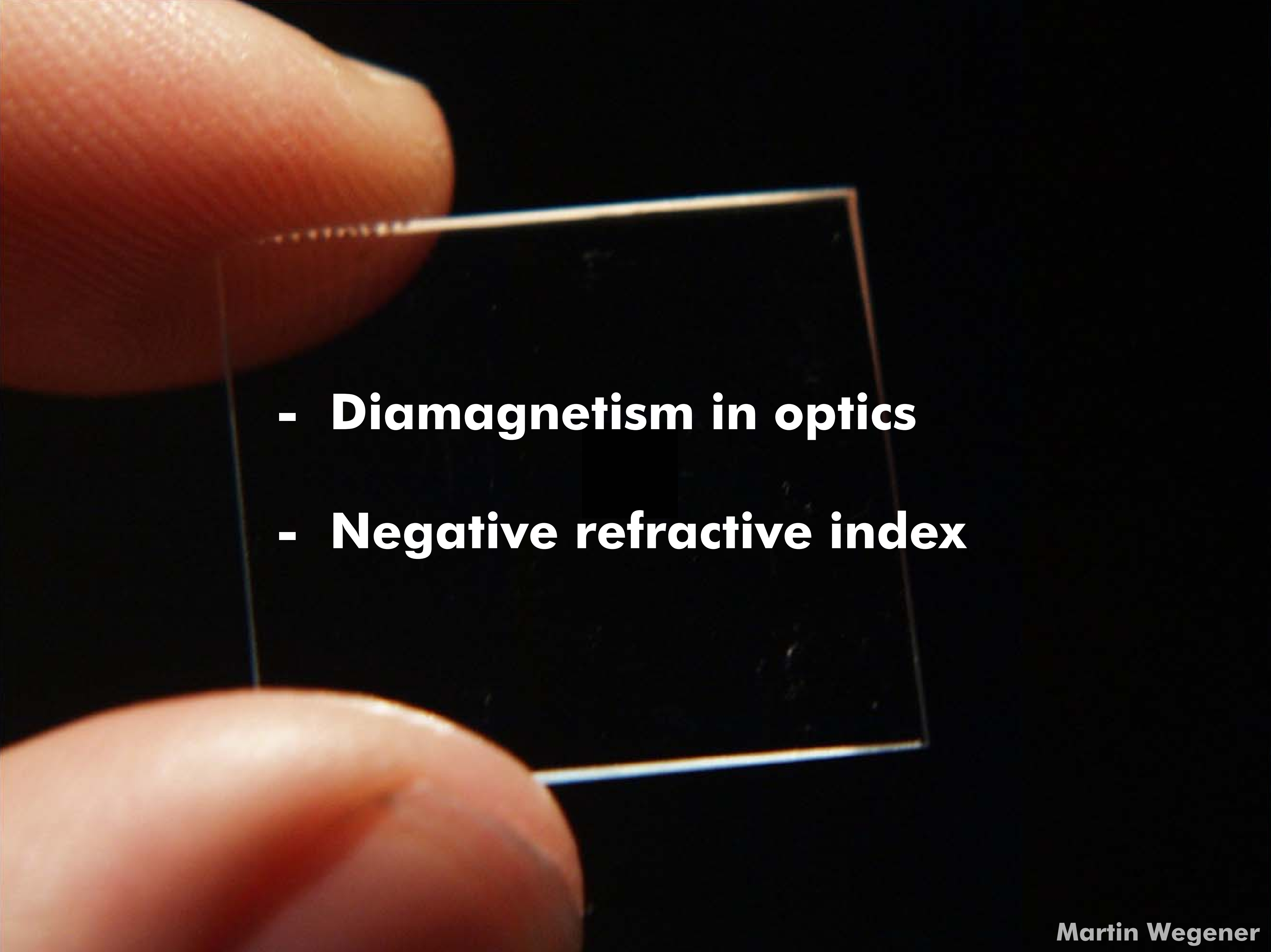
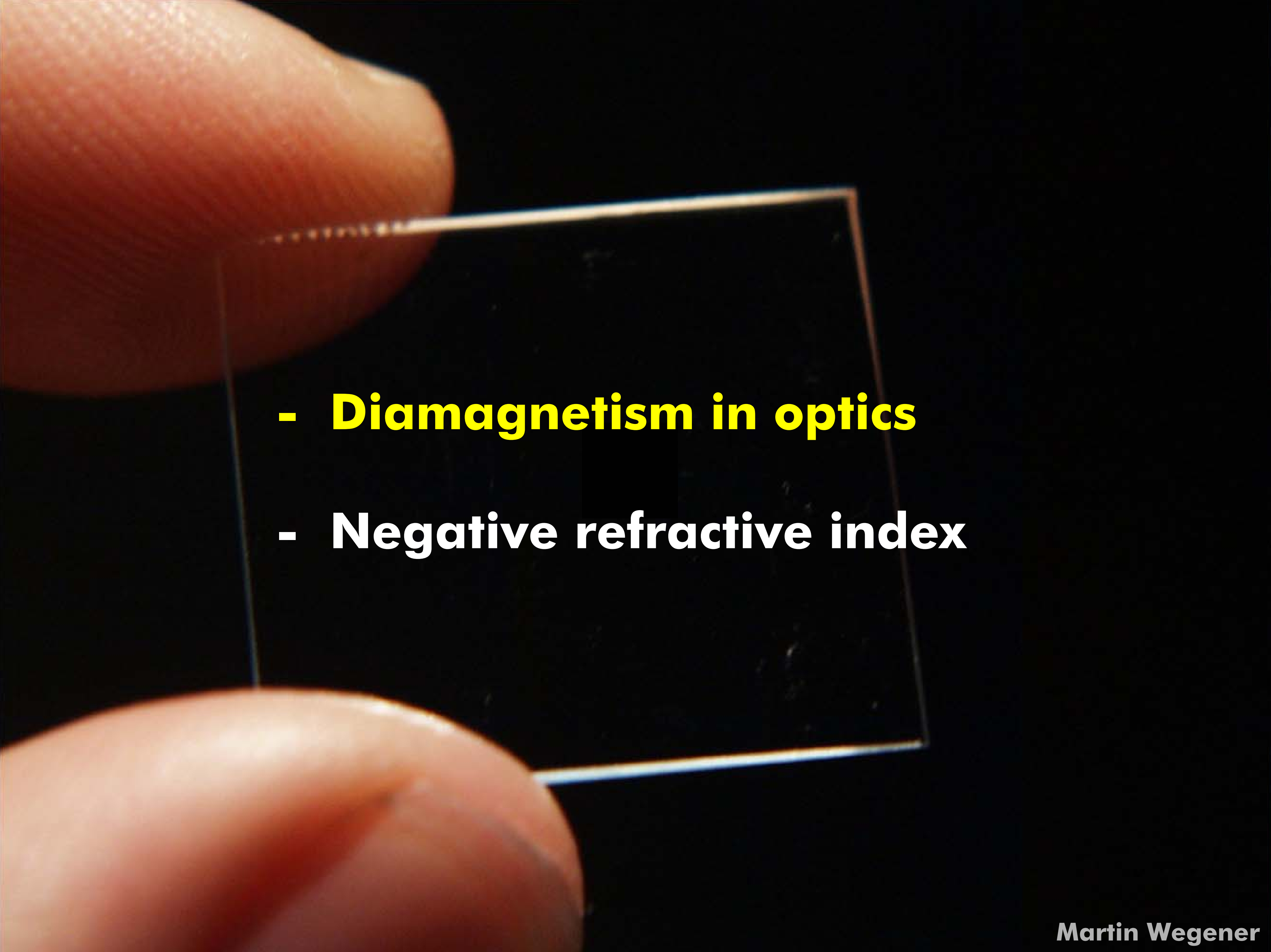


- Diamagnetism in optics
- Negative refractive index

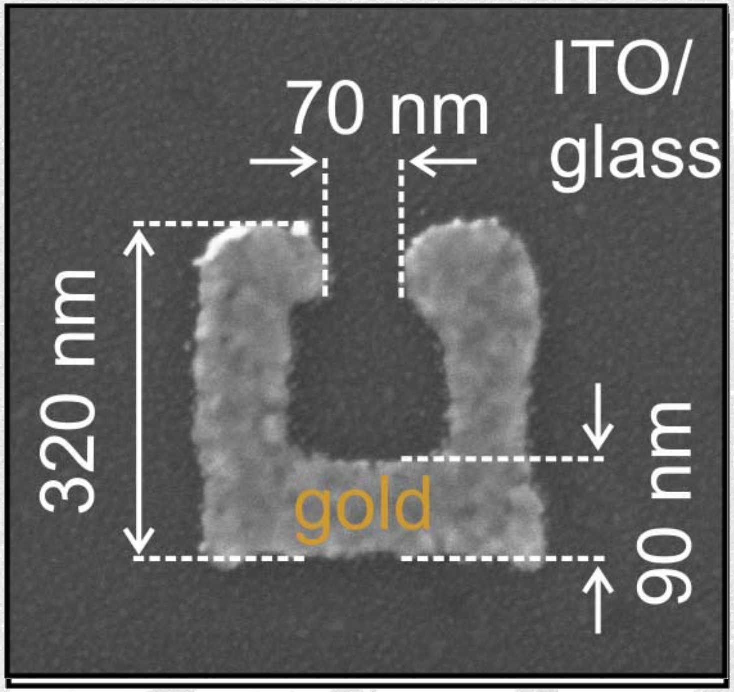
Martin Wegener



- Diamagnetism in optics
- Negative refractive index

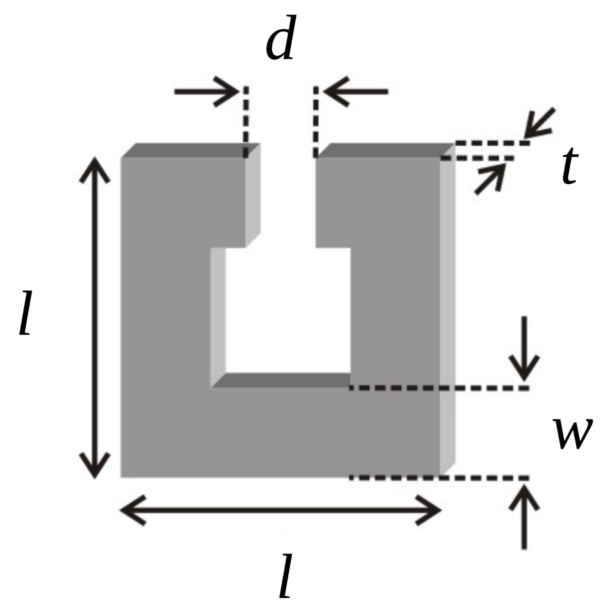
Martin Wegener

LC-circuits @ 100 THz (3 μm)



Stefan Linden, Christian Enkrich & Martin Wegener

Estimating the LC-resonance



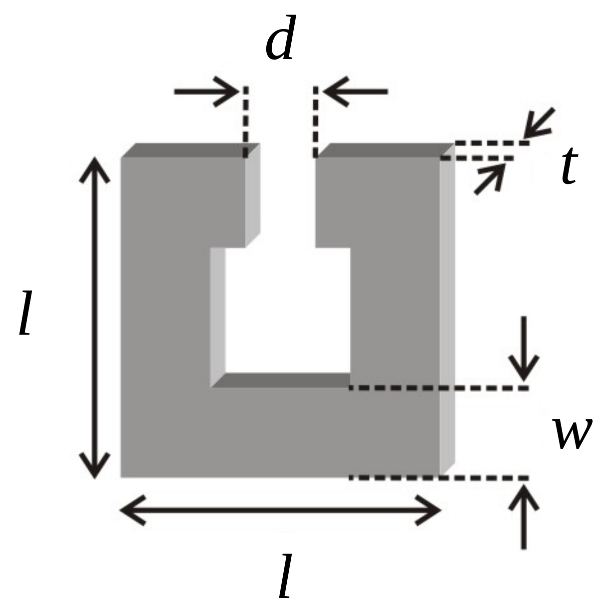
$$L = \mu_0 \frac{l^2}{t} = 5.6\,\mathrm{pH}$$

$$C = \varepsilon_0 \varepsilon_C \frac{wt}{d} = 0.5\,\mathrm{aF}$$

$$\omega_{LC} = \frac{1}{\sqrt{LC}} \approx 2\pi\, 100\,\mathrm{THz}$$

$$\boxed{\Rightarrow \lambda_{LC} = l\, 2\pi \sqrt{\varepsilon_C} \sqrt{\frac{w}{d}} \approx 3\,\mu\mathrm{m}}$$

Magnetic permeability



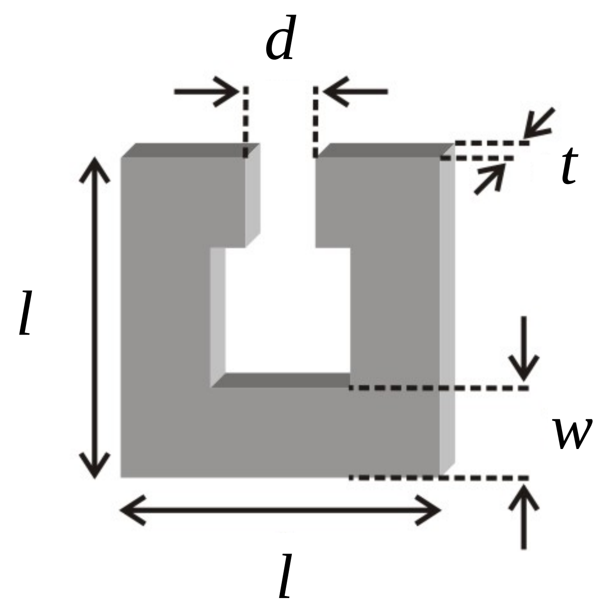
$$U_C + U_L = U_{\text{ind}} = -\dot{\phi}$$

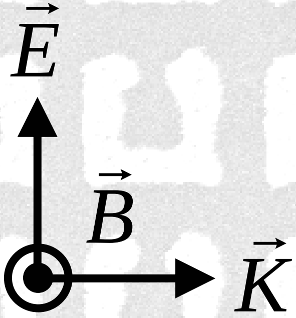
$$\Rightarrow I(t) \Rightarrow l^2 I(t) \Rightarrow M(t)$$

$$0 \le F := \frac{l^2 t}{a_{xy}\, a_z} \le 1$$

$$\boxed{\Rightarrow \mu(\omega) = 1 + \frac{F\,\omega^2}{\omega_{LC}^2 - \omega^2}}$$

Polarization dependence

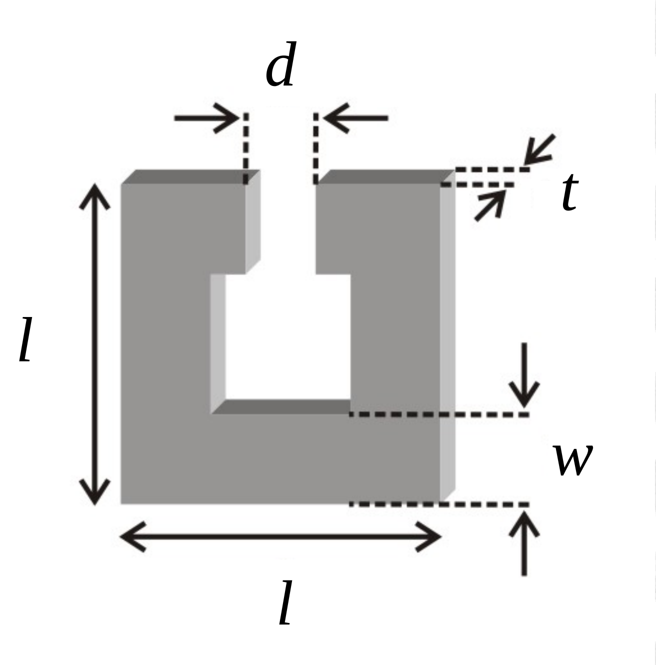


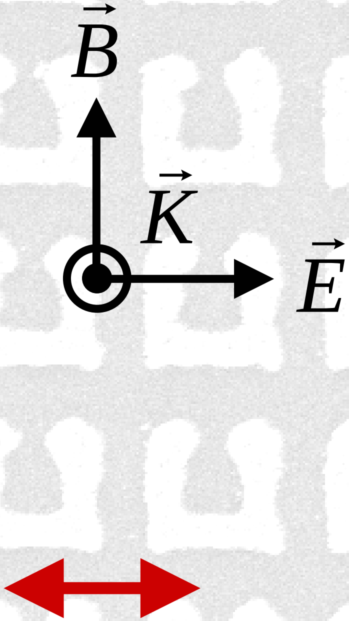


Coupling to the LC-resonance & magnetic response but not accessible for normal incidence.

N. Katsarakis et al., Appl. Phys. Lett. 84, 2943 (2004)

Polarization dependence

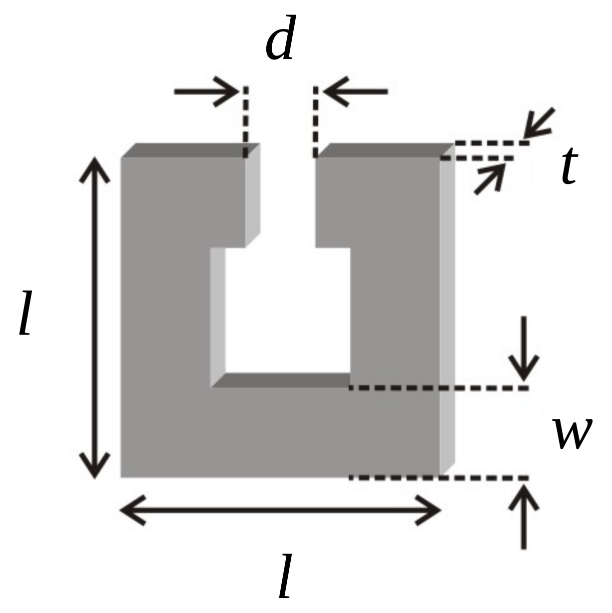


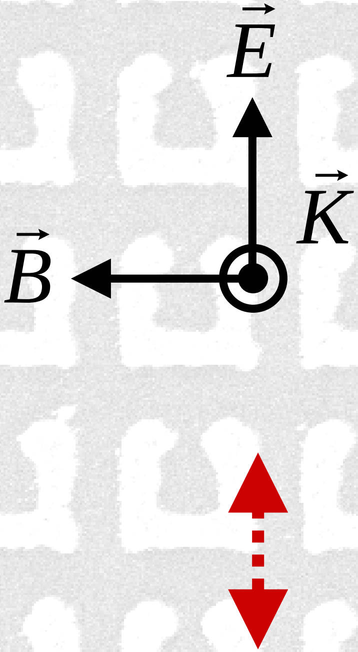


Coupling to the LC-resonance even for normal incidence.

N. Katsarakis et al., Appl. Phys. Lett. 84, 2943 (2004)

Polarization dependence

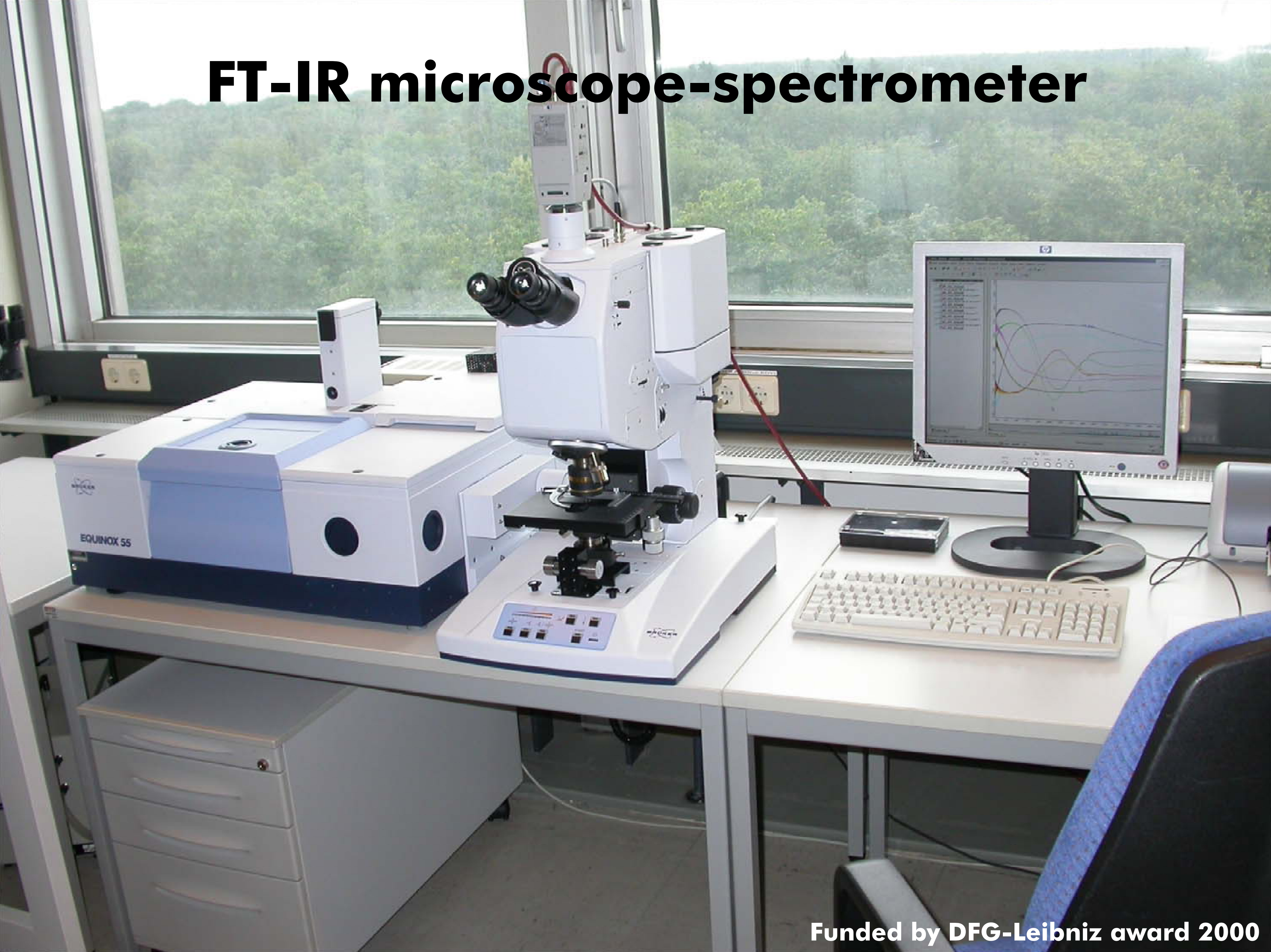




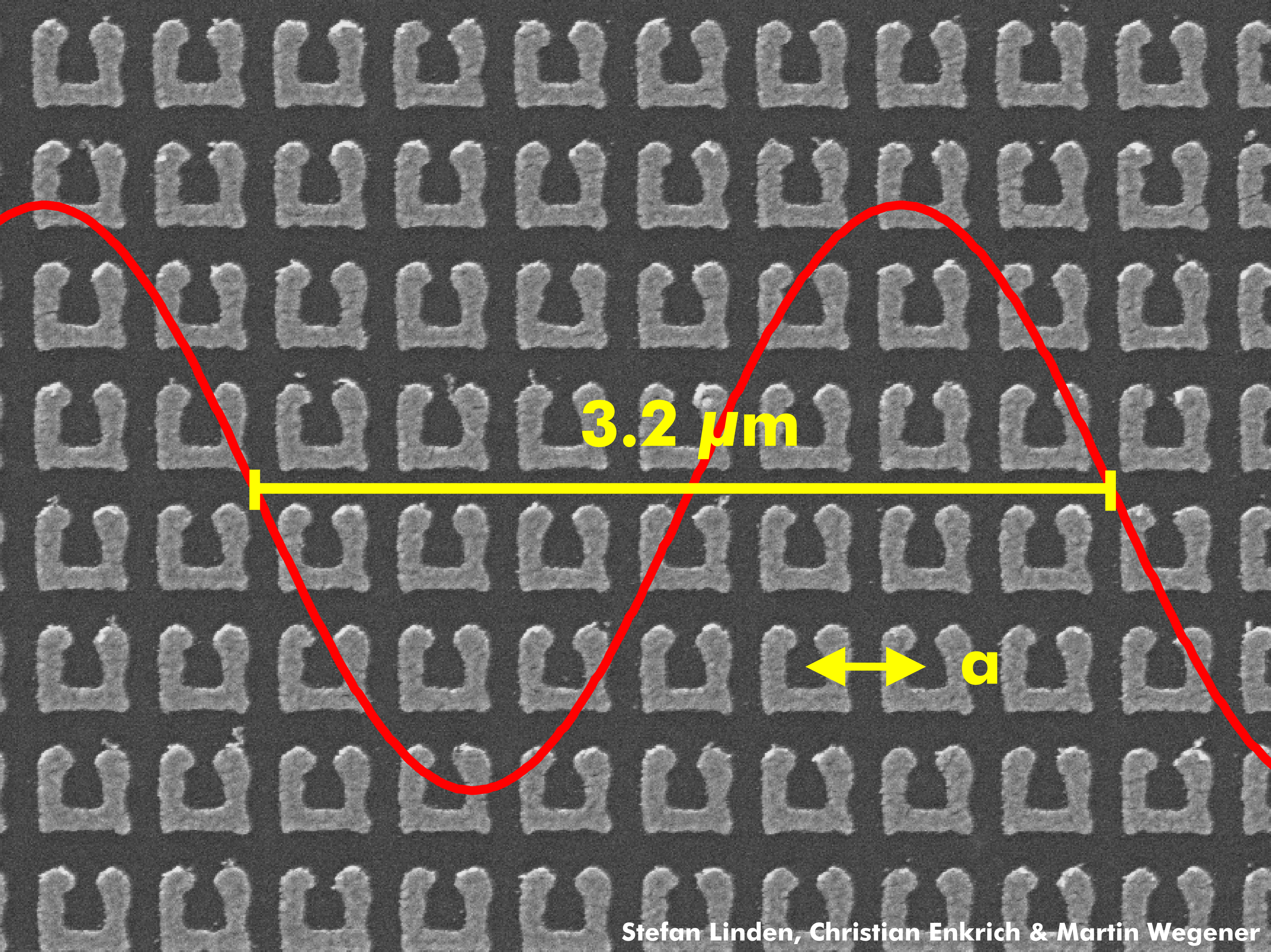
No coupling to the LC-resonance, normal incidence.

N. Katsarakis et al., Appl. Phys. Lett. 84, 2943 (2004)

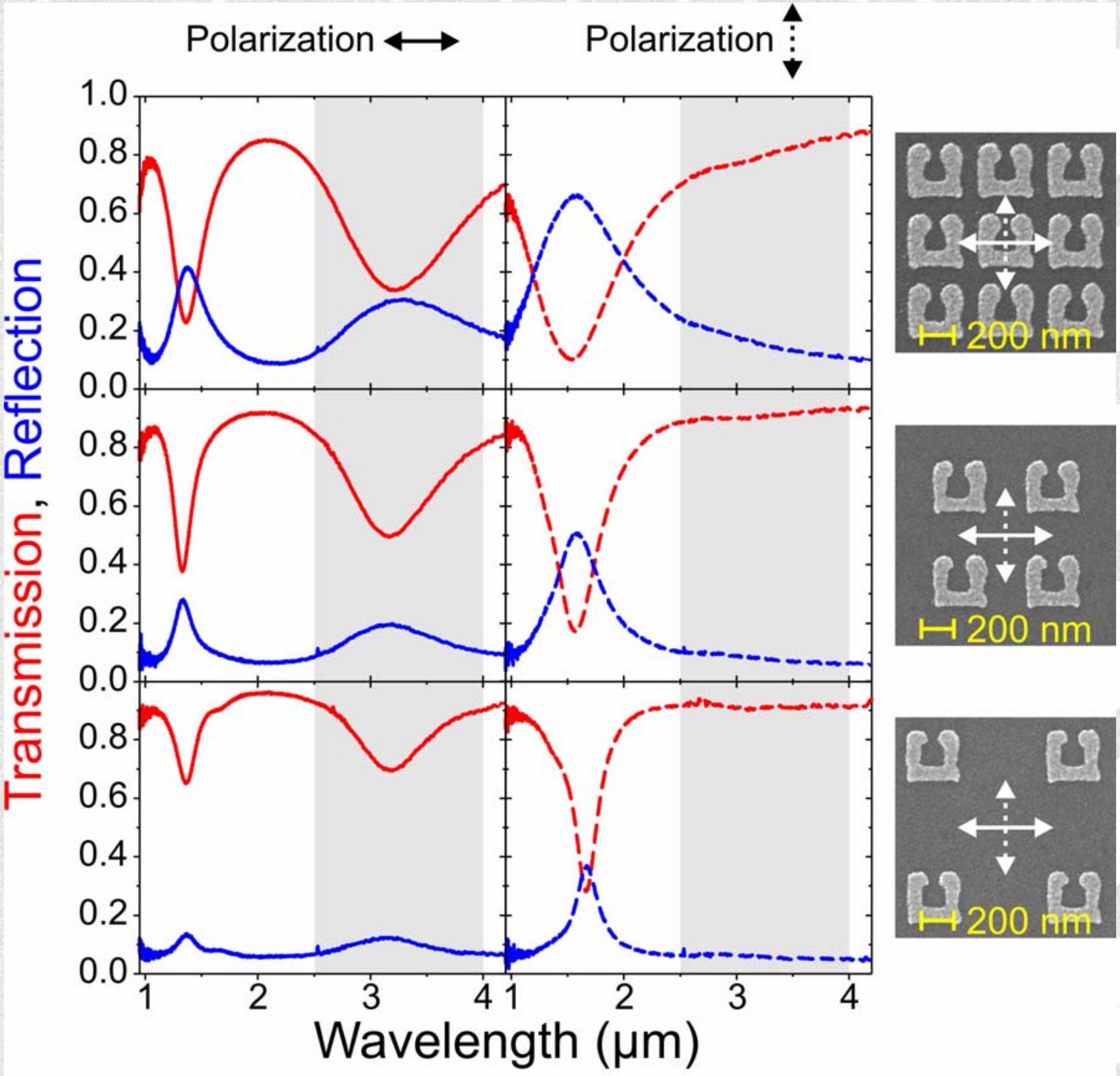
FT-IR microscope-spectrometer



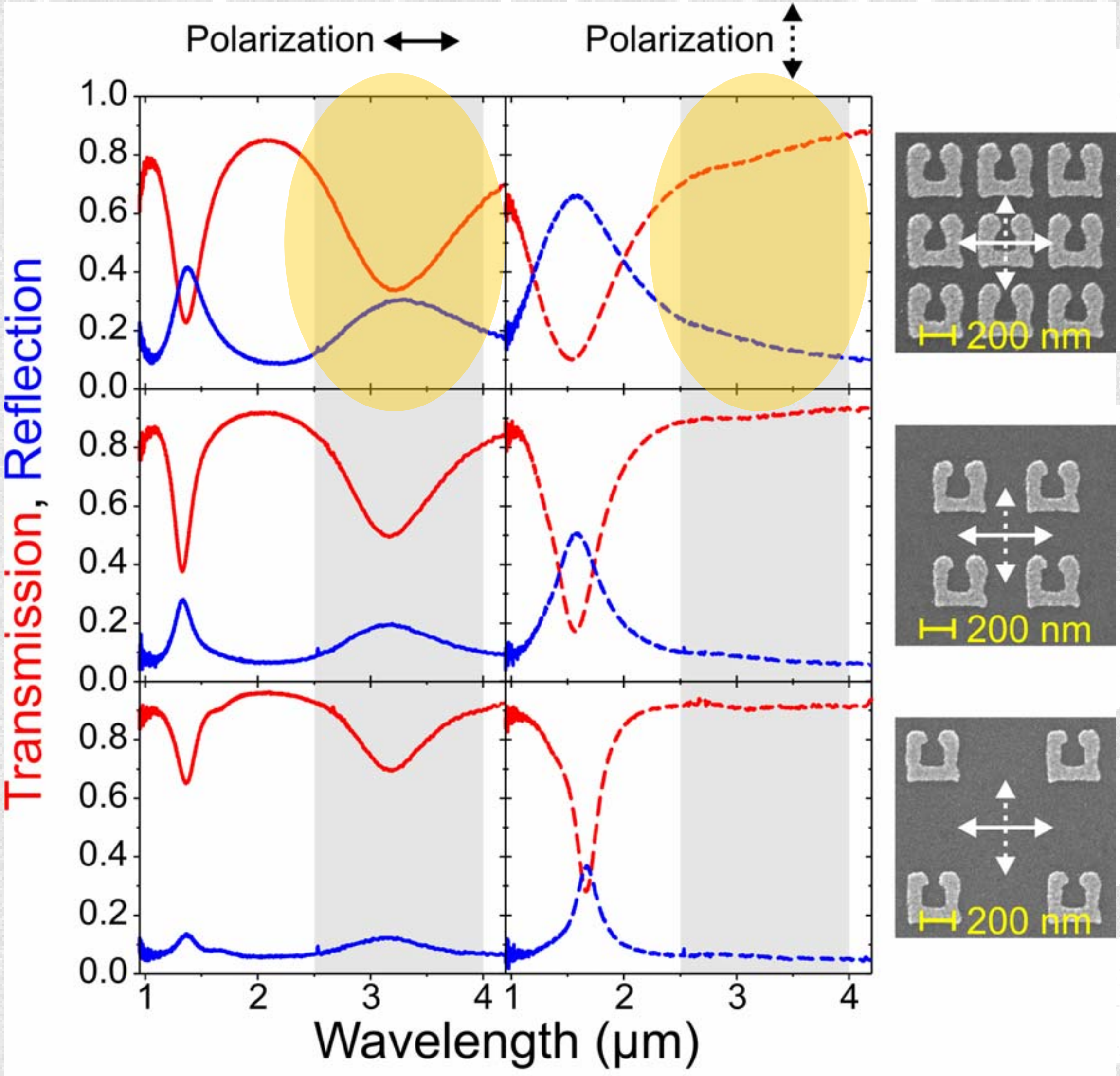
Funded by DFG-Leibniz award 2000



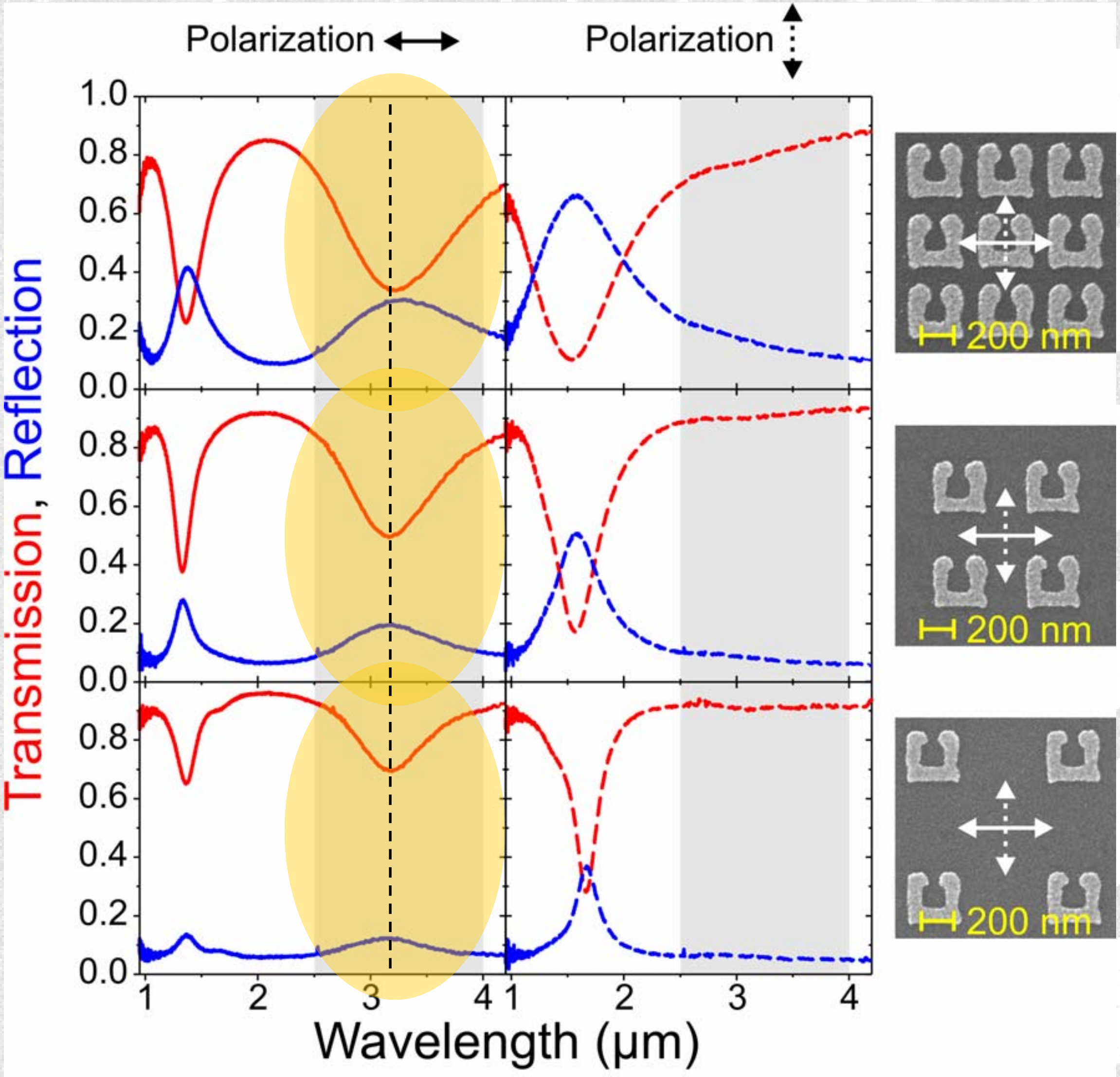
Stefan Linden, Christian Enkrich & Martin Wegener



Science 306, 1351 (2004)



Science 306, 1351 (2004)

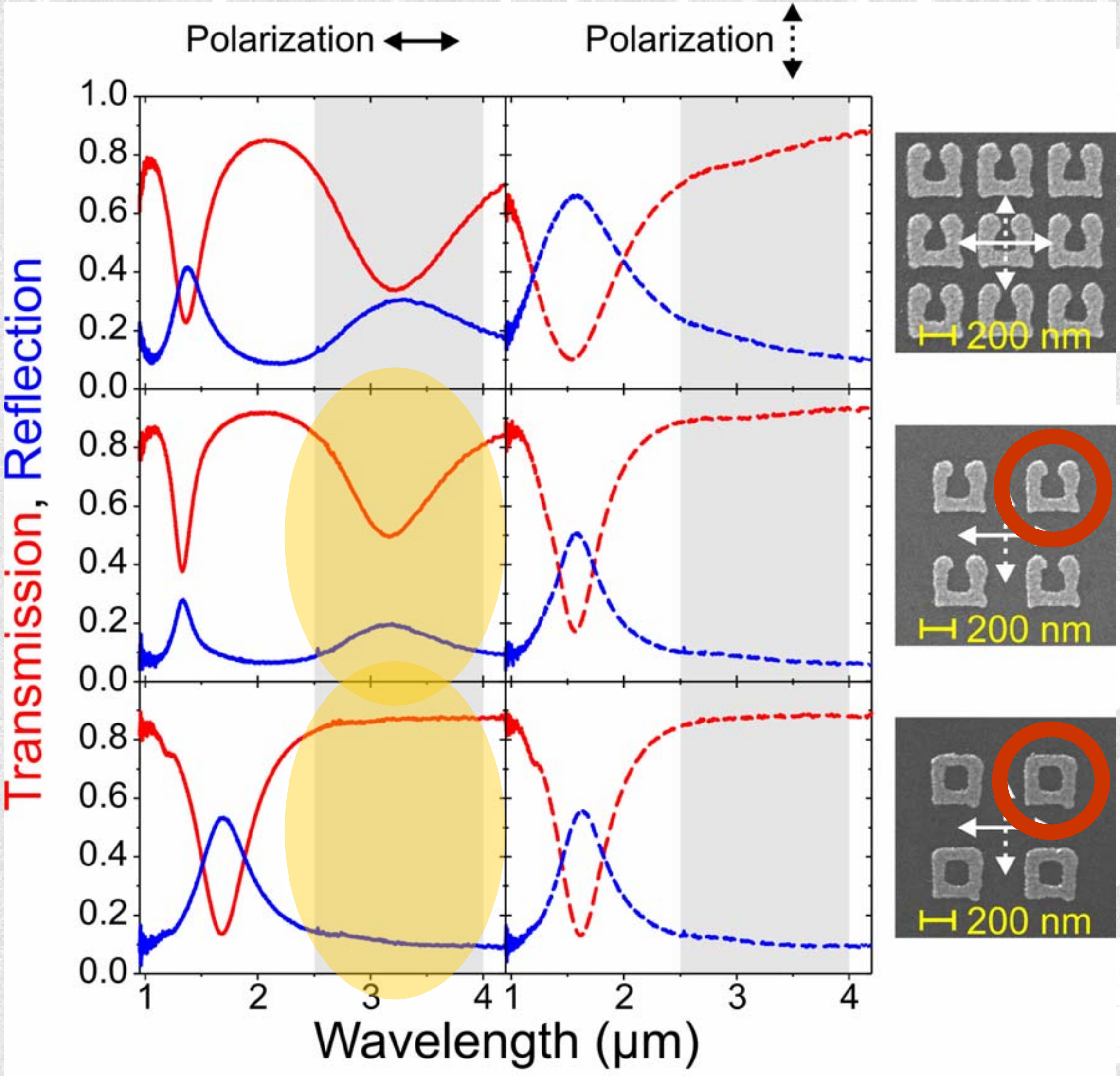


a=450 nm

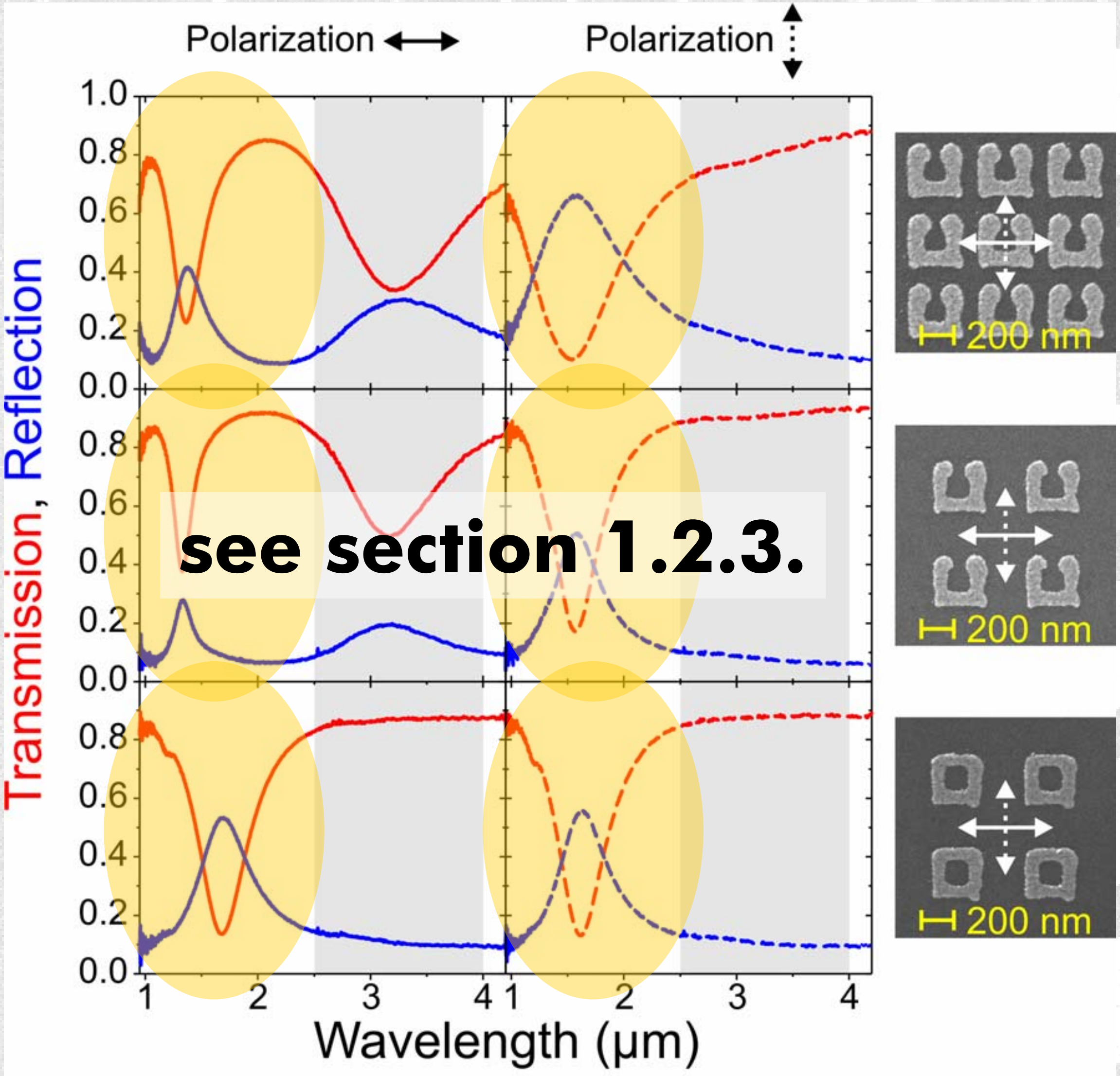
a=600 nm

a=900 nm

Science 306, 1351 (2004)

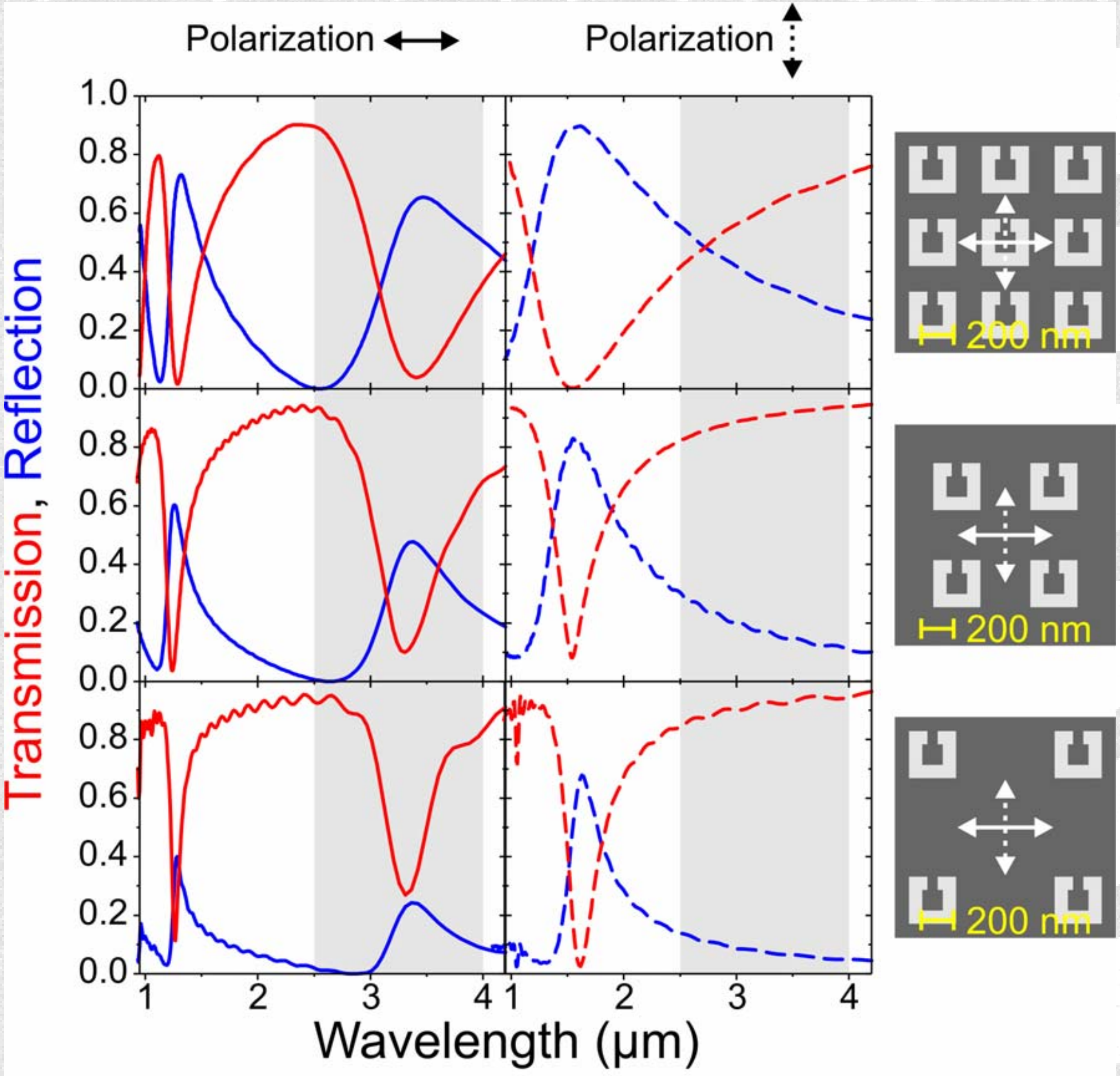


Science 306, 1351 (2004)

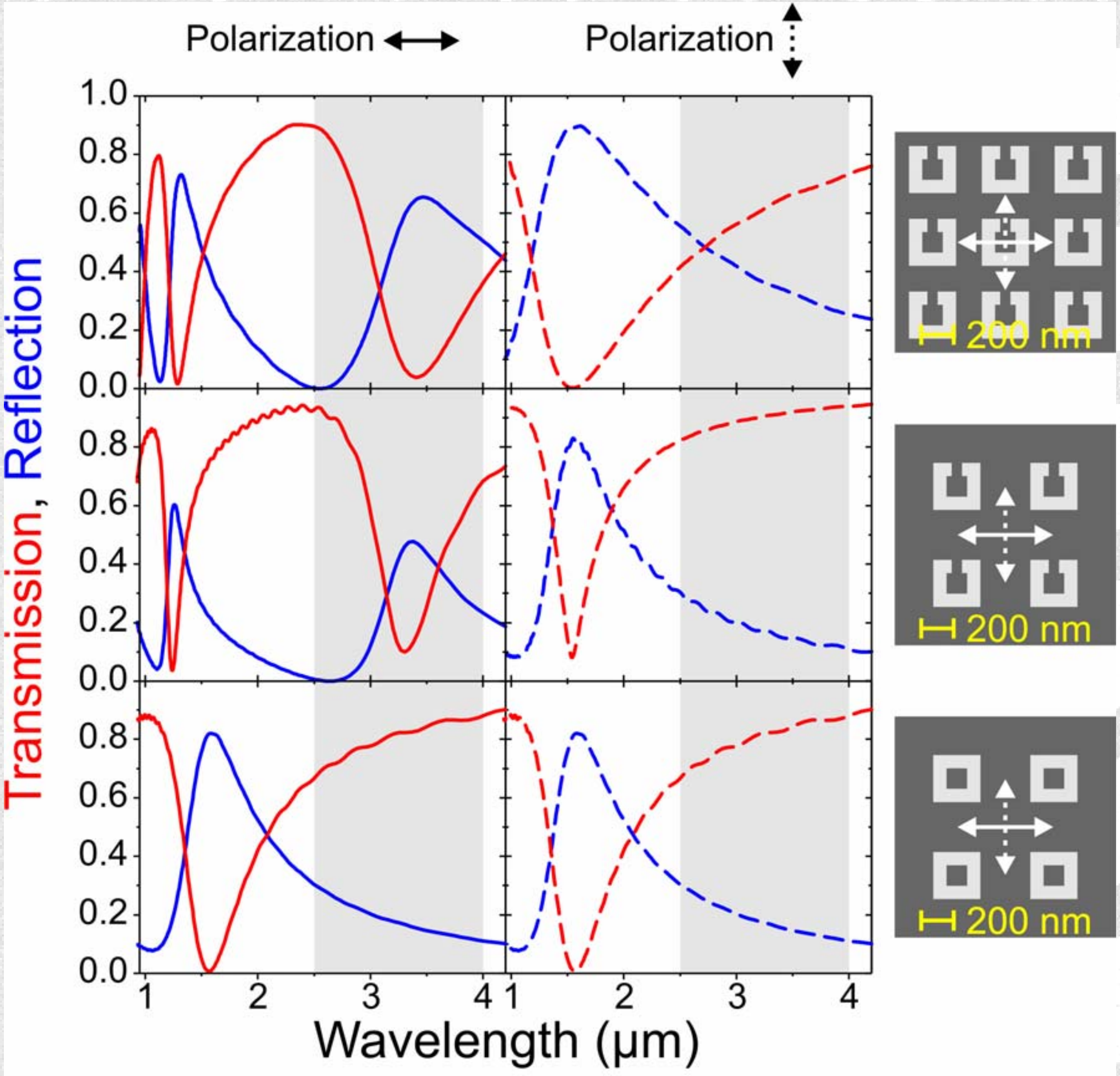


see section 1.2.3.

Science 306, 1351 (2004)



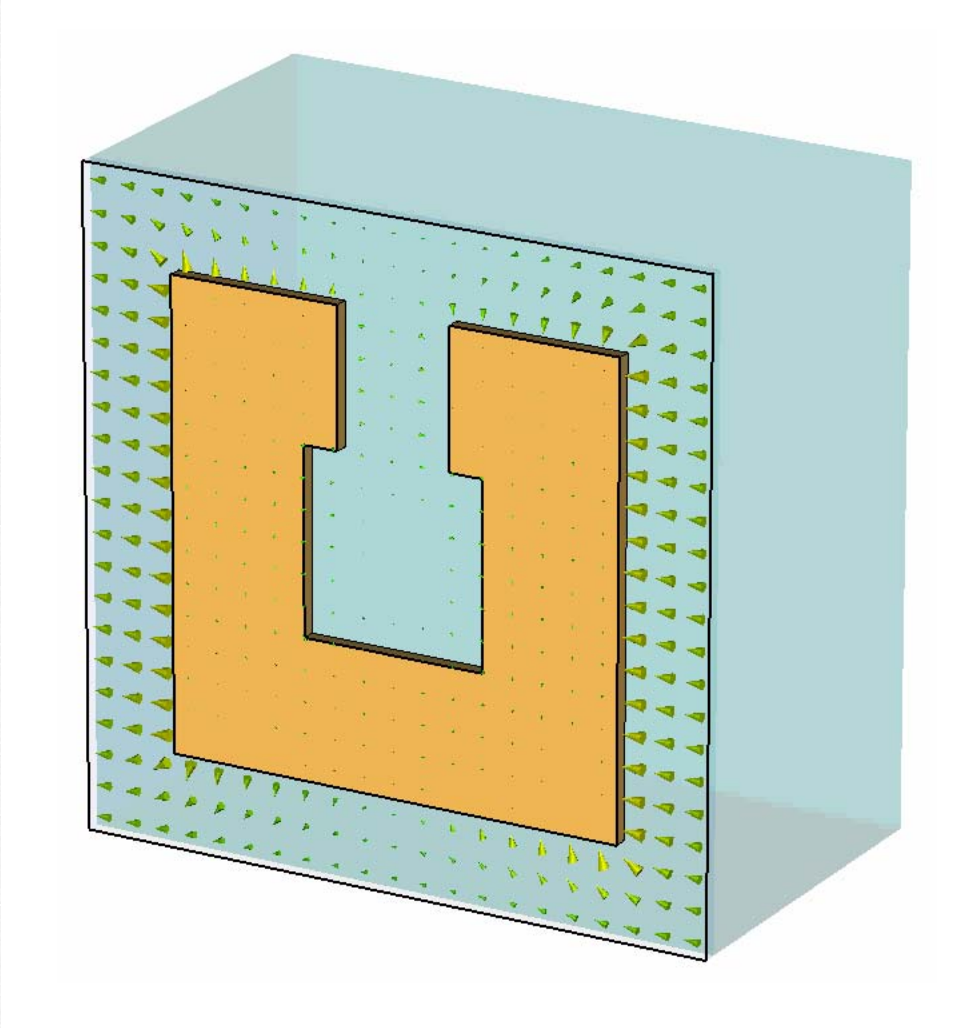
Theory: Iowa State



Theory: Iowa State

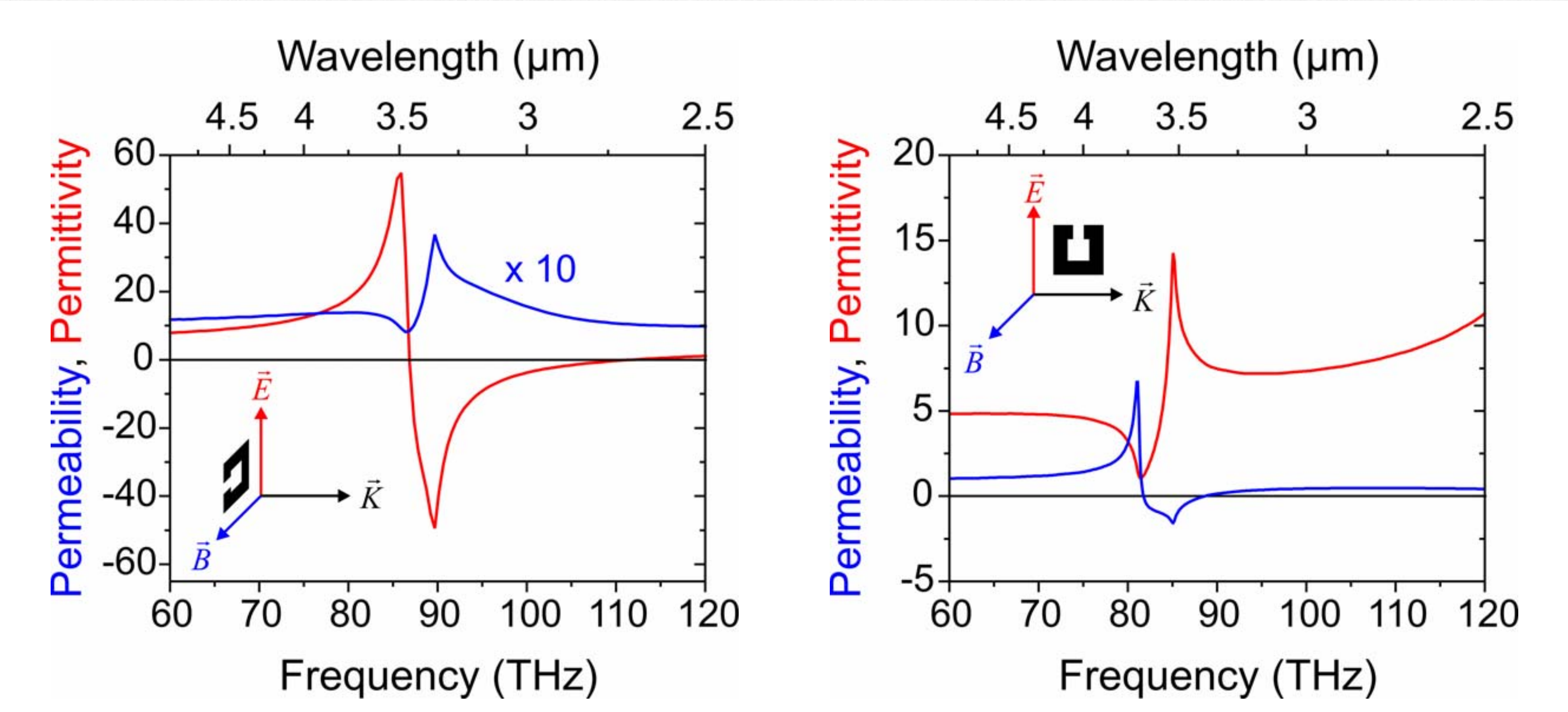
Numerical calculations (E-field)

@ fundamental magnetic resonance



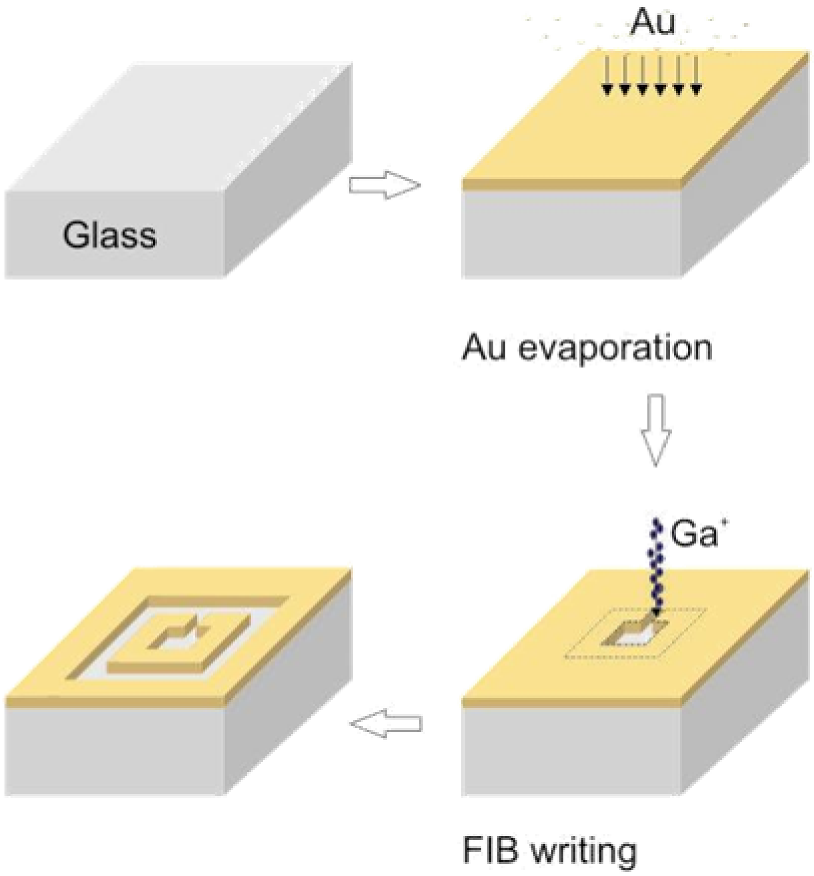
3D-FDTD calculations with *CST Microwave Studio*

Retrieved metamaterial parameters



Jiangfeng Zhou, Thomas Koschny & Costas M. Soukoulis

Focused-ion-beam writing (FIB)



Christian Enkrich

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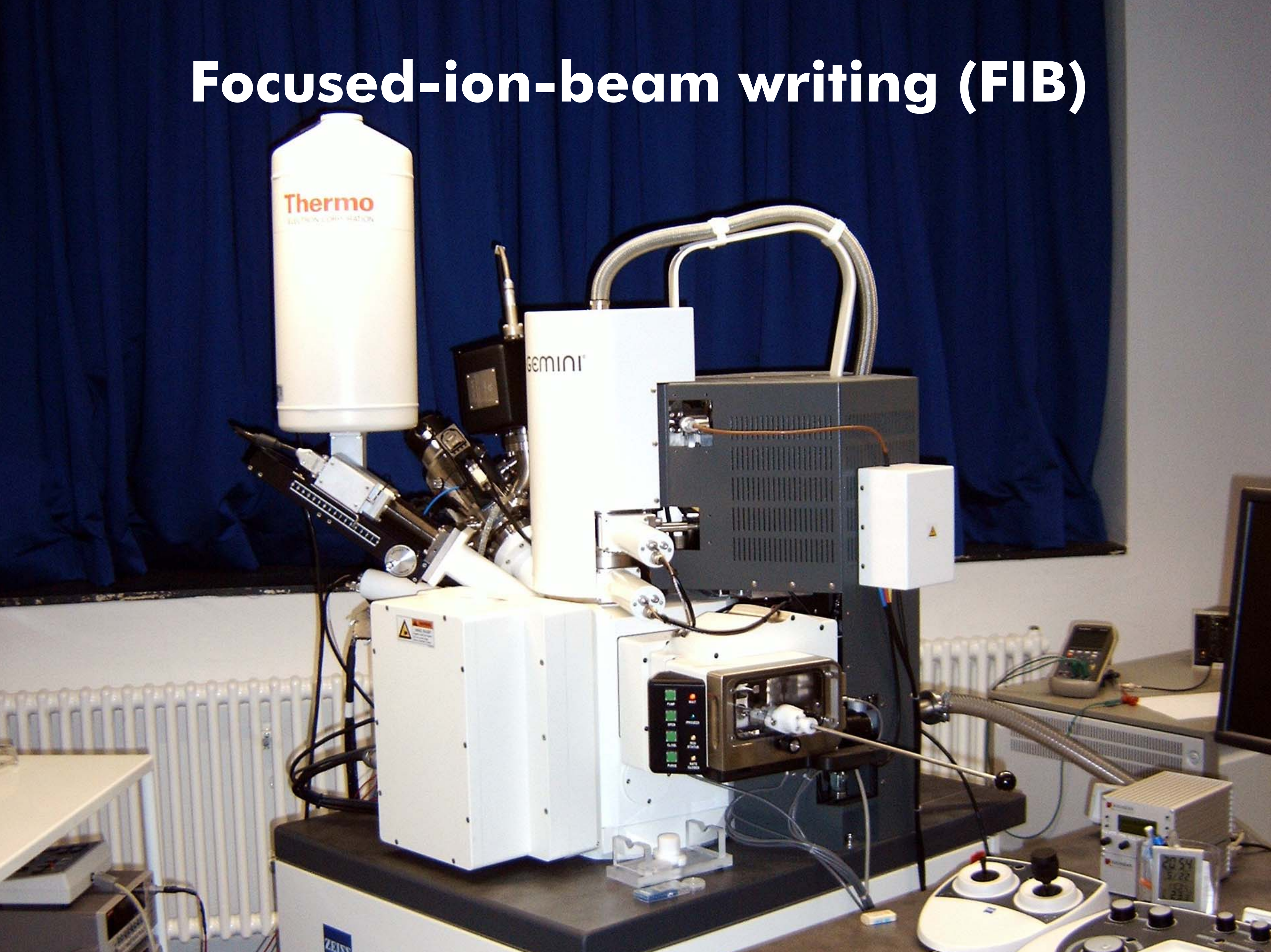
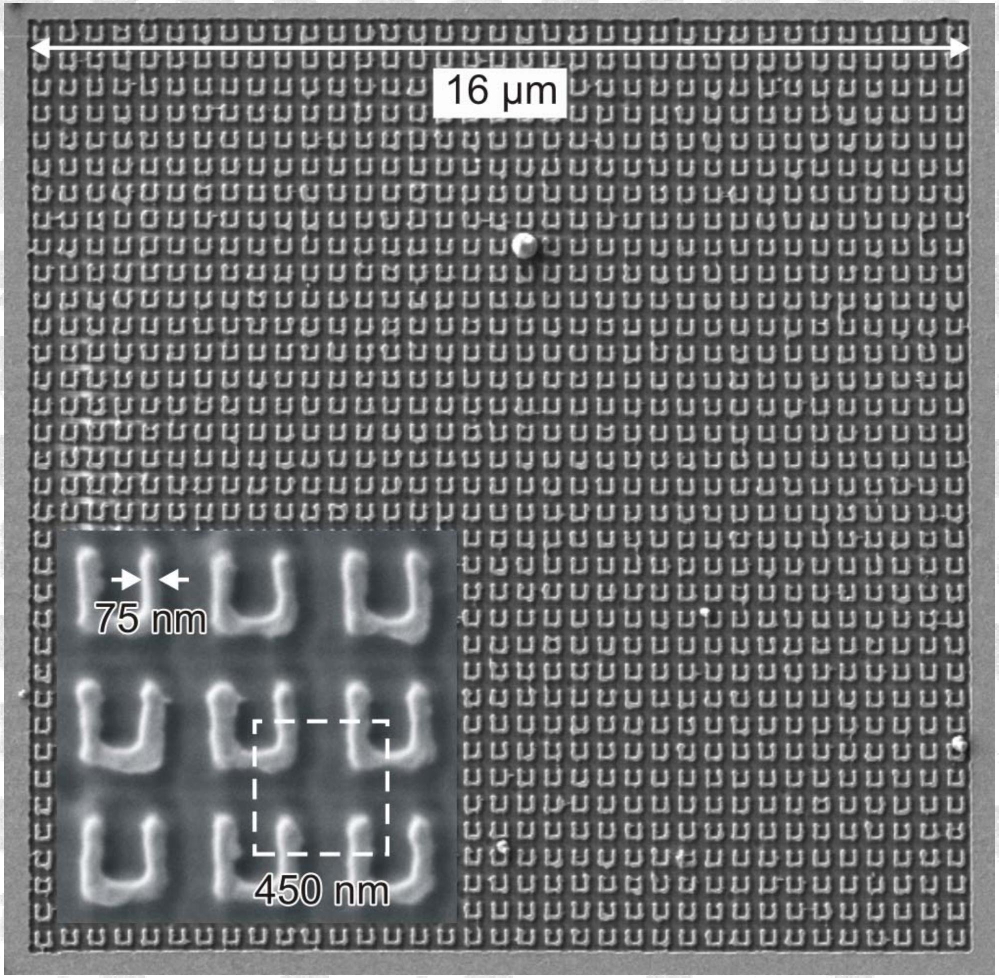
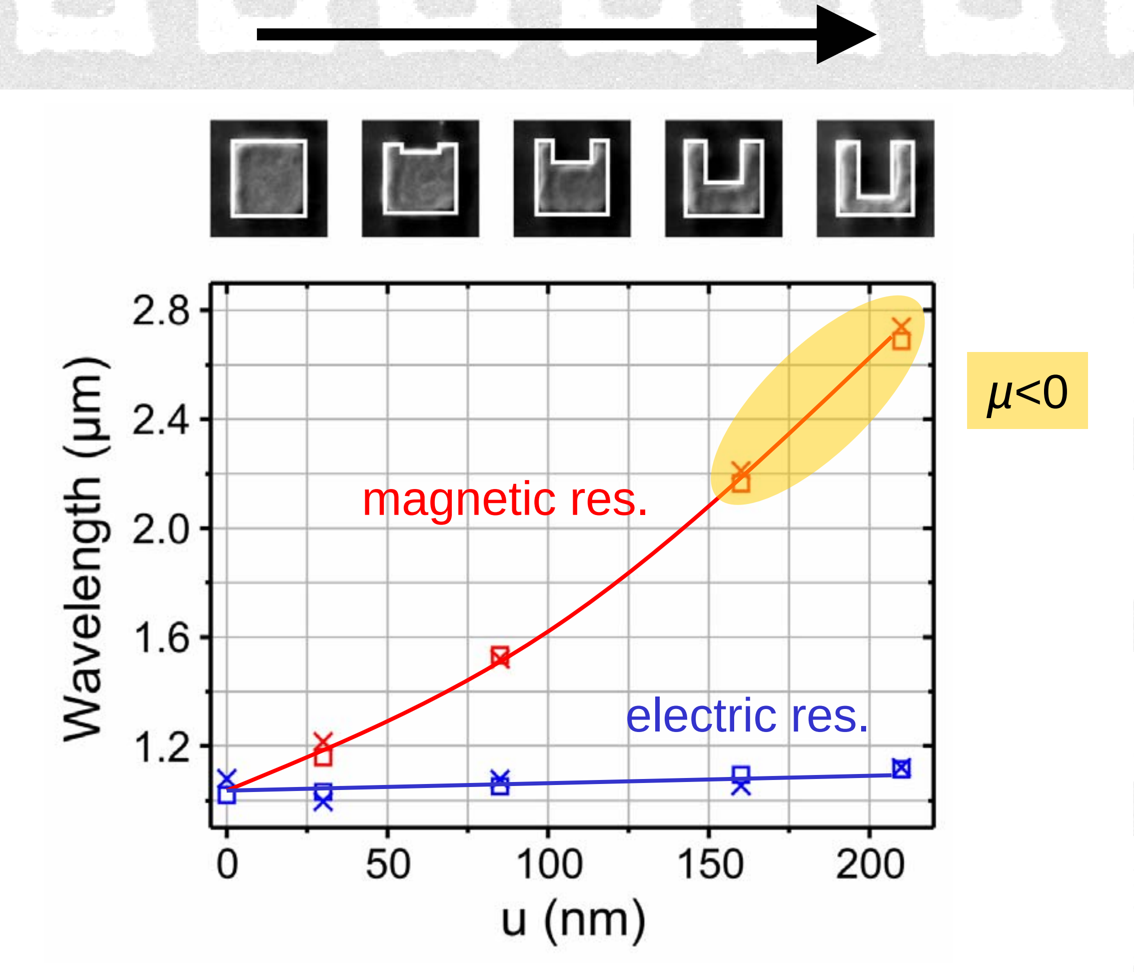


“Rapid prototyping“ with FIB



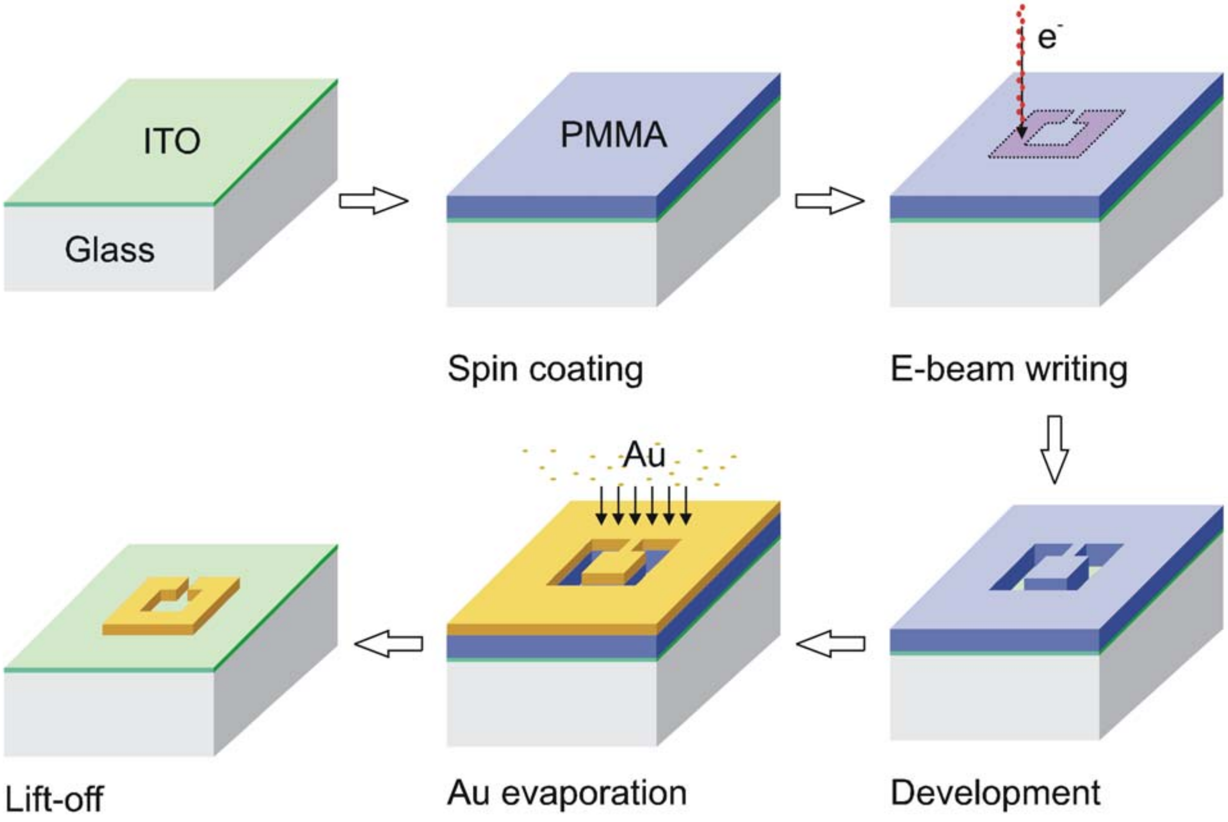
20 min. fabrication time

Mie resonance - SRR

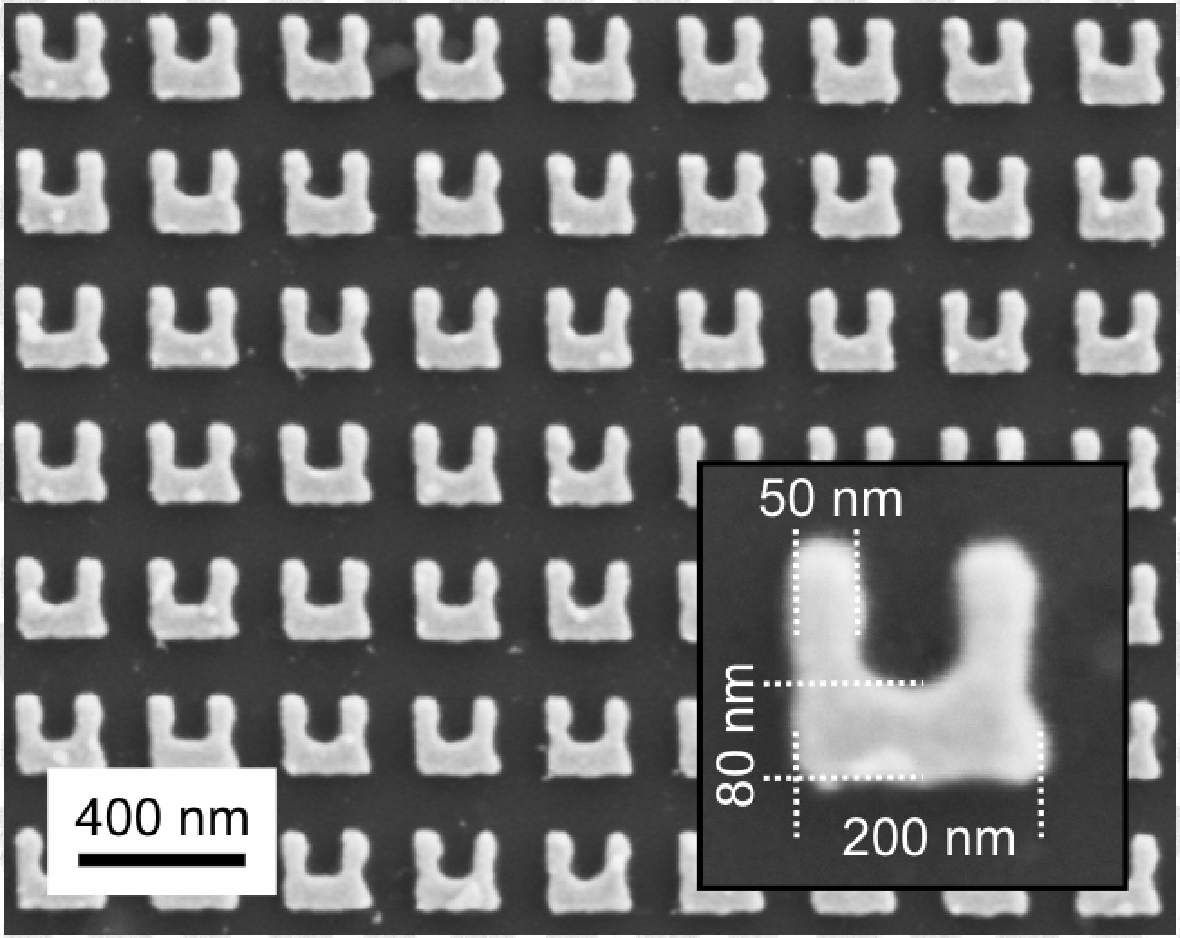


C. Enkrich et al., Adv. Mater. 17, 2547 (2005)

Electron-beam lithography

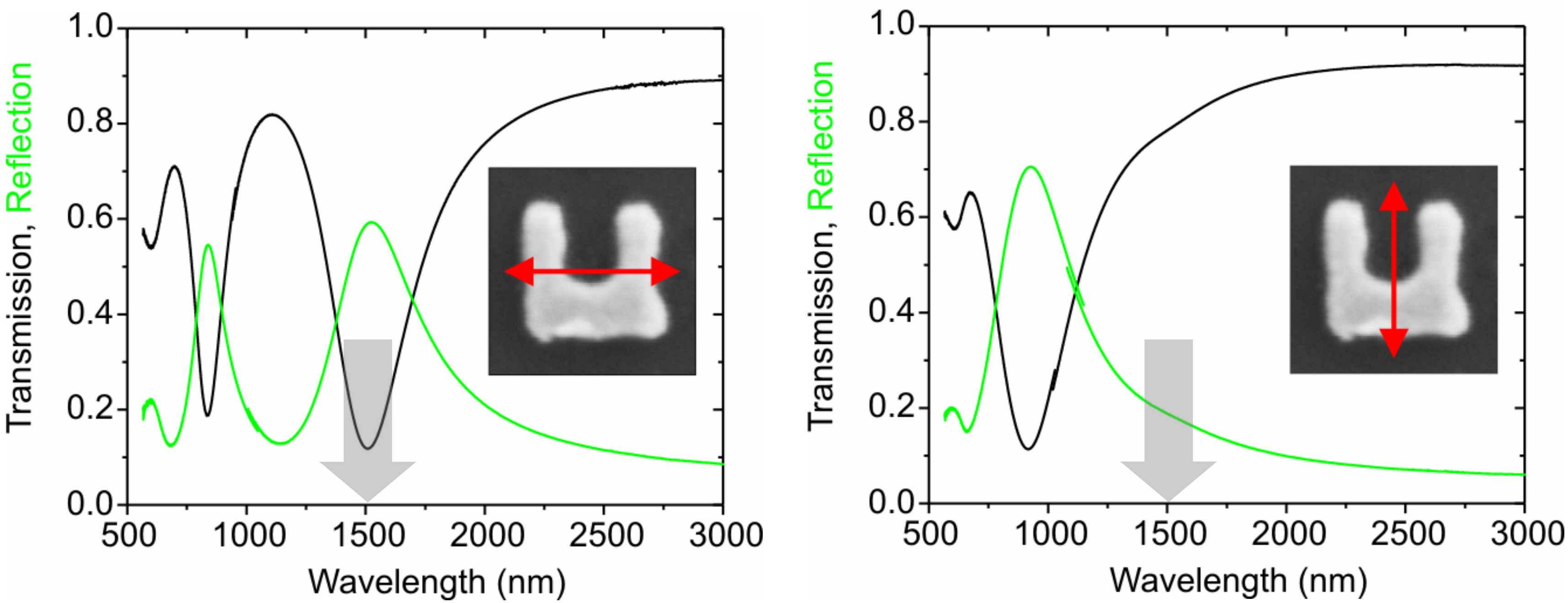


Magnetic resonance @ 1.5 μm



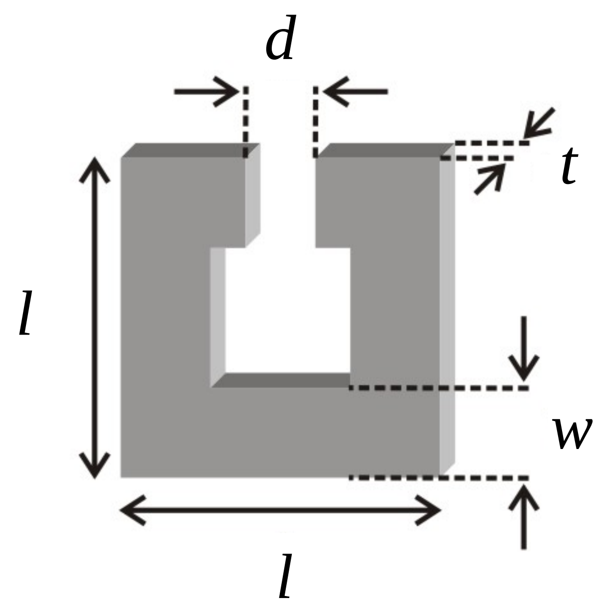
30nm Au on glass, e-beam lithography

Magnetic resonance @ 1.5 μm



retrieval yields $\mu<0$ @ 1.5 μm

SRR size scaling for ever?



$$L = \mu_0 \frac{l^2}{t} \propto \text{size}$$

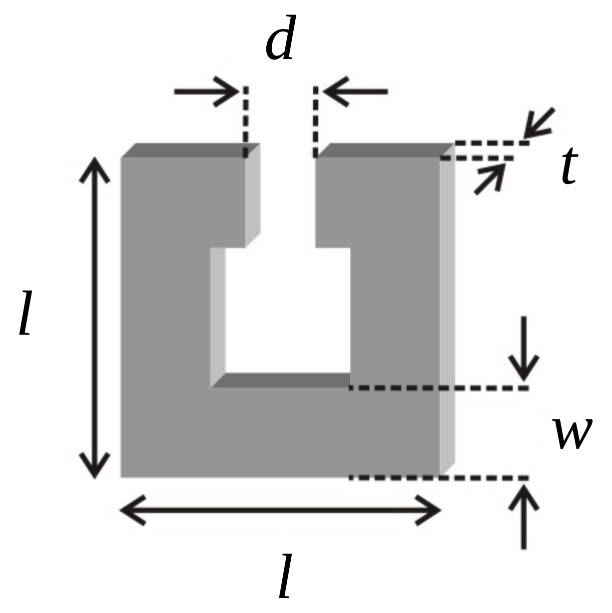
$$C = \varepsilon_0 \varepsilon_C \frac{wt}{d} \propto \text{size}$$

$$\omega_{LC} = \frac{1}{\sqrt{LC}}$$

$$\boxed{\Rightarrow \omega_{LC} \propto \frac{1}{\text{size}}}$$

H.J. Schneider and P. Dullenkopf, Rev. Sci. Instrum. 48, 68 (1977)

Limits of size scaling



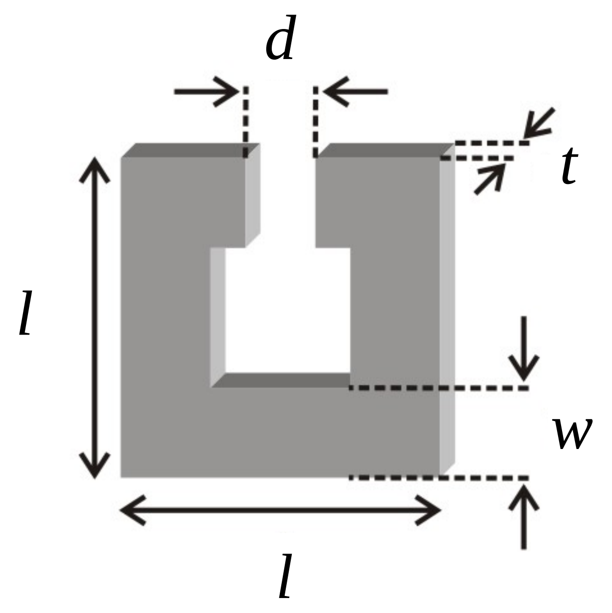
$$E_{\text{kin}} = N_{\text{e}} \frac{m_{\text{e}}}{2} v_{\text{e}}^2 = \frac{1}{2} L_{\text{kin}} I^2$$

$$L_{\text{kin}} = \frac{m_{\text{e}}}{n_{\text{e}}\, \text{e}^2} \frac{4(l-w)-d}{wt}$$

$$\boxed{\Rightarrow \omega_{LC} \propto \frac{1}{\sqrt{\text{size}^2 + \text{const.}}}}$$

J. Zhou et al., Phys. Rev. Lett. 95, 223902 (2005)

Limits of size scaling



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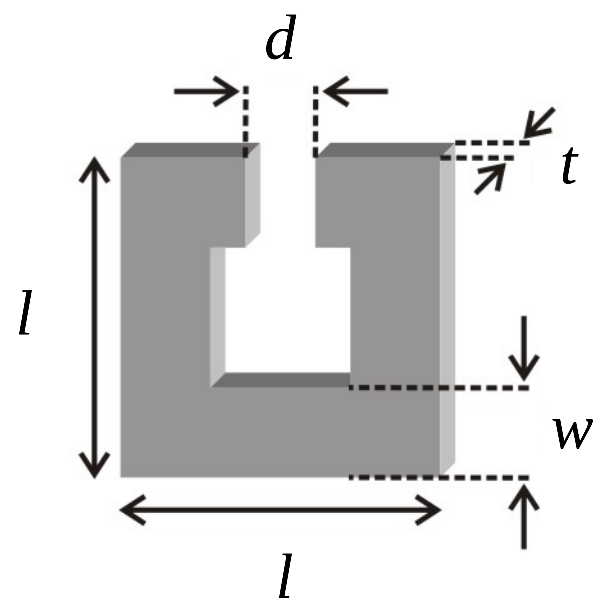
$$L_{\mathrm{kin}} = \frac{m_{\mathrm{e}}}{n_{\mathrm{e}}\,\mathrm{e}^2} \frac{4(l-w)-d}{wt}$$

$$\Rightarrow \omega_{LC}^{\max} \approx \omega_{\mathrm{pl}} \sqrt{\frac{d}{4l}}$$

A. Ishikawa et al., Phys. Rev. Lett. 95, 237401 (2005)

Ohmic damping & size scaling



$$L = \mu_0 \frac{l^2}{t} \propto \text{size}$$

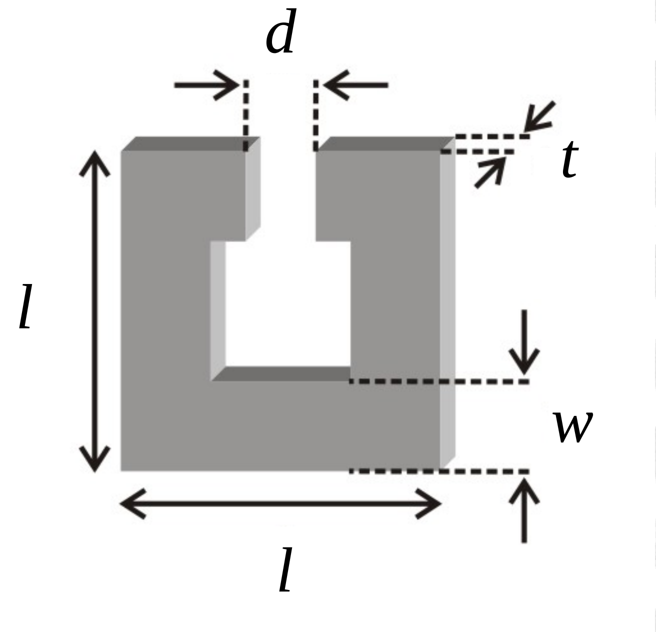
$$C = \varepsilon_0 \varepsilon_C \frac{wt}{d} \propto \text{size}$$

$$R = \frac{1}{\sigma} \frac{4(l-w)-d}{wt} \propto \frac{1}{\text{size}}$$

and $$\boxed{\ddot{I} + \frac{R}{L}\dot{I} + \frac{1}{LC} I = 0}$$

Martin Wegener

Ohmic damping & size scaling



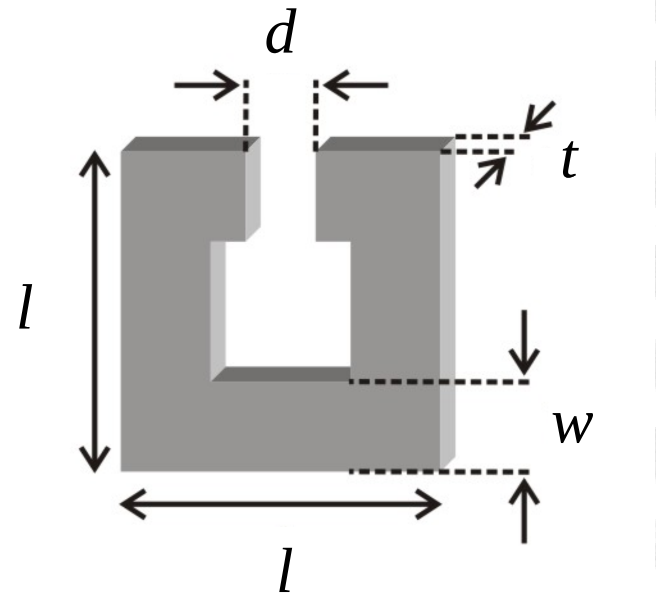
$$L = \mu_0 \frac{l^2}{t} \propto \text{size}$$

$$C = \varepsilon_0 \varepsilon_C \frac{wt}{d} \propto \text{size}$$

$$R = \frac{1}{\sigma} \frac{4(l-w)-d}{wt} \propto \frac{1}{\text{size}}$$

$$\boxed{\Rightarrow \frac{\text{damping}}{\text{frequency}} = \frac{R/L}{\dfrac{1}{\sqrt{LC}}} \propto \frac{1}{\text{size}}}$$

Size scaling, constant thickness



$$L = \mu_0 \frac{l^2}{t} \propto (\text{size})^2$$

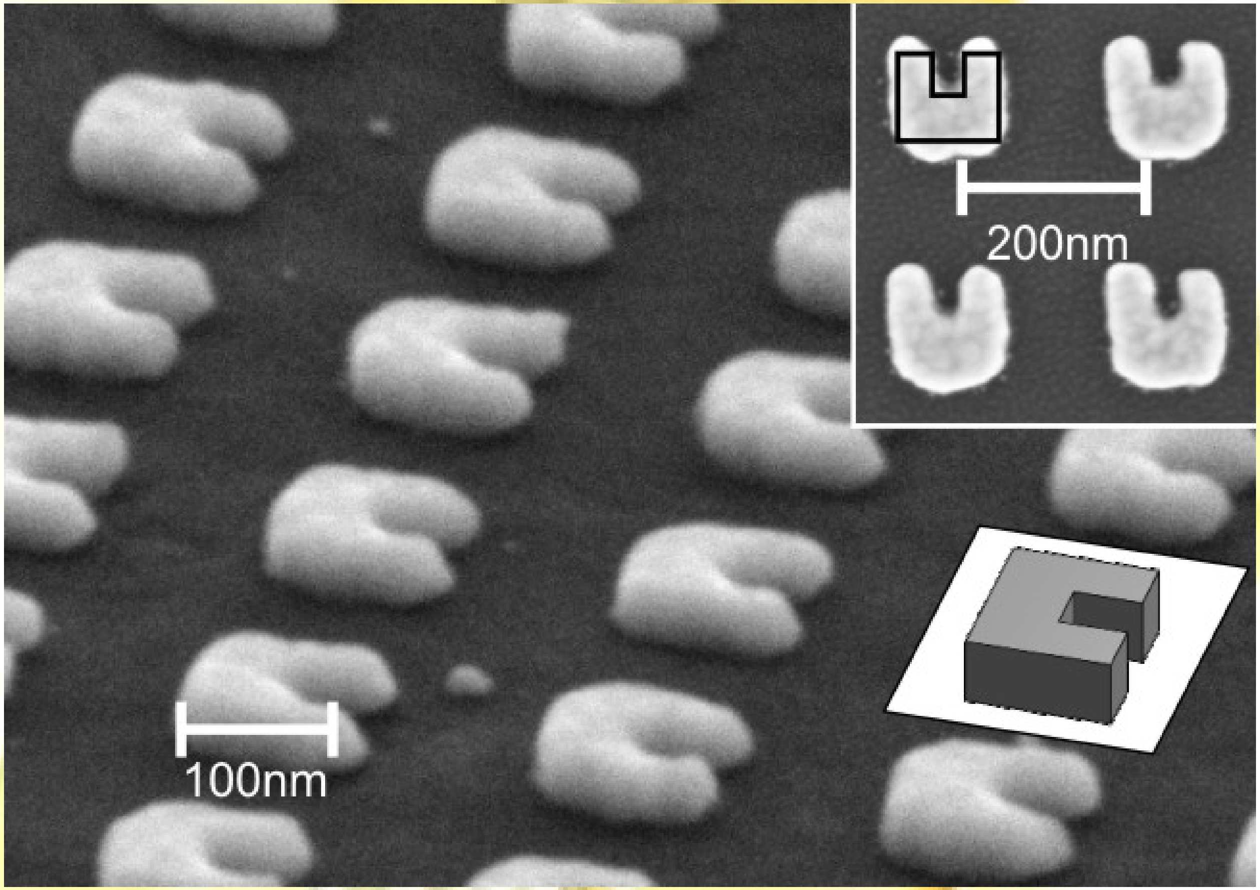
$$C = \varepsilon_0 \varepsilon_C \frac{wt}{d} \propto \text{const.}$$

$$R = \frac{1}{\sigma} \frac{4(l-w)-d}{wt} \propto \text{const.}$$

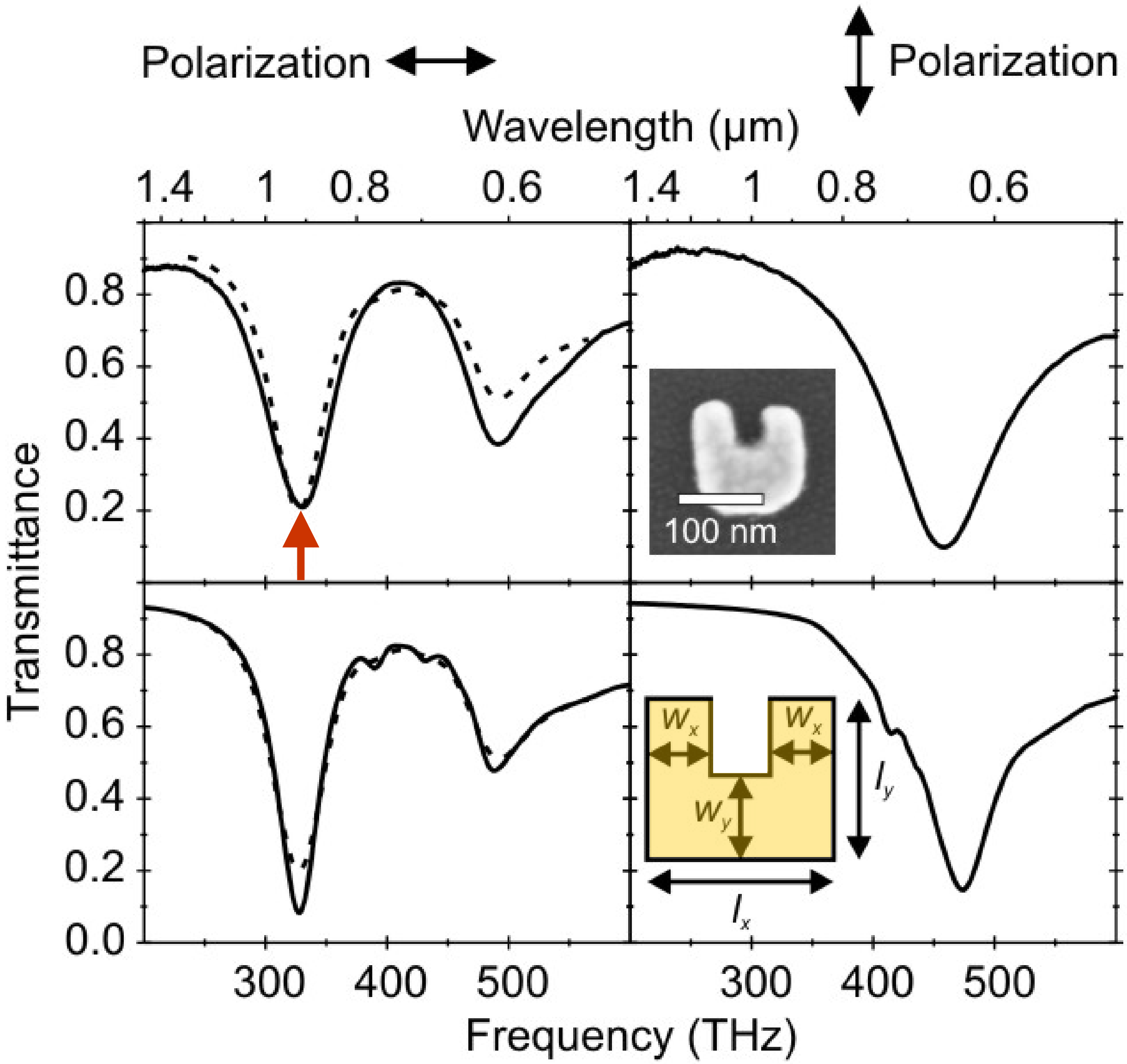
$$\Rightarrow \frac{\text{damping}}{\text{frequency}} = \frac{R/L}{\frac{1}{\sqrt{LC}}} \propto \frac{1}{\text{size}}$$

Martin Wegener

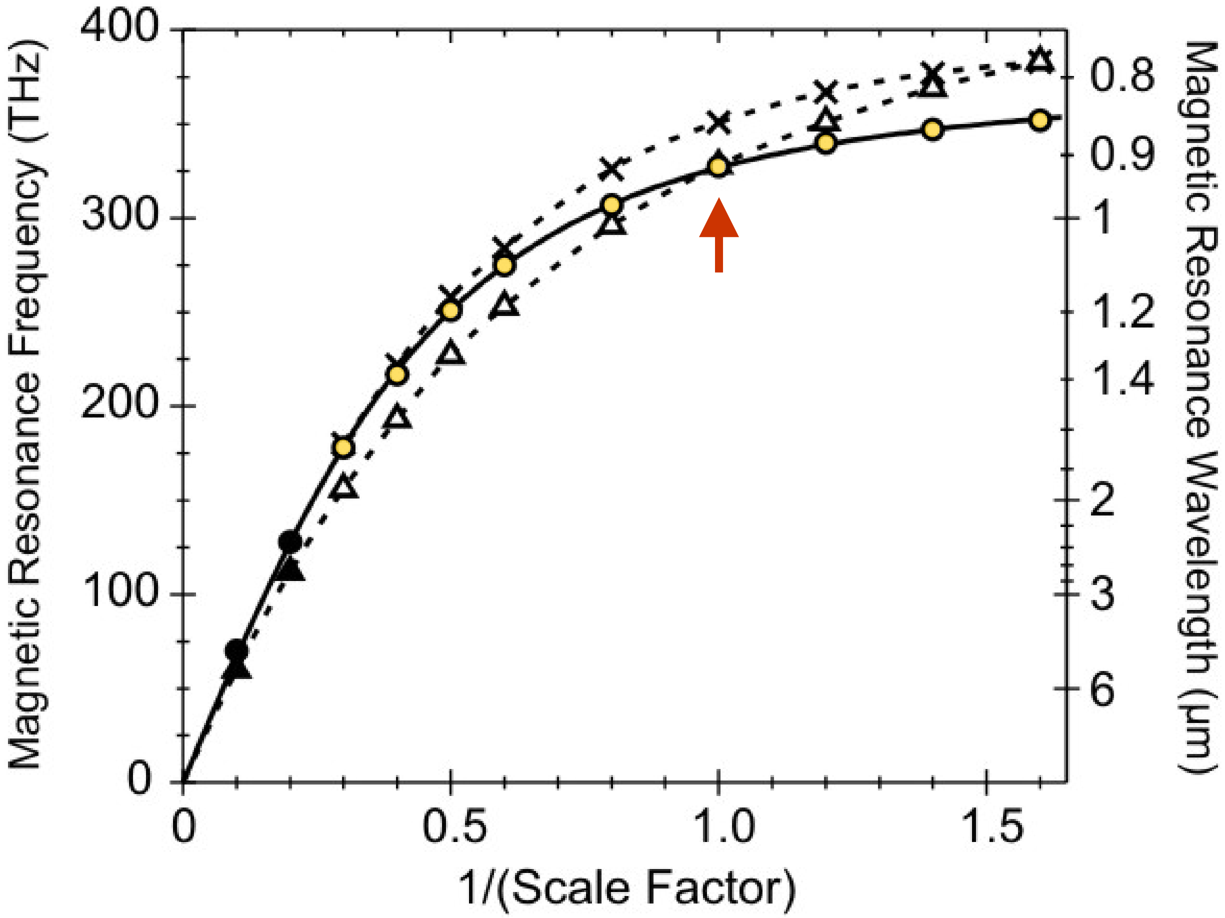
Limits of size scaling



M.W. Klein et al., Opt. Lett. 31, 1259 (2006)

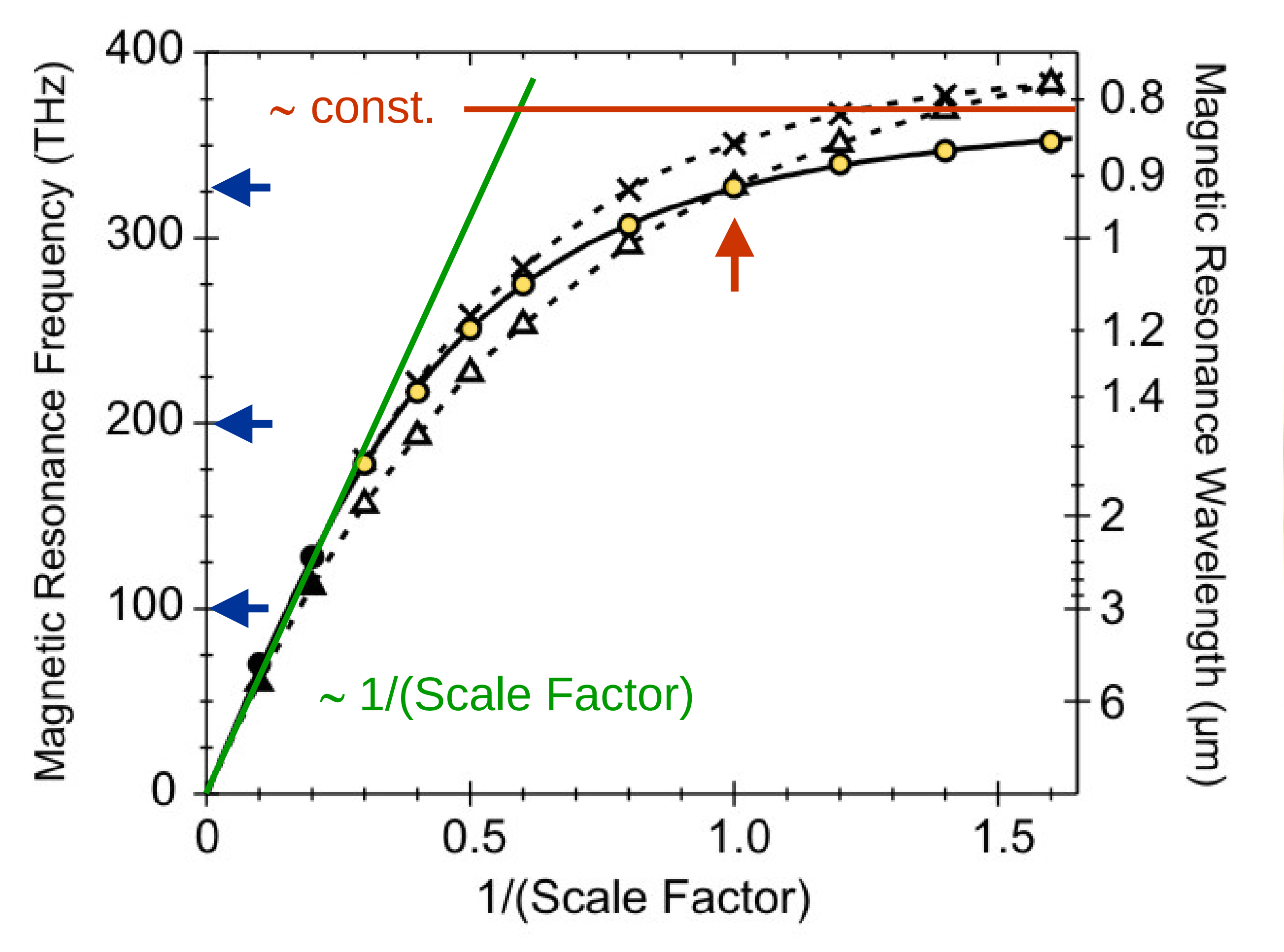

Polarization
Polarization
Wavelength (µm)
1.4
1
0.8
0.6
1.4
1
0.8
0.6
Transmittance
0.8
0.6
0.4
0.2
0.8
0.6
0.4
0.2
0.0
100 nm
w_x
w_x
w_y
l_y
l_x
300
400
500
300
400
500
Frequency (THz)

Limits of size scaling



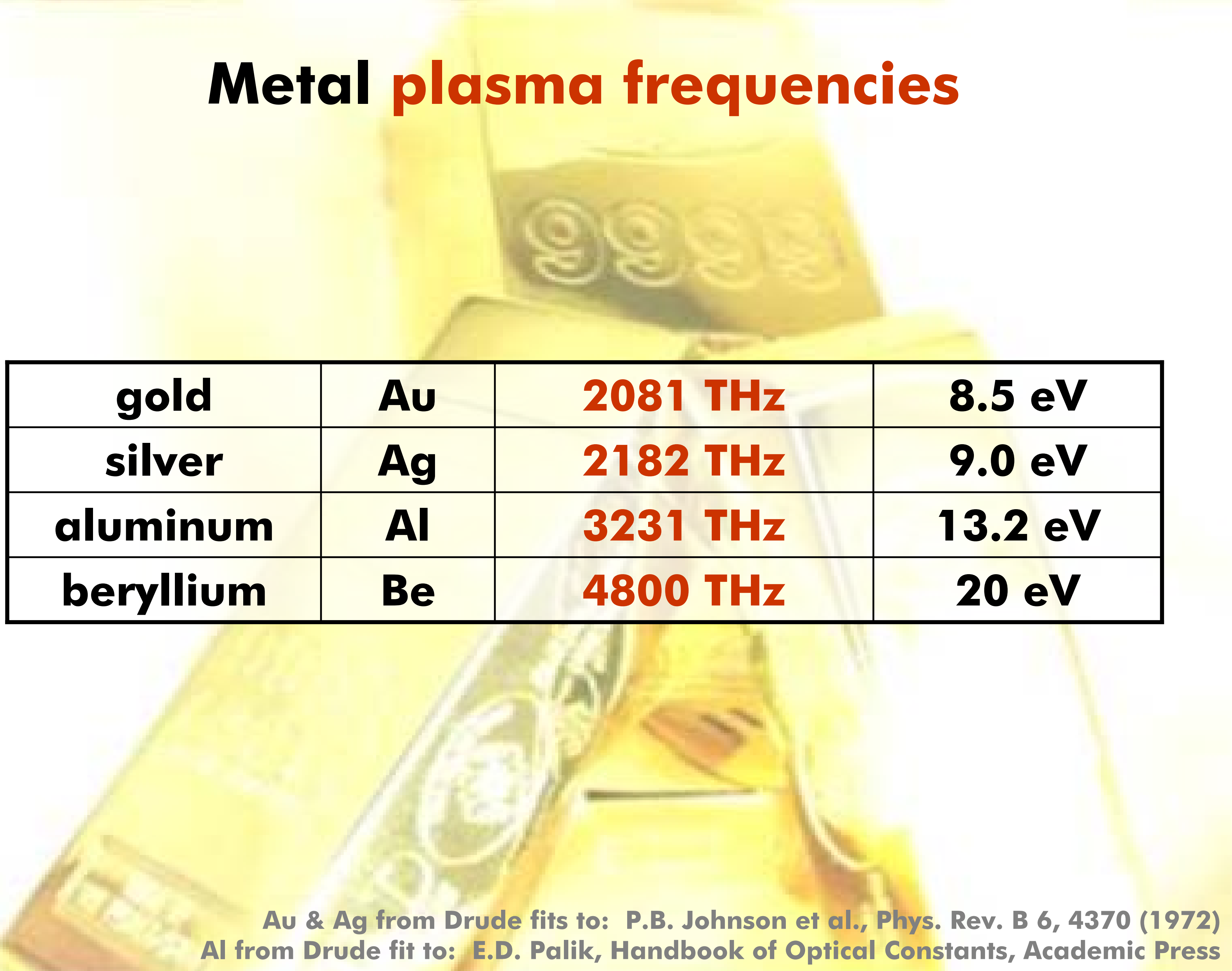
M.W. Klein et al., Opt. Lett. 31, 1259 (2006)

Limits of size scaling



M.W. Klein et al., Opt. Lett. 31, 1259 (2006)

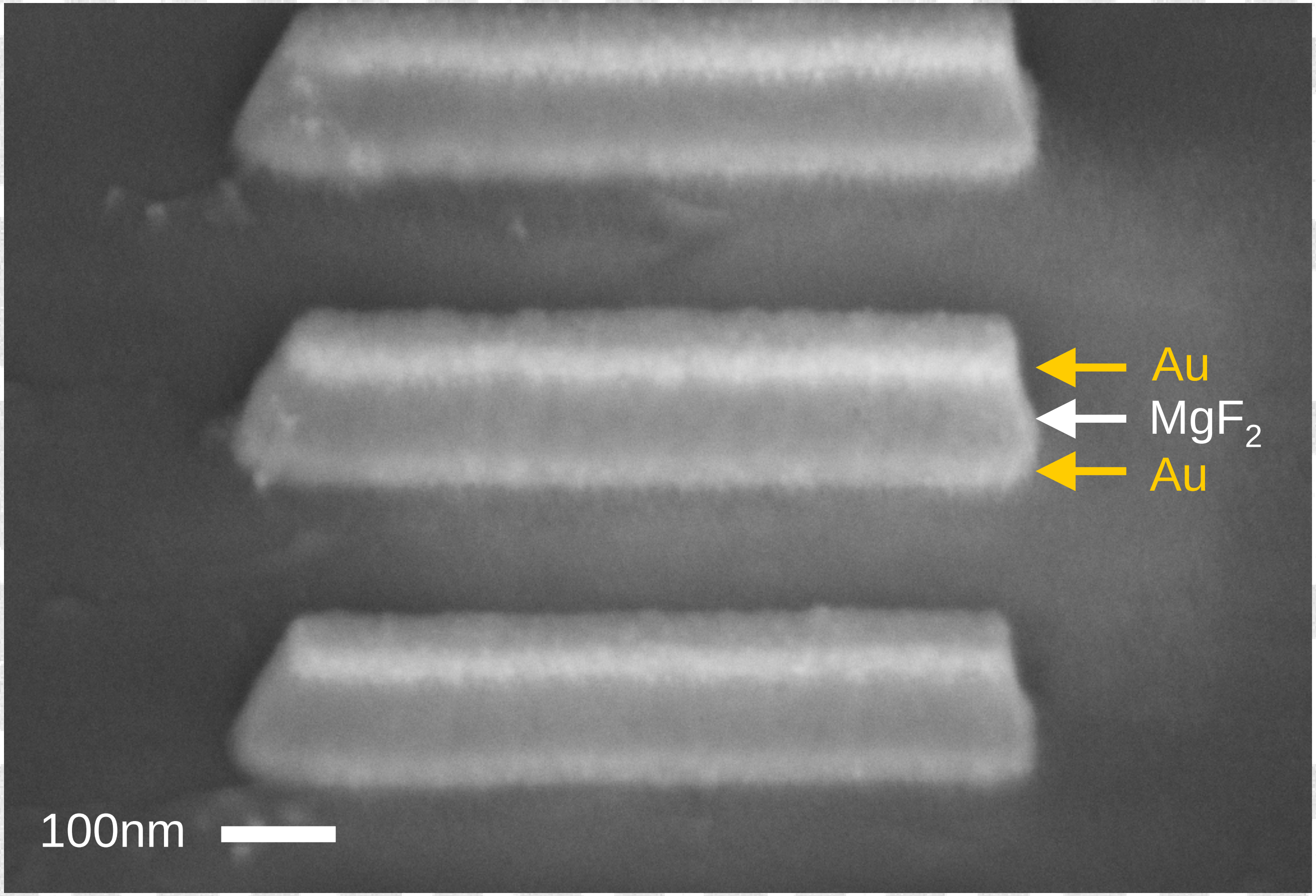
Metal plasma frequencies



gold	Au	2081 THz	8.5 eV
silver	Ag	2182 THz	9.0 eV
aluminum	Al	3231 THz	13.2 eV
beryllium	Be	4800 THz	20 eV

Au & Ag from Drude fits to: P.B. Johnson et al., Phys. Rev. B 6, 4370 (1972)
Al from Drude fit to: E.D. Palik, Handbook of Optical Constants, Academic Press

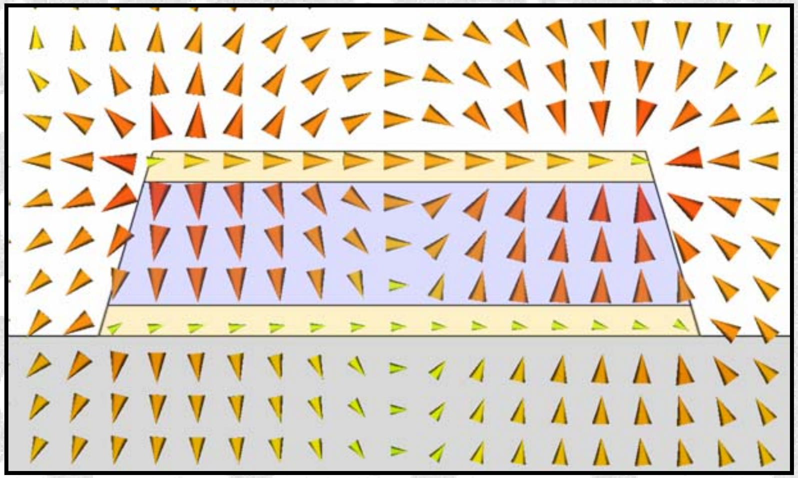
... brings us to cut-wire pairs



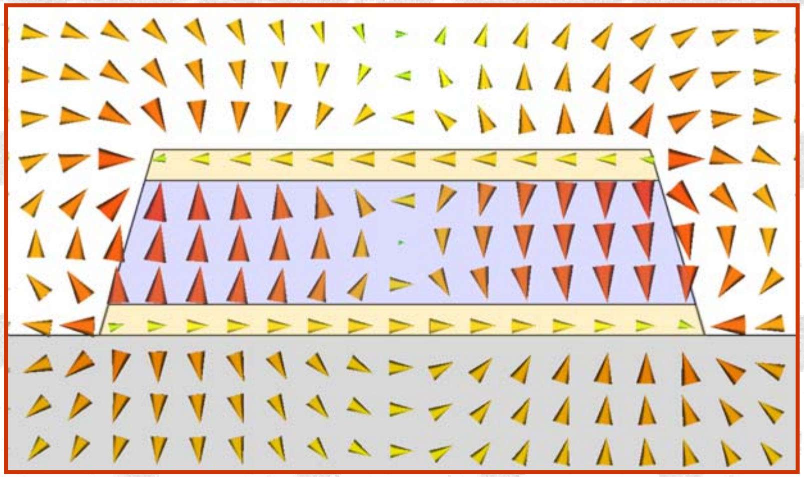
Theory: A.N. Lagarkov et al., Phys. Rev. B 53, 6318 (1996)

Calculated electric fields

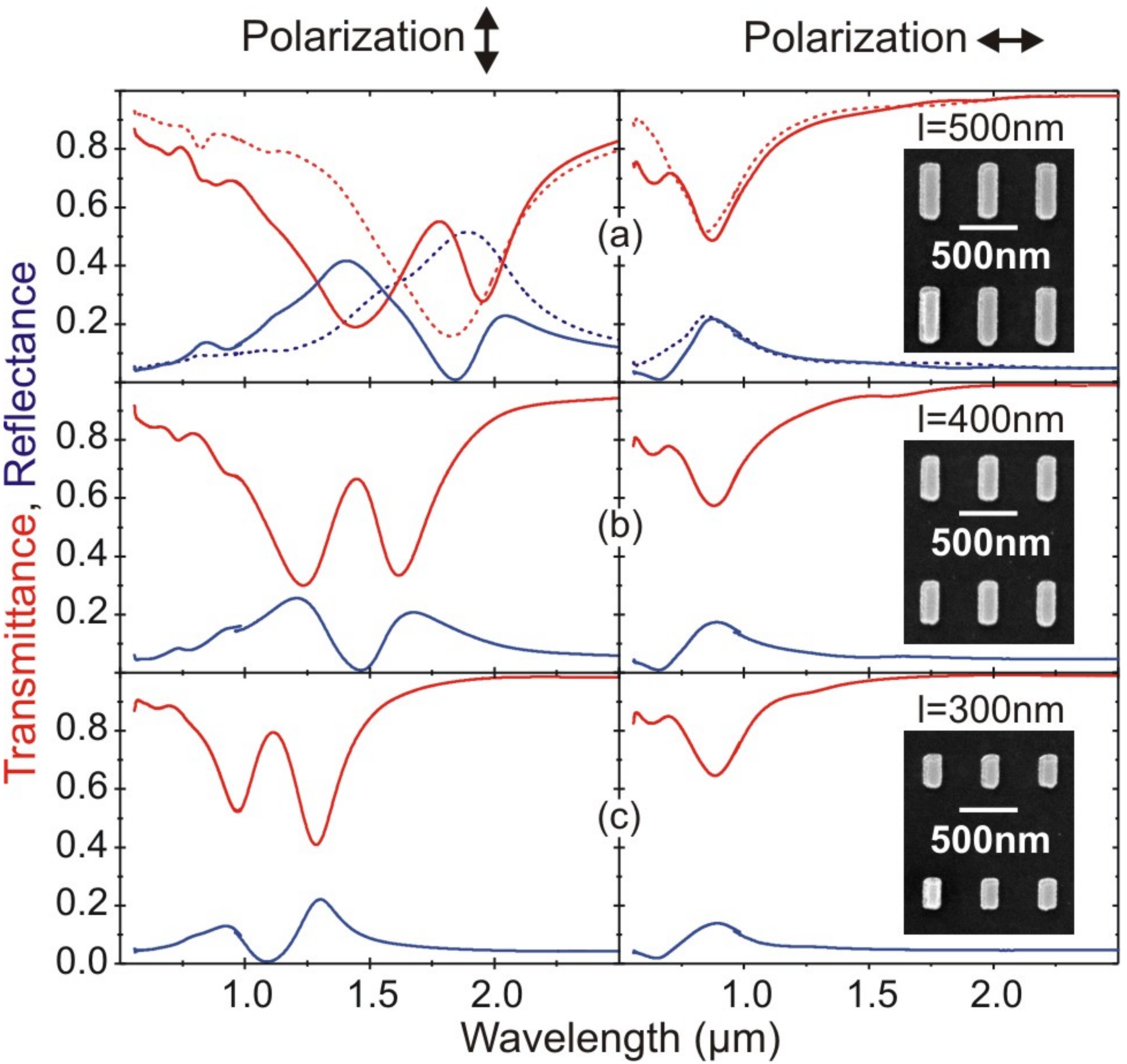
Short-wavelength resonance



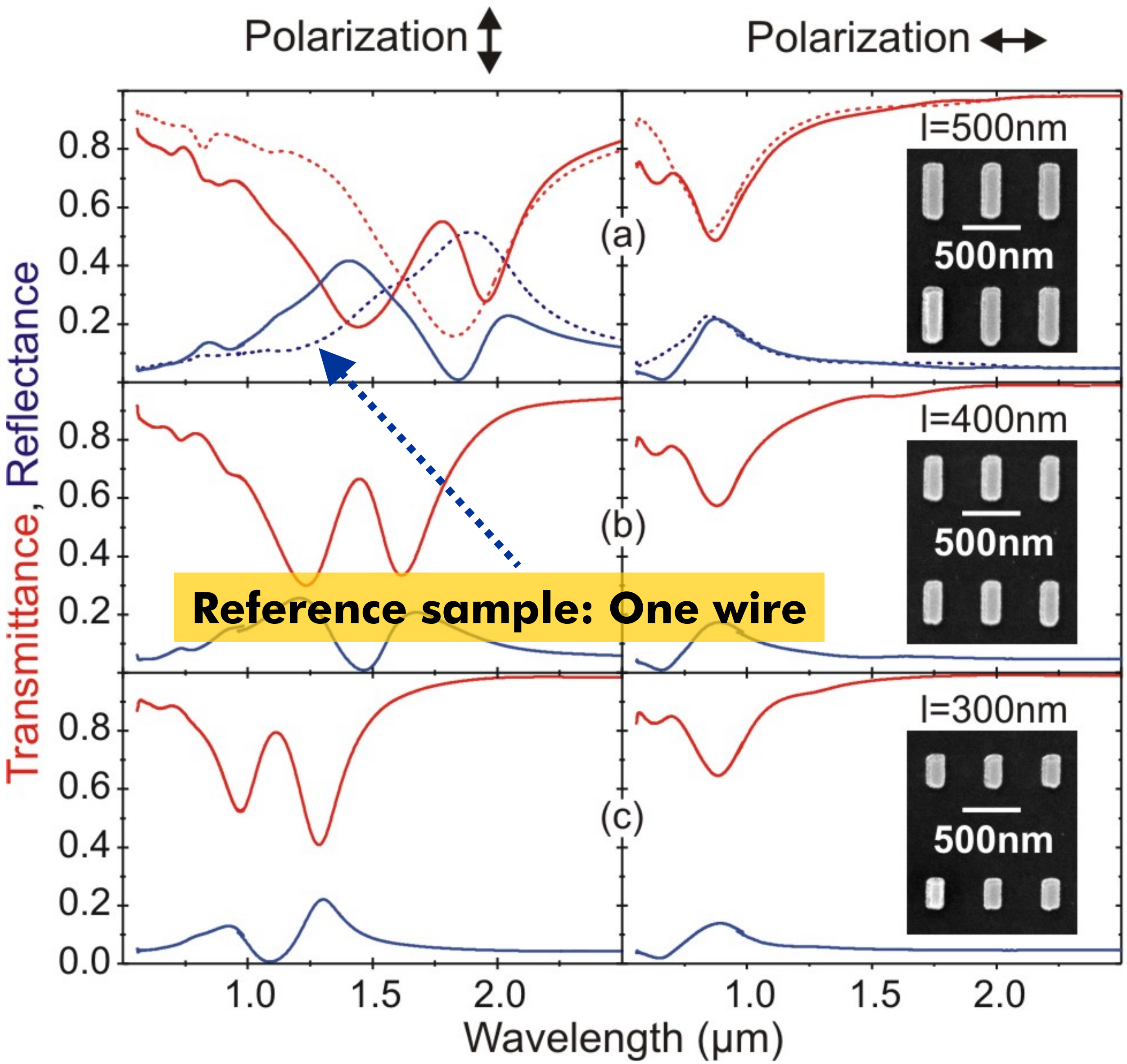
Long-wavelength resonance



l=500nm, w=150nm, d=80nm

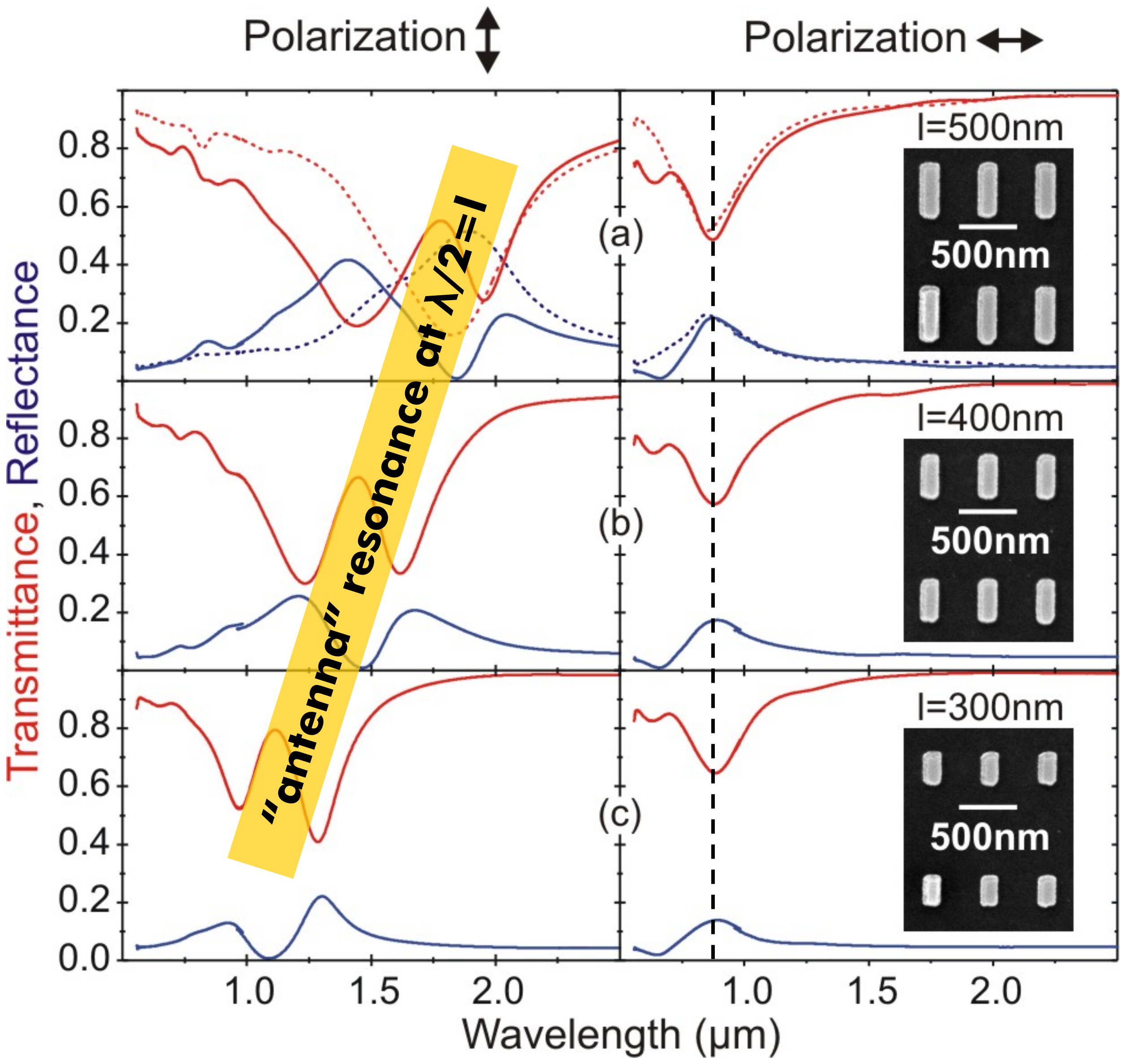


fixed: w=150nm, d=80nm



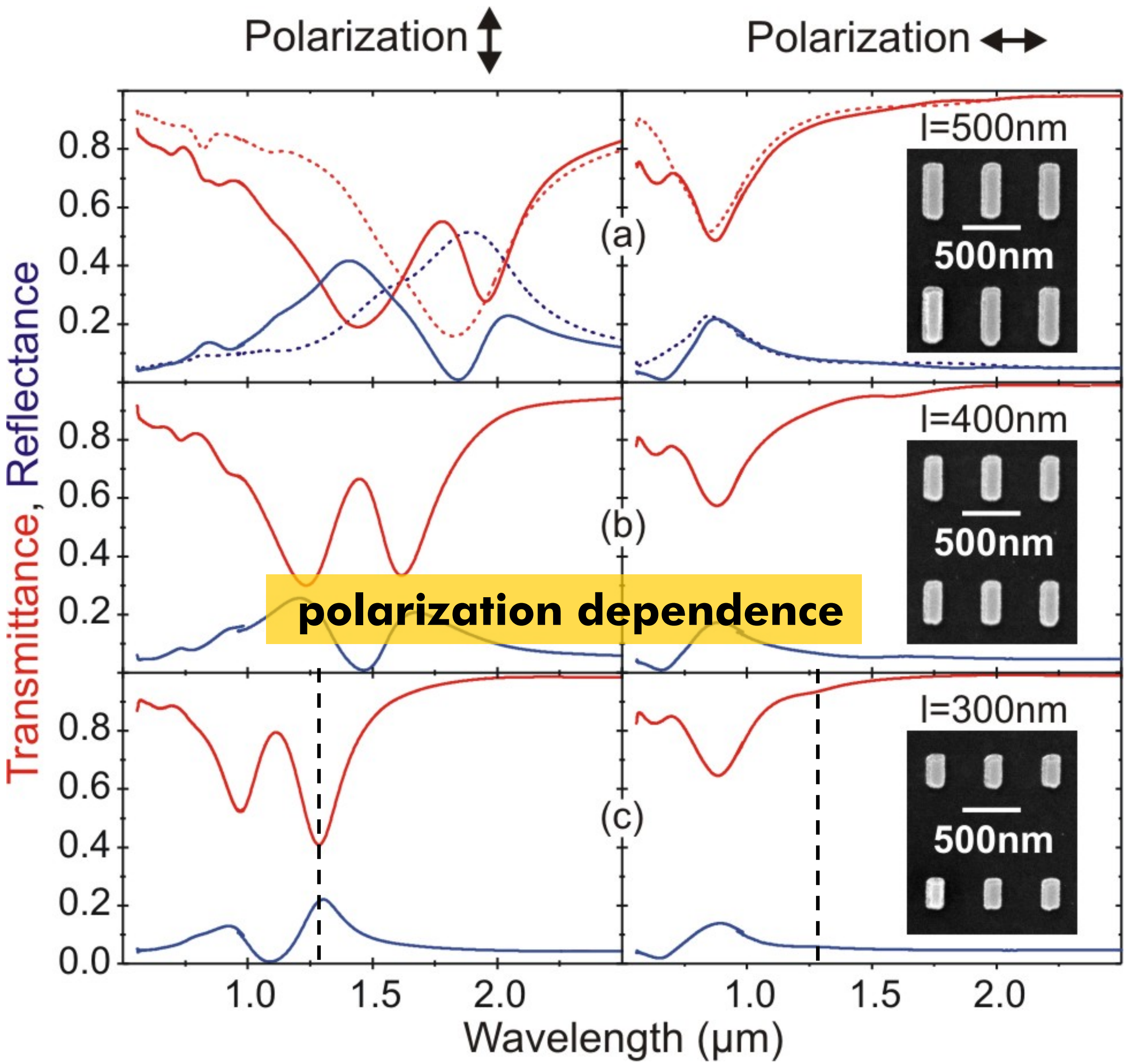
Reference sample: One wire

fixed: w=150nm, d=80nm



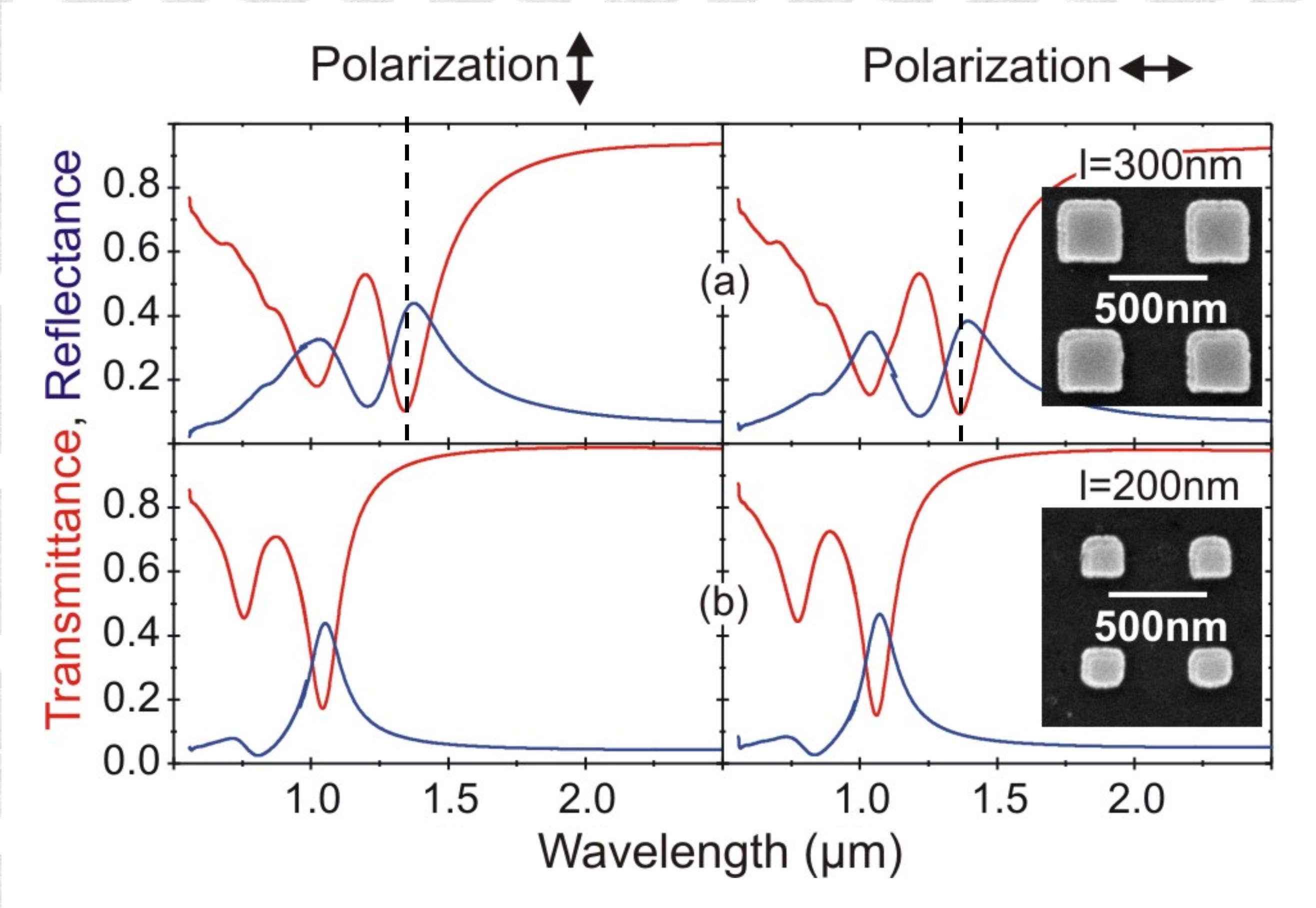
"antenna" resonance at λ/2=l

fixed: w=150nm, d=80nm

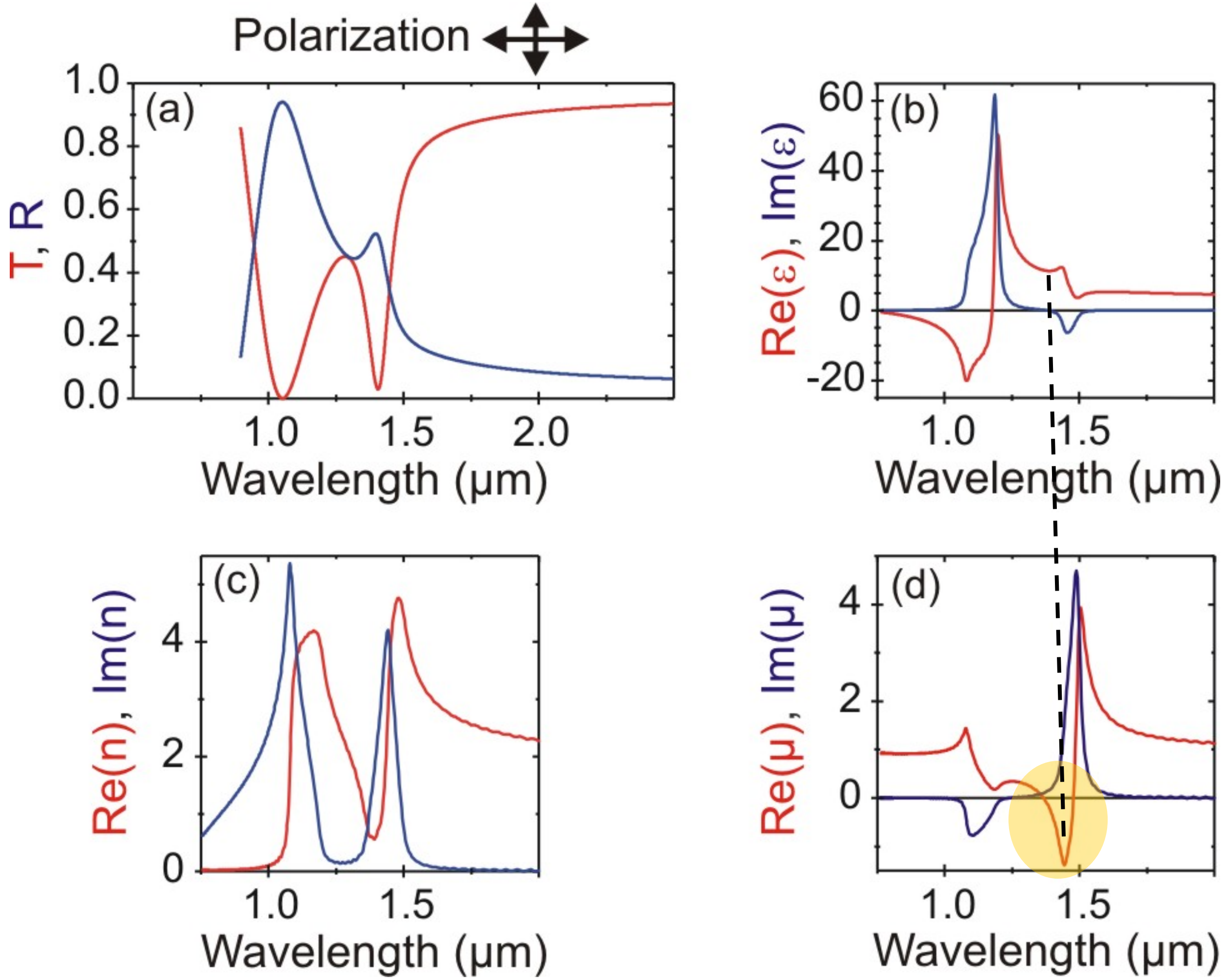


polarization dependence

fixed: w=150nm, d=80nm

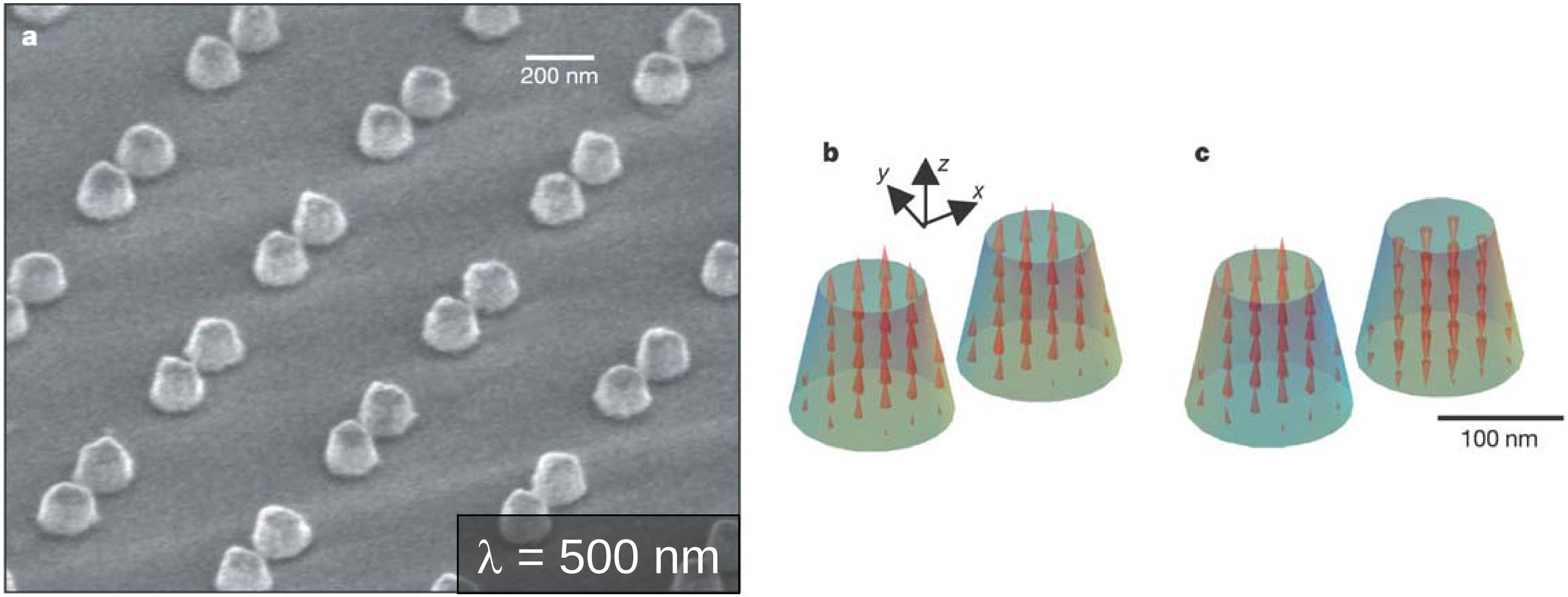


G. Dolling et al., Opt. Lett. 30, 3198 (2005)



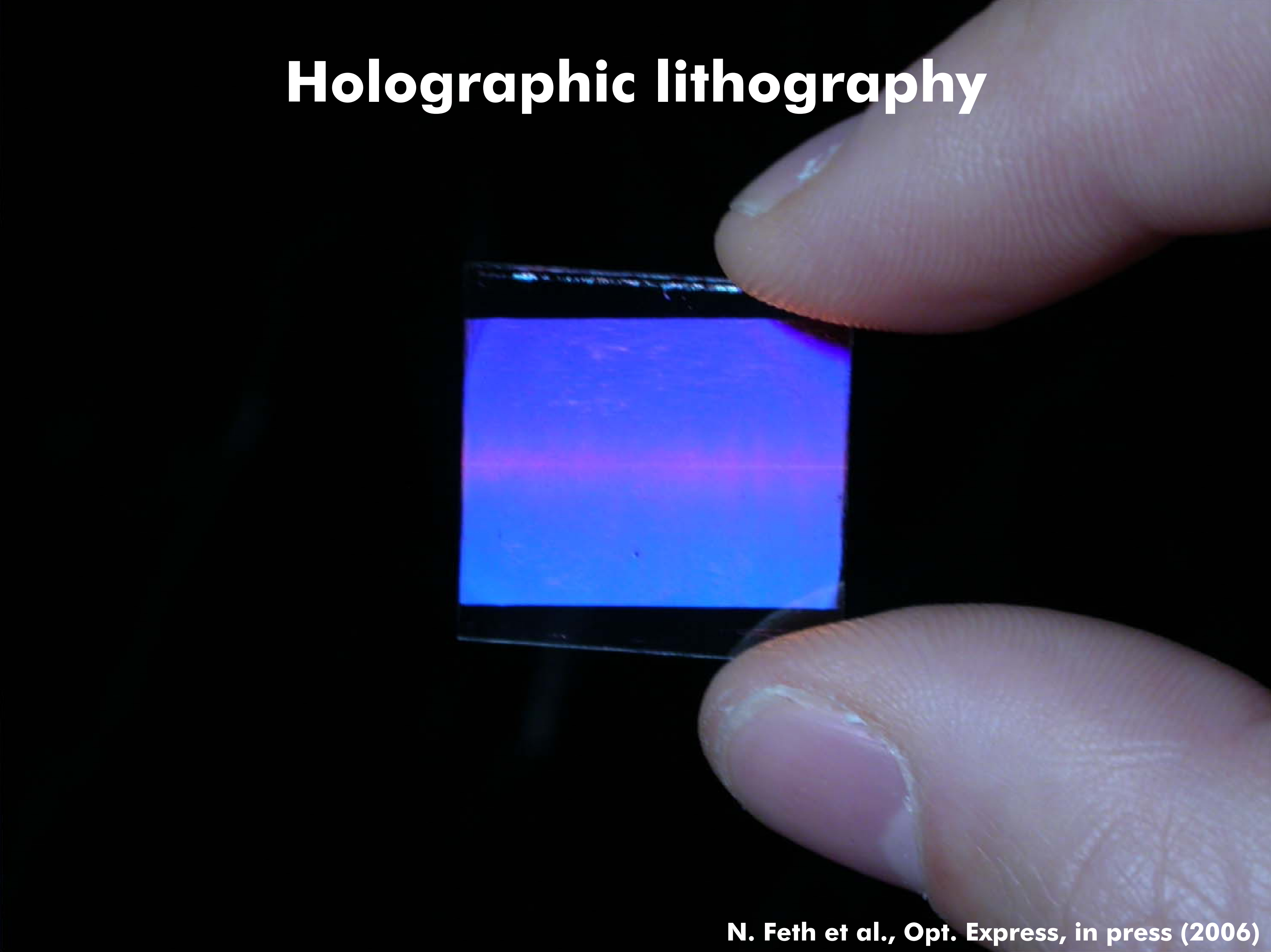
G. Dolling et al., Opt. Lett. 30, 3198 (2005)

Alternative orientation



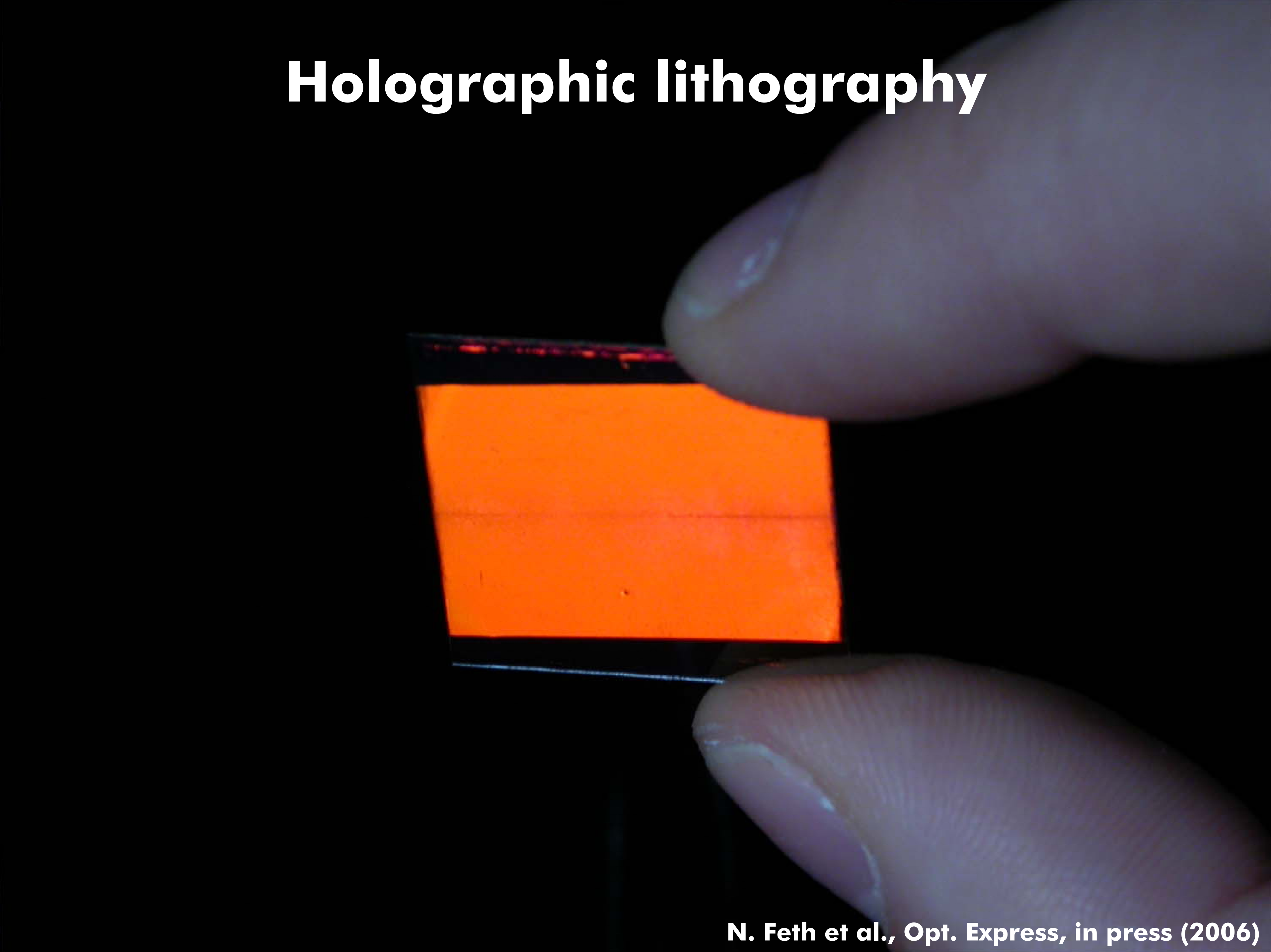
A.N. Grigorenko et al., Nature 438, 335 (2005)

Holographic lithography



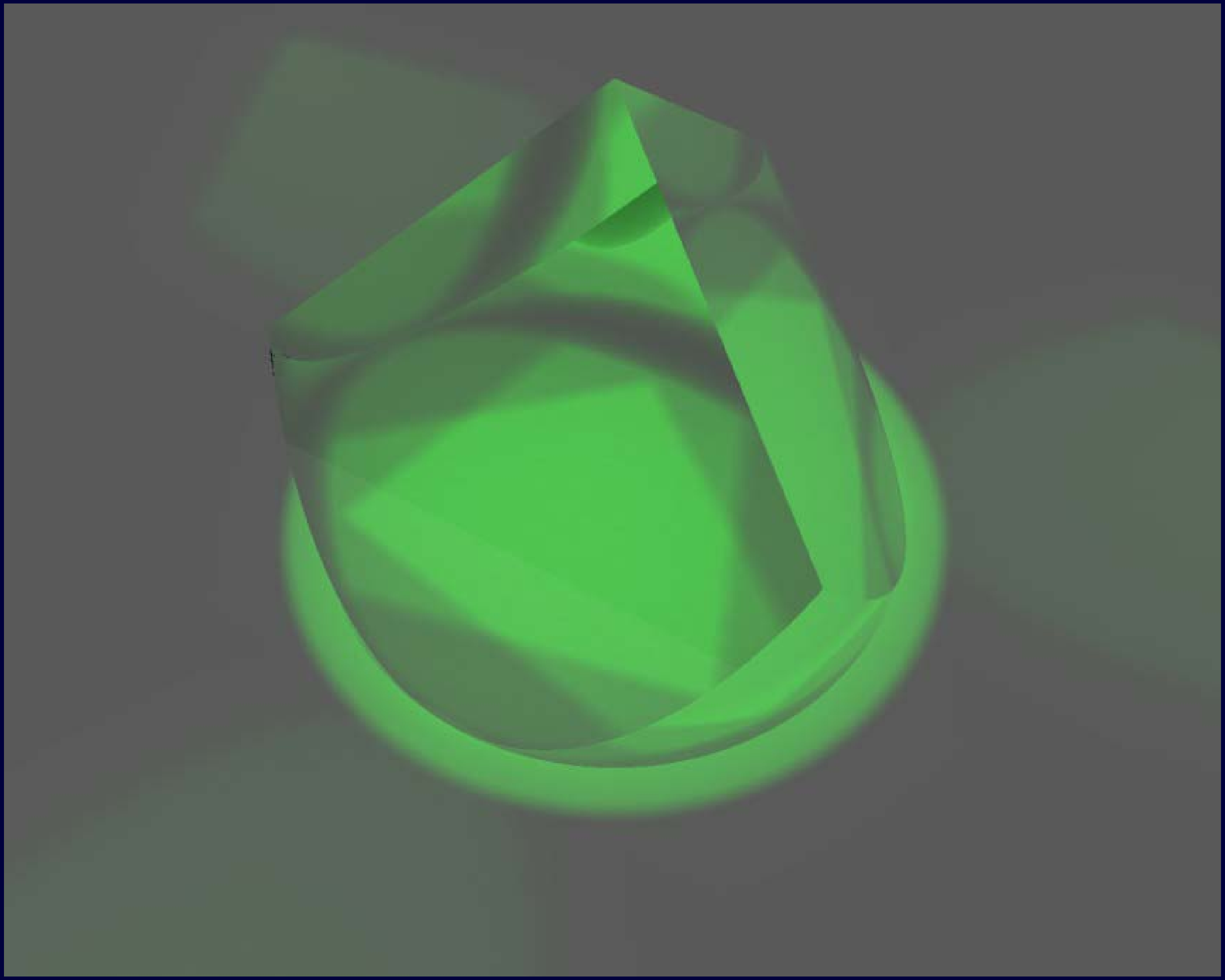
N. Feth et al., Opt. Express, in press (2006)

Holographic lithography



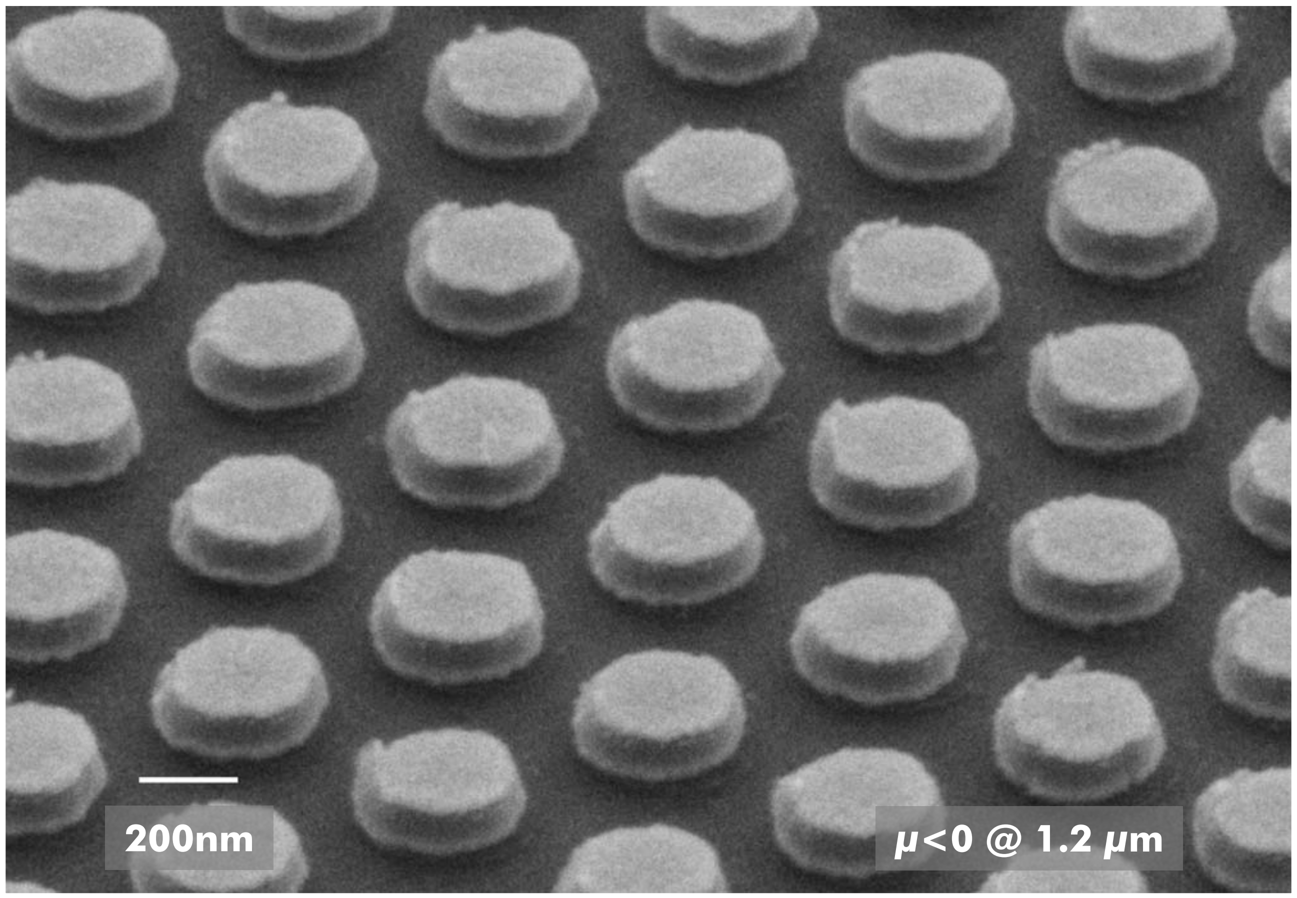
N. Feth et al., Opt. Express, in press (2006)

One incident beam does the job

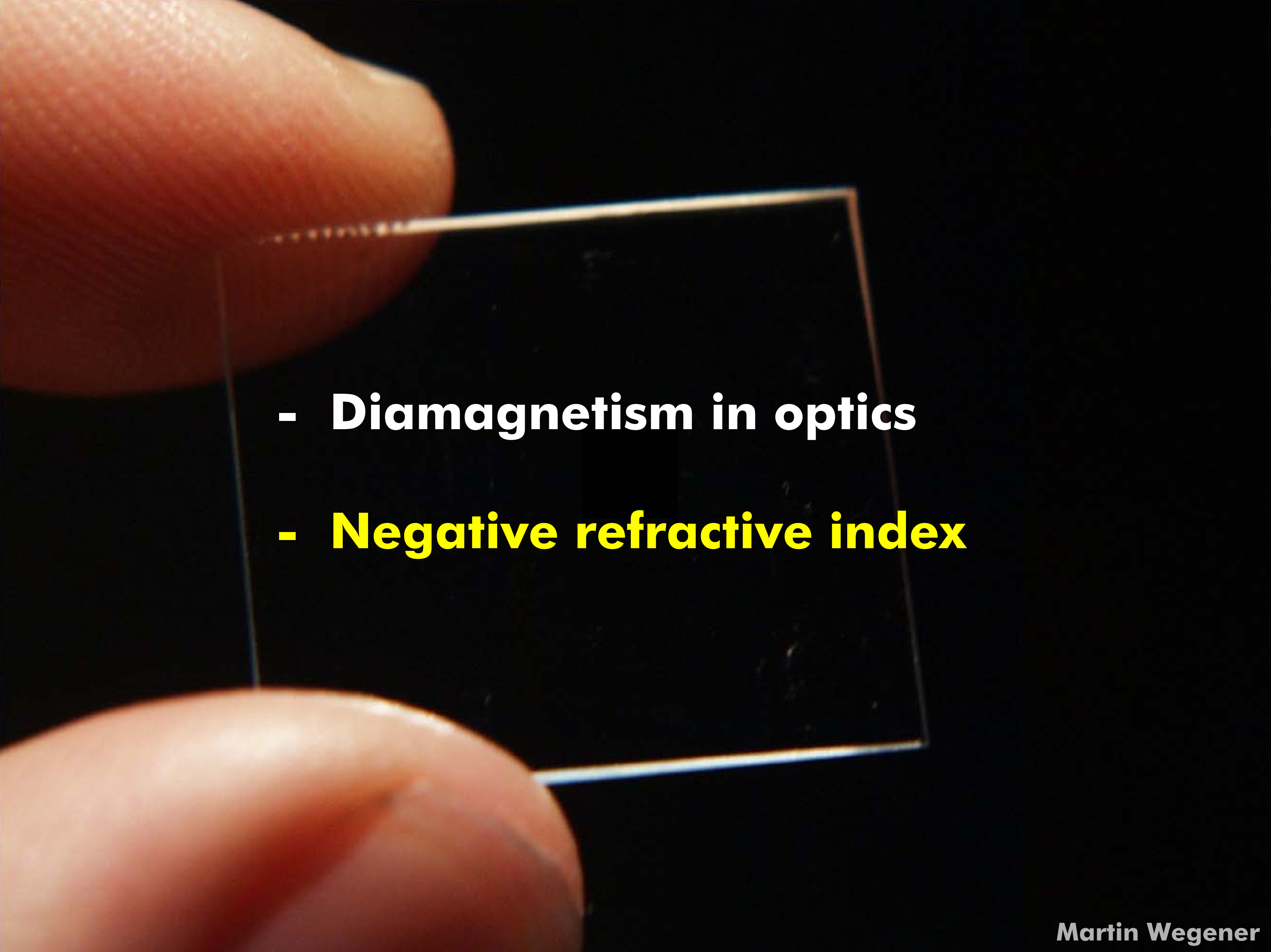


exposure @ 532 nm

Billion “magnetic atoms“



N. Feth et al., Opt. Express, in press (2006)

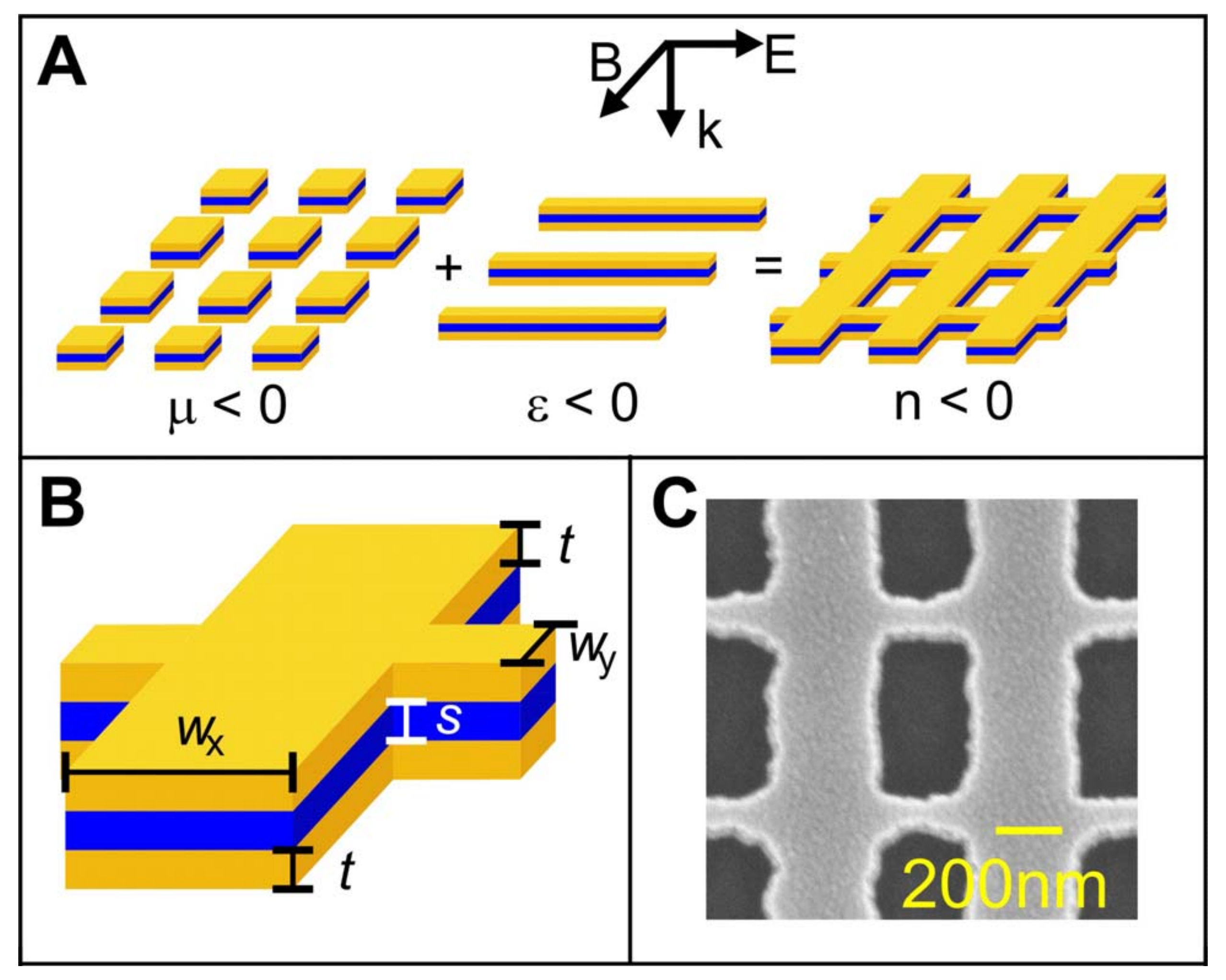


- Diamagnetism in optics
- Negative refractive index

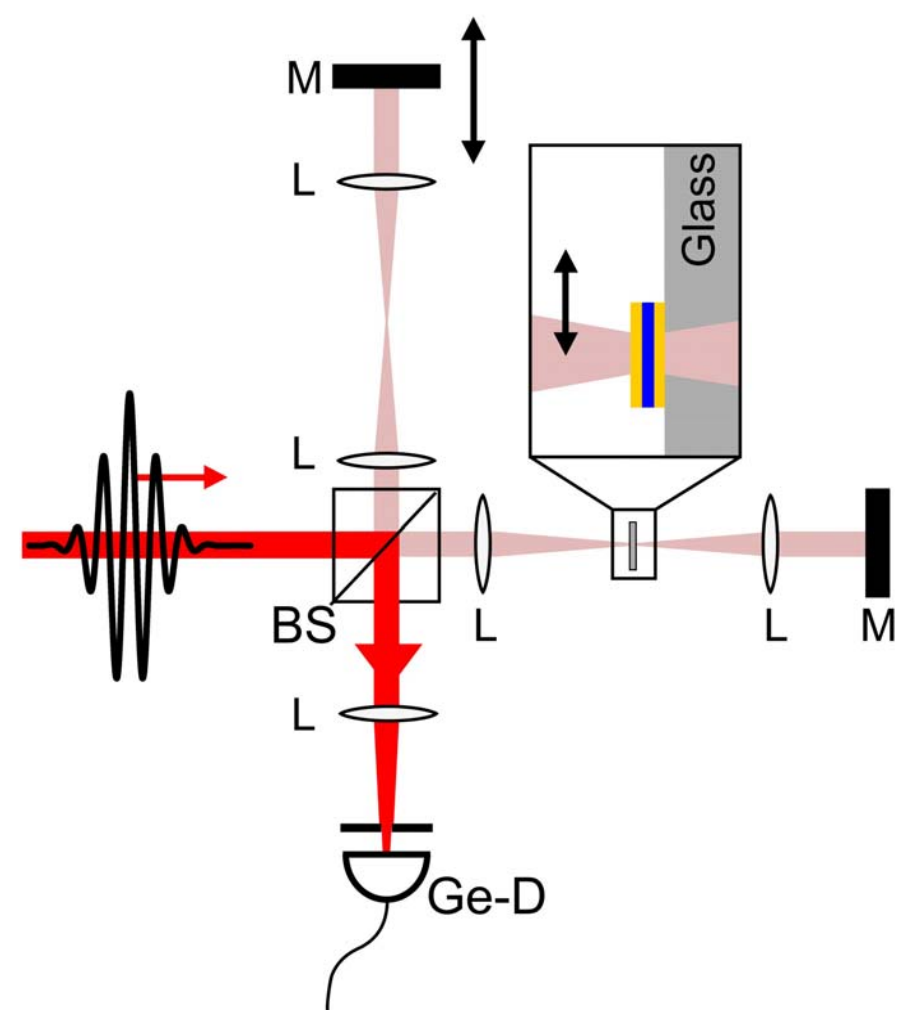
Martin Wegener

A double-negative metamaterial

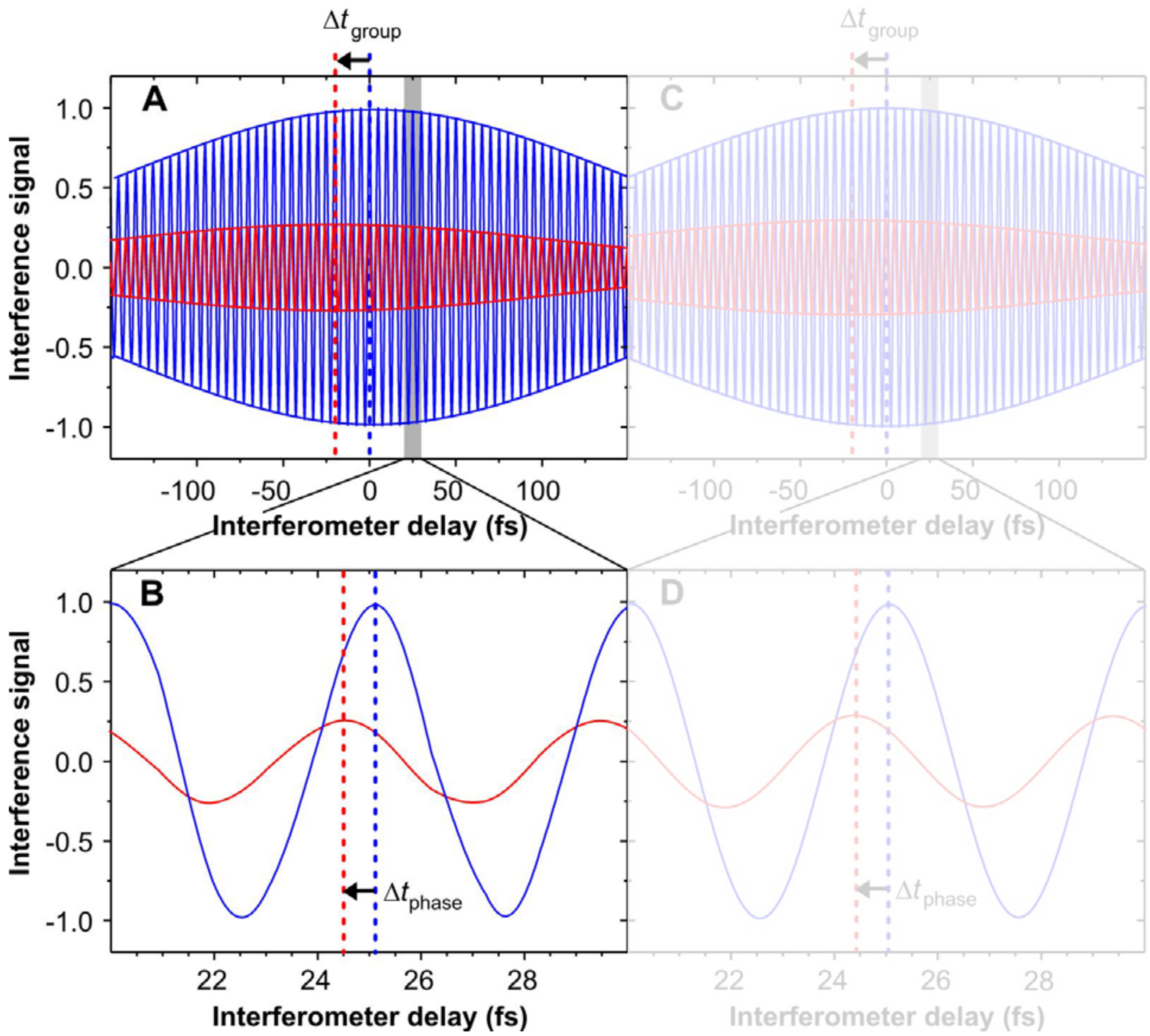
Proposal: S. Zhang et al., Opt. Express 13, 4922 (2005)



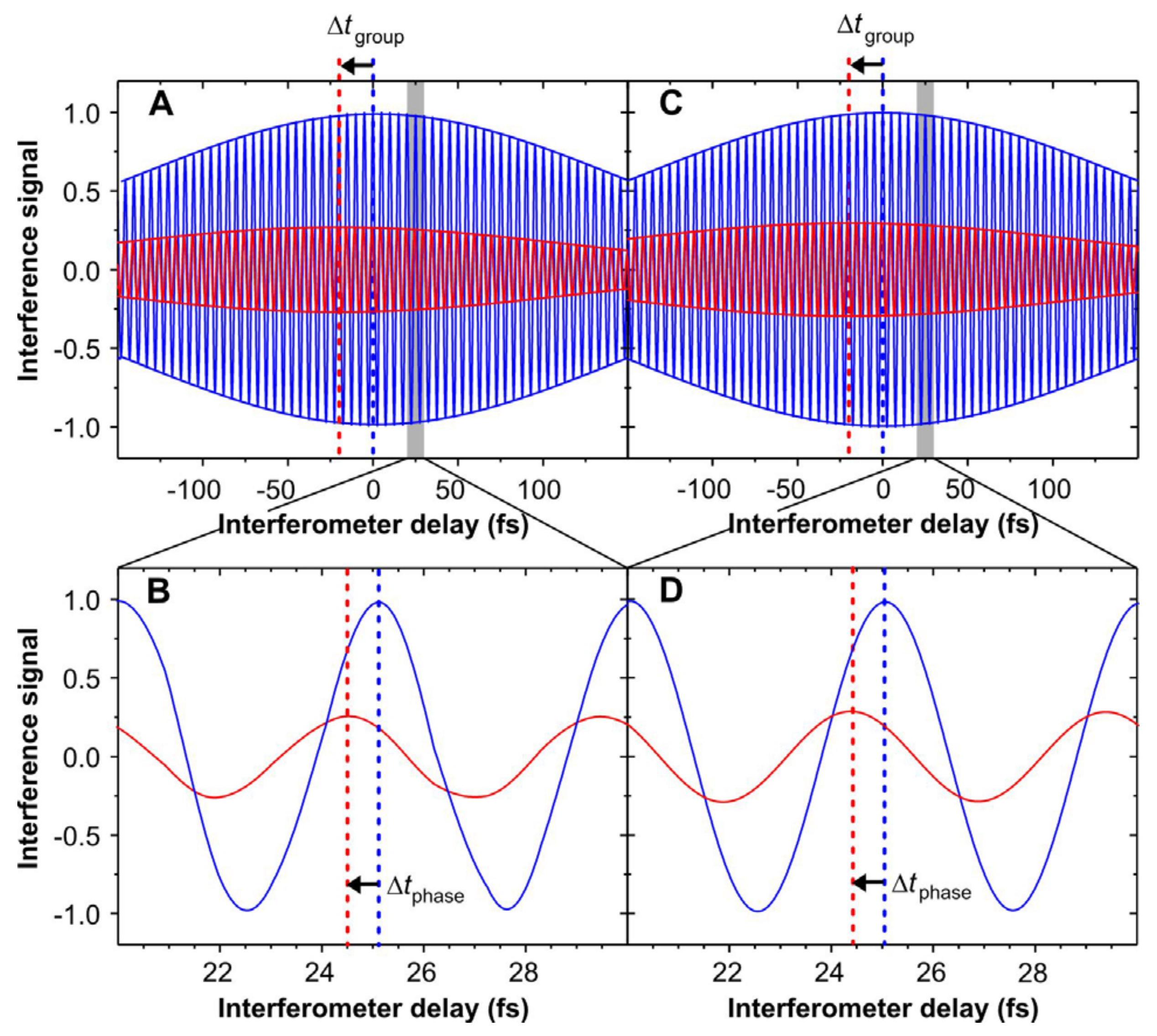
G. Dolling et al., Science 312, 892 (2006)



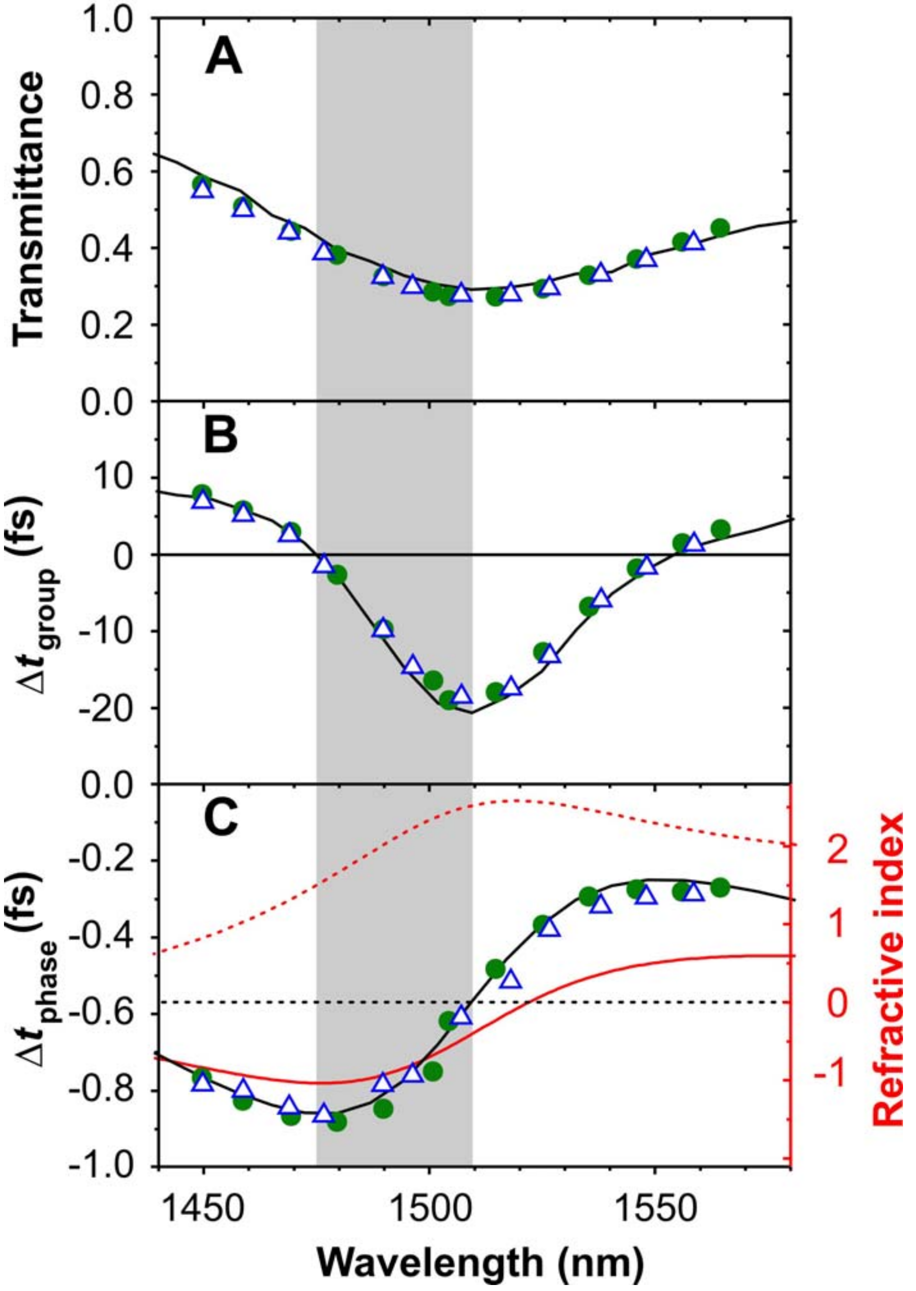
G. Dolling et al., Science 312, 892 (2006)



G. Dolling et al., Science 312, 892 (2006)

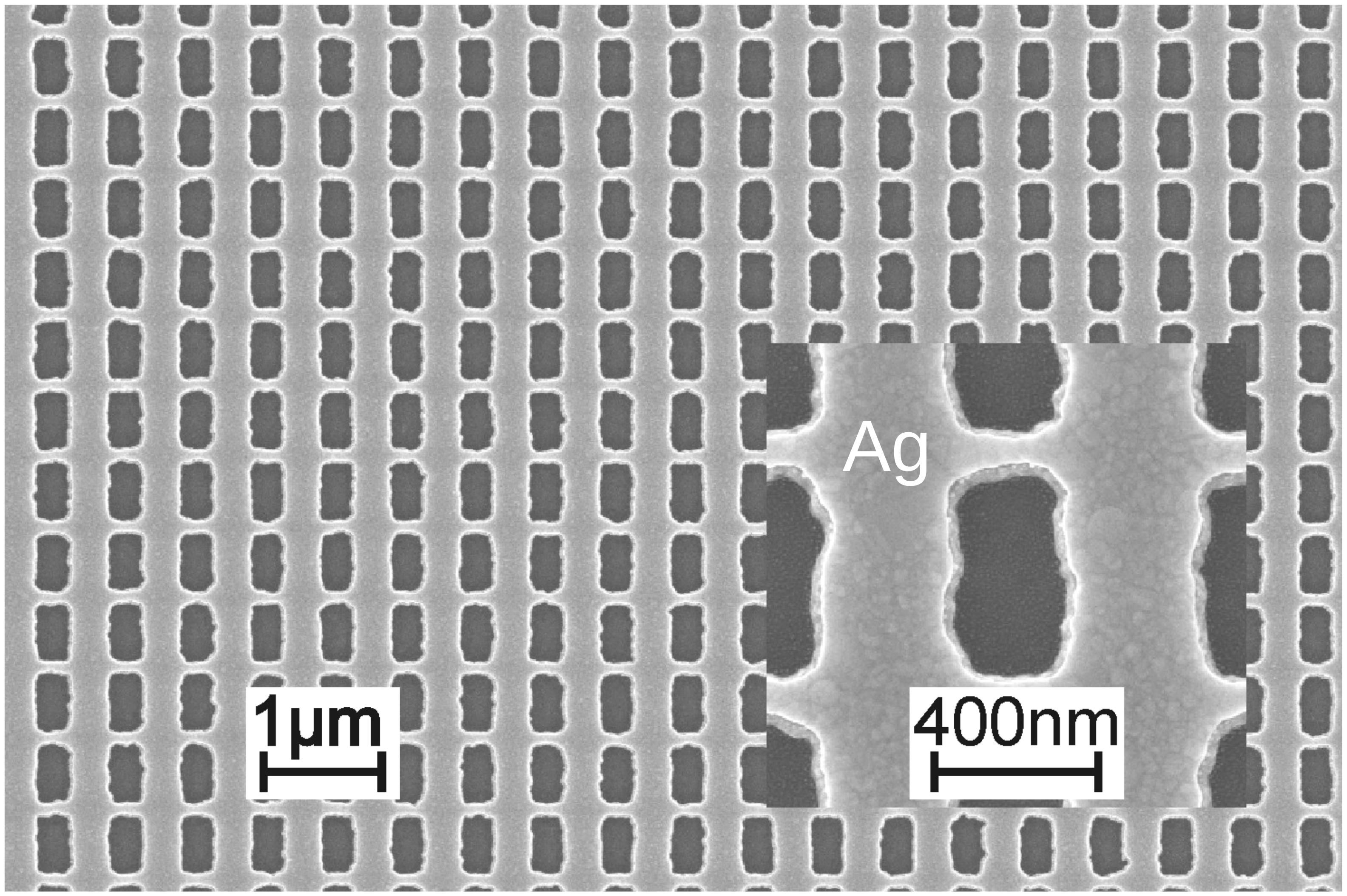


G. Dolling et al., Science 312, 892 (2006)

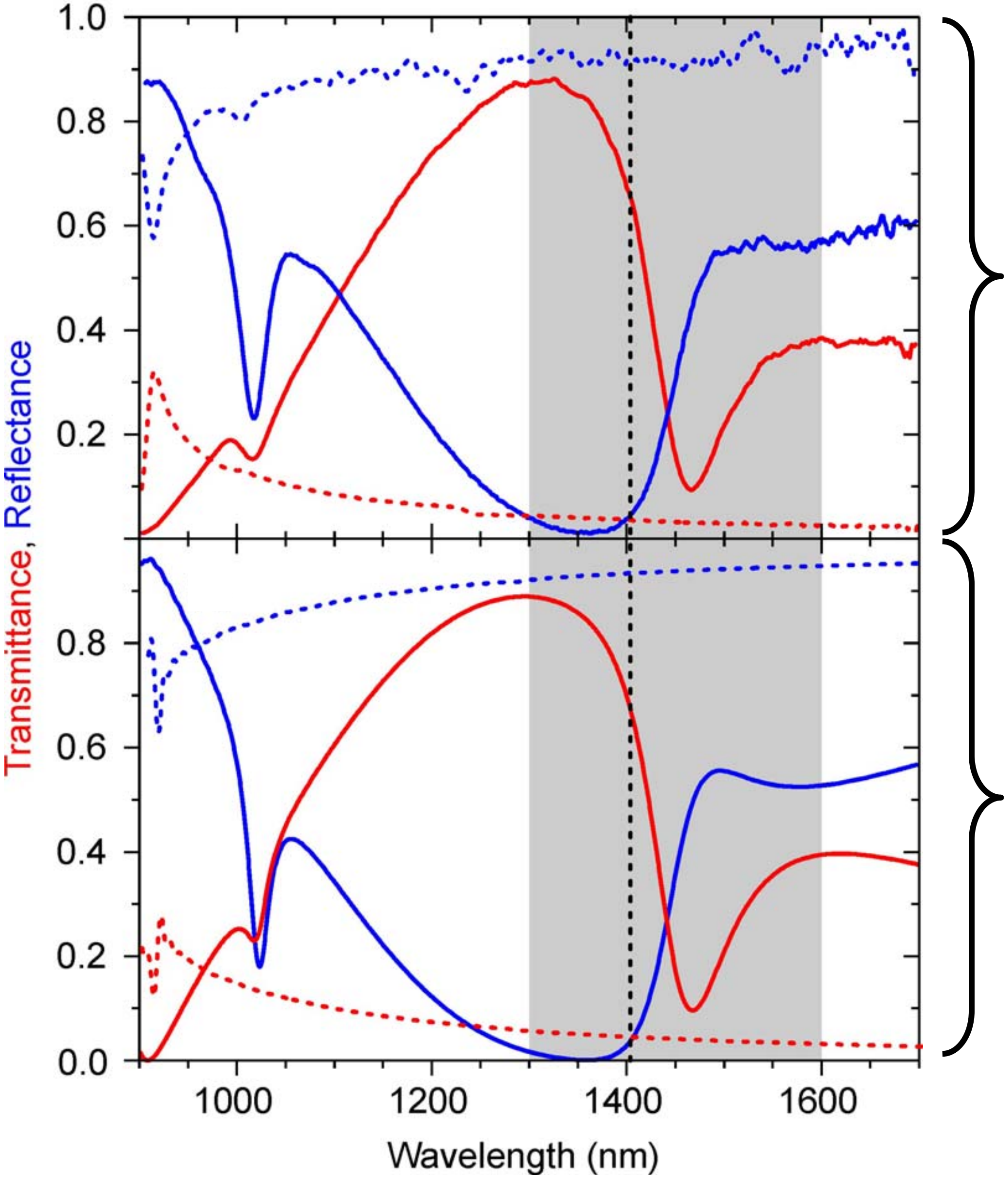

A
B
C
Transmittance
1.0
0.8
0.6
0.4
0.2
0.0
$\Delta t_{\mathrm{group}}$ (fs)
10
0
-10
-20
0.0
$\Delta t_{\mathrm{phase}}$ (fs)
-0.2
-0.4
-0.6
-0.8
-1.0
Refractive index
2
1
0
-1
1450
1500
1550
Wavelength (nm)

G. Dolling et al., Science 312, 892 (2006)

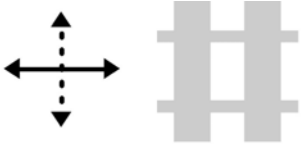
G. Dolling et al., Opt. Lett. 31, 1800 (2006)



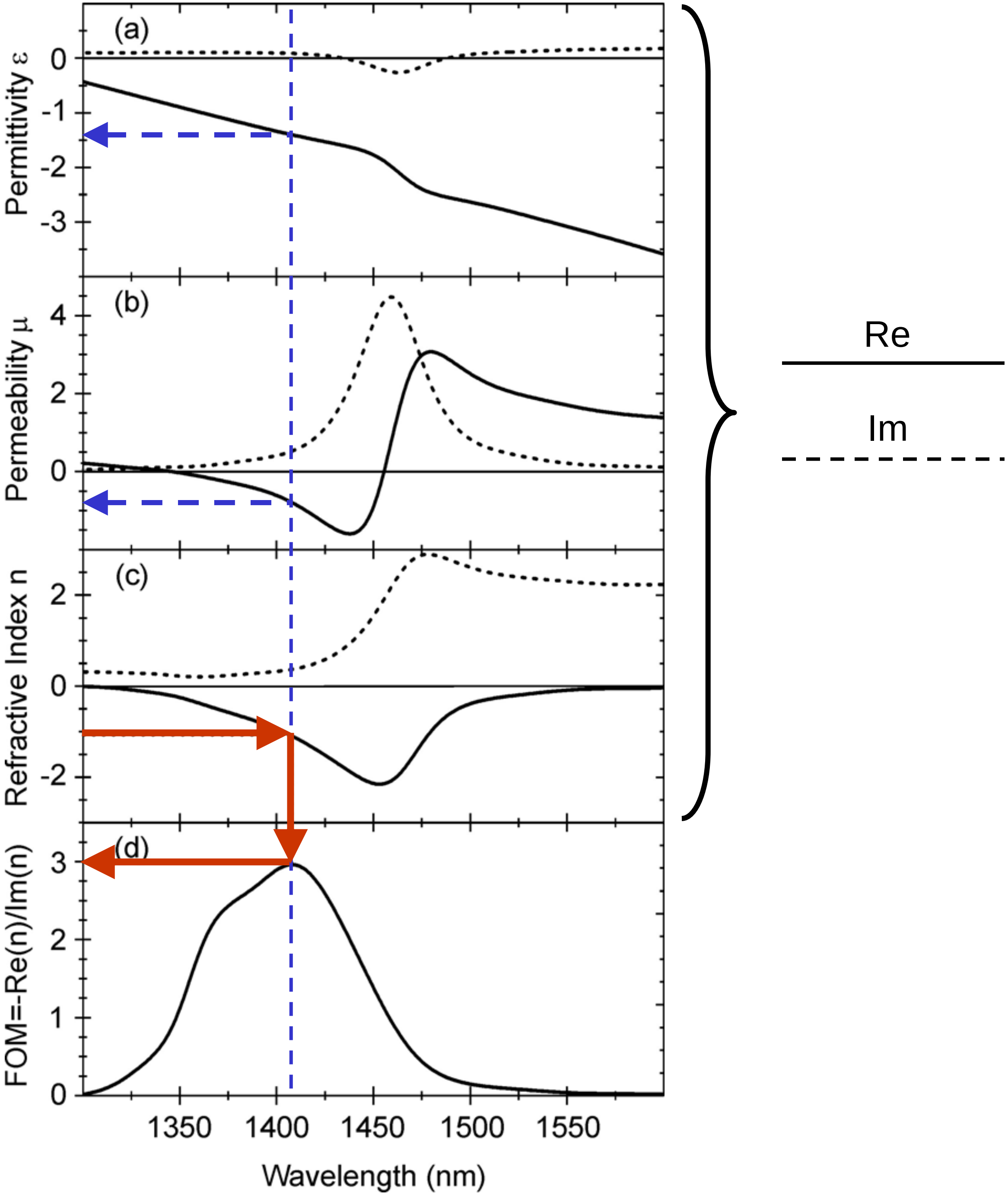
G. Dolling et al., Opt. Lett. 31, 1800 (2006)



Experiment



Theory


(a)
Permittivity ε
0
-1
-2
-3
(b)
Permeability μ
4
2
0
(c)
Refractive Index n
2
0
-2
(d)
FOM=-Re(n)/Im(n)
3
2
1
0
1350
1400
1450
1500
1550
Wavelength (nm)
Re
Im

FOM=0.5 @ $\mathrm{Re}(n)=-1$ & 2 µm & Au:
S. Zhang et al., Phys. Rev. Lett. 95, 137404 (2005)

FOM=0.1 @ $\mathrm{Re}(n)=-0.2$ & 1.5 µm & Au:
V.M. Shalaev et al., Opt. Lett. 30, 3356 (2005)

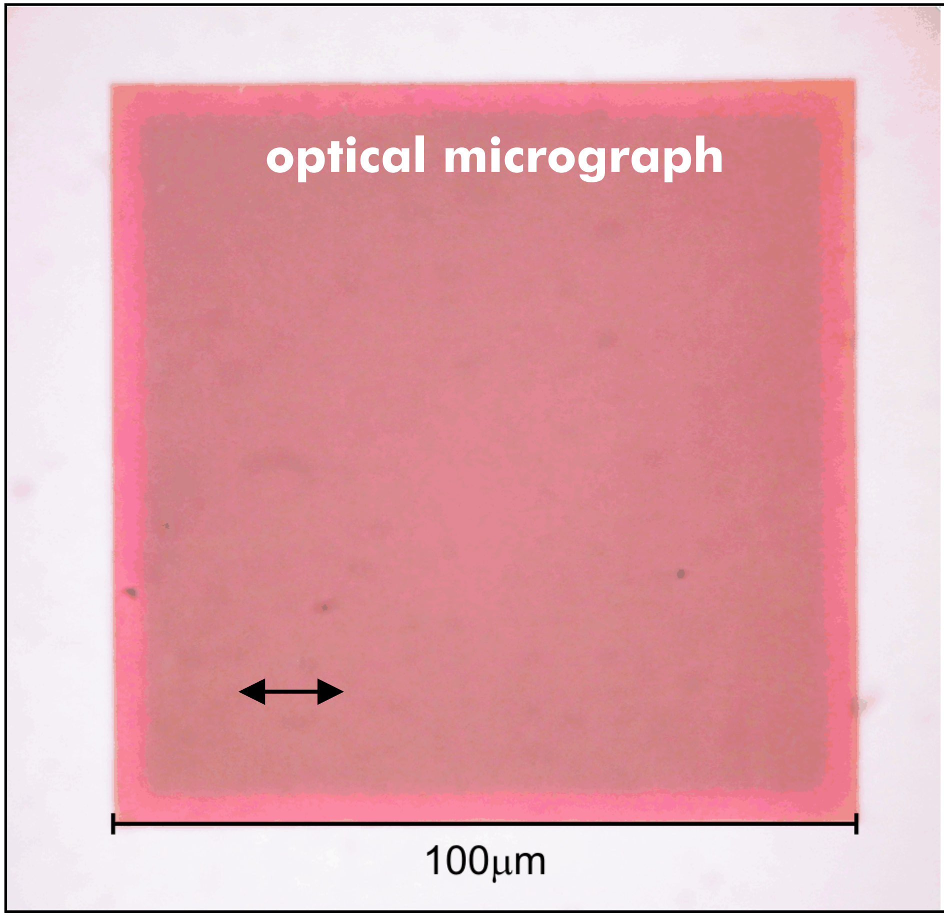
FOM=1-2 @ $\mathrm{Re}(n)\approx-2$ & 1.7 µm & Au:
S. Zhang et al., J. Opt. Soc. Am. B 23, 434 (2006)

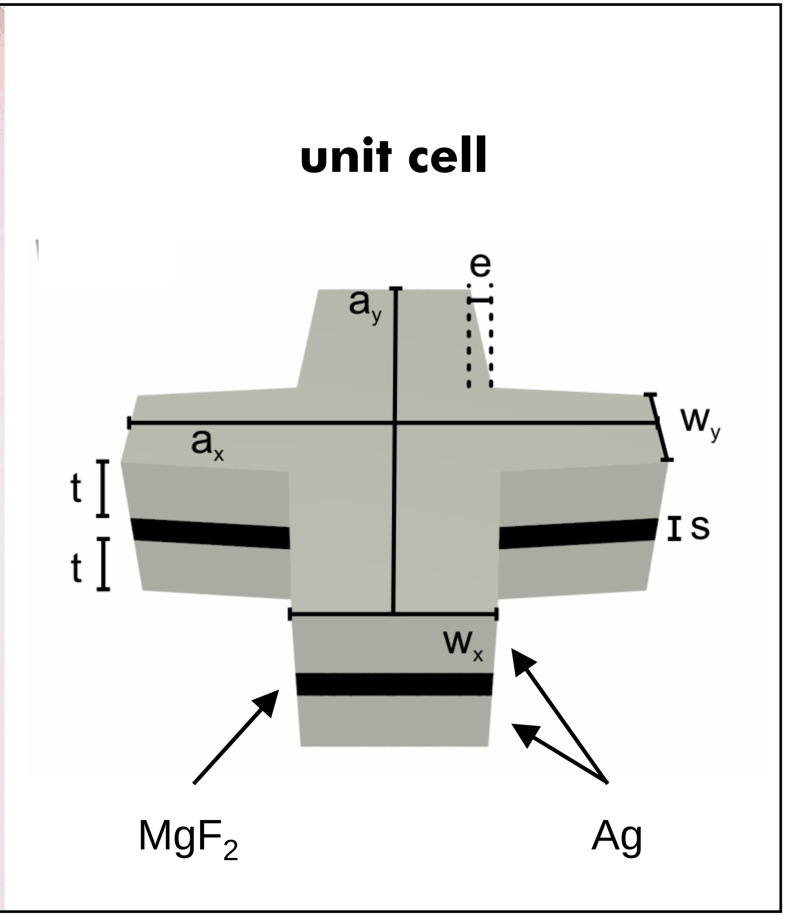
FOM=0.7 @ $\mathrm{Re}(n)=-1$ & 1.5 µm & Au:
G. Dolling et al., Science 312, 892 (2006)

FOM=3 @ $\mathrm{Re}(n)=-1$ & 1.4 µm & Ag:
G. Dolling et al., Opt. Lett. 31, 1800 (2006)

FOM=0.5 @ $\mathrm{Re}(n)=-0.6$ & 780 nm & Ag
G. Dolling et al., submitted (2006)

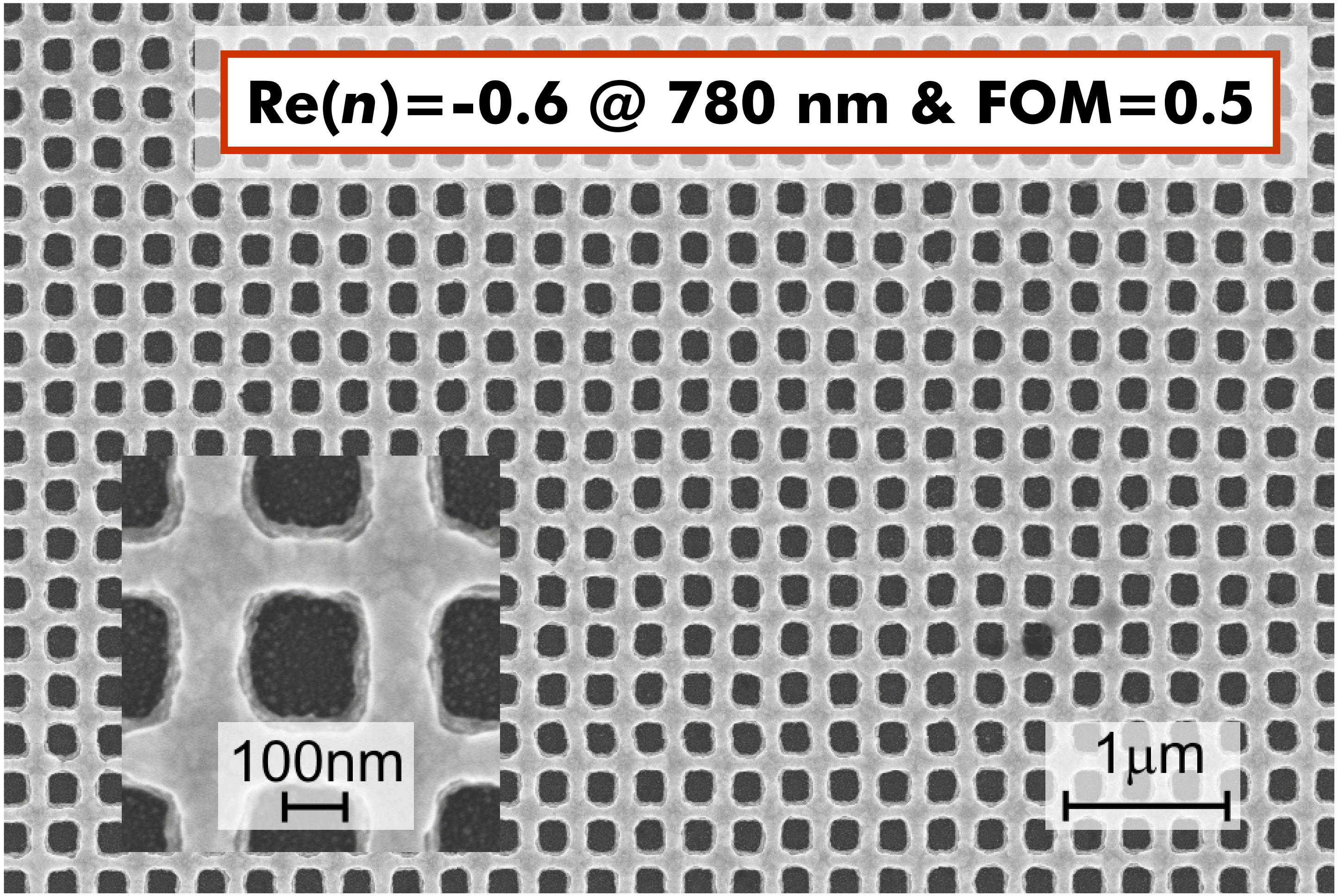
Ag-based visible metamaterial





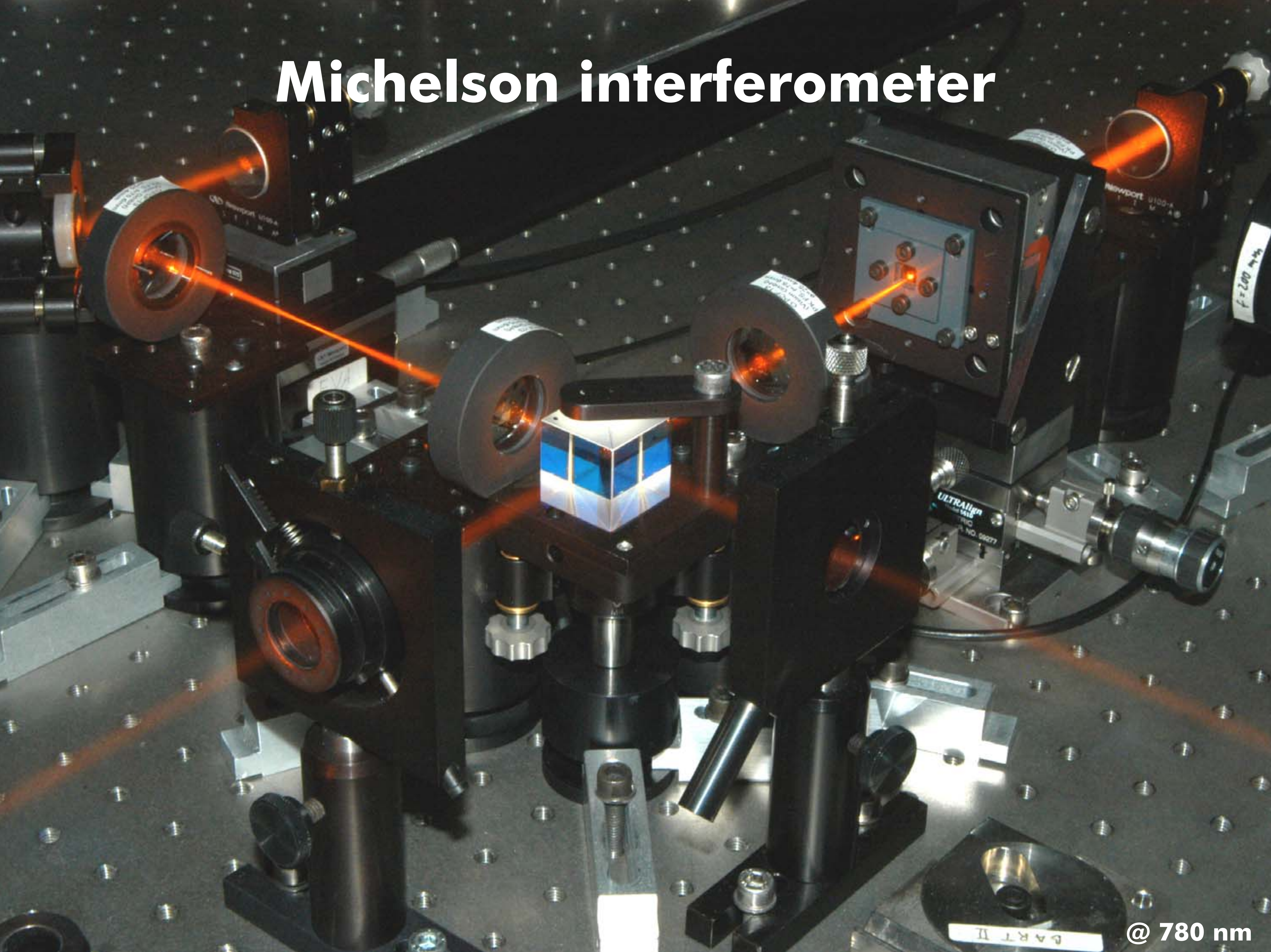
$s = 17$ nm, $t = 40$ nm, $a_x = a_y = 300$ nm, $w_x = 102$ nm, $w_y = 68$ nm, $e = 8$ nm

Ag-based visible metamaterial

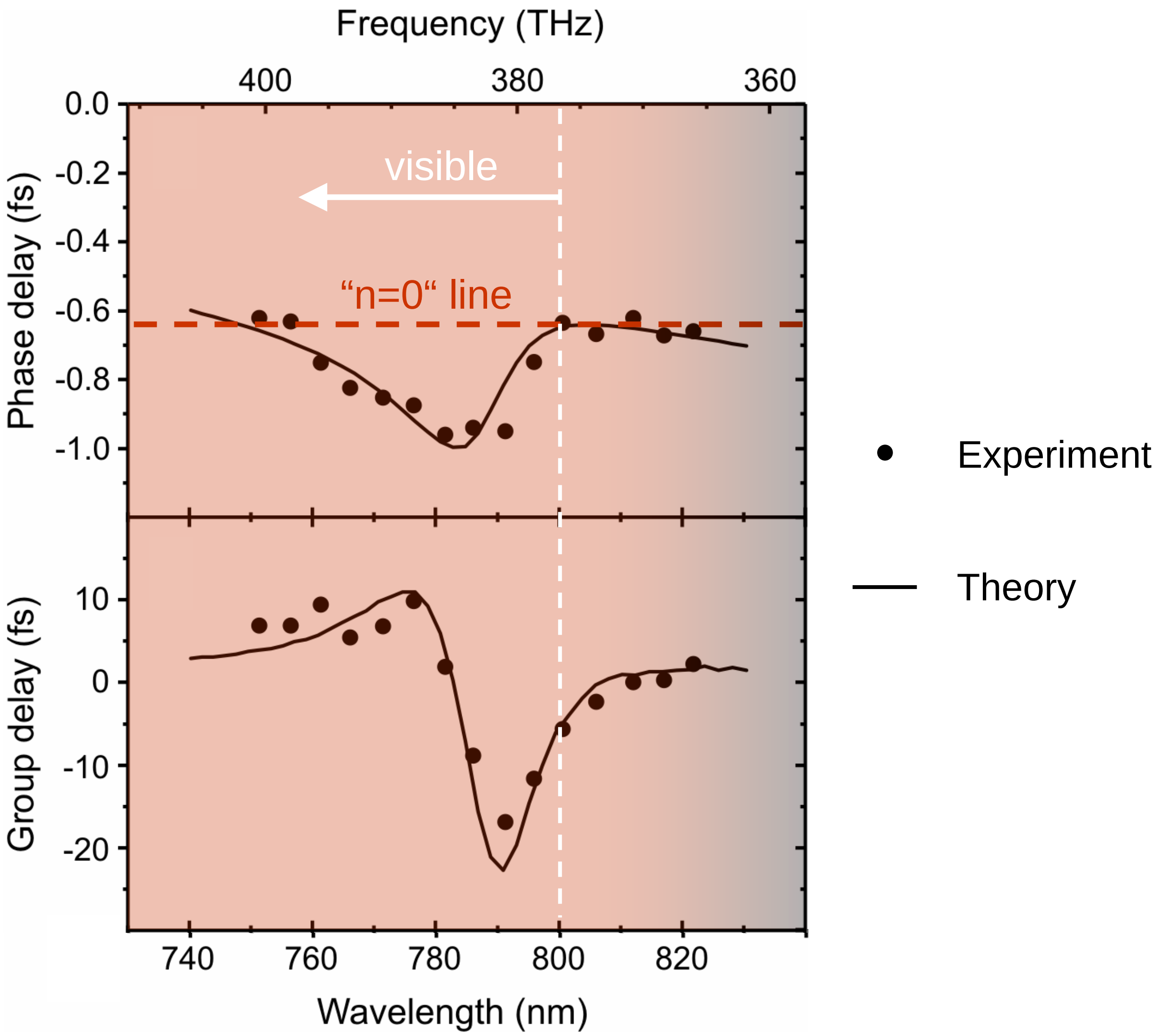

Re(*n*)=-0.6 @ 780 nm & FOM=0.5

G. Dolling et al., Opt. Lett., in press (2007)

Michelson interferometer



@ 780 nm



G. Dolling et al., Opt. Lett., in press (2007)

Towards 3D metamaterials

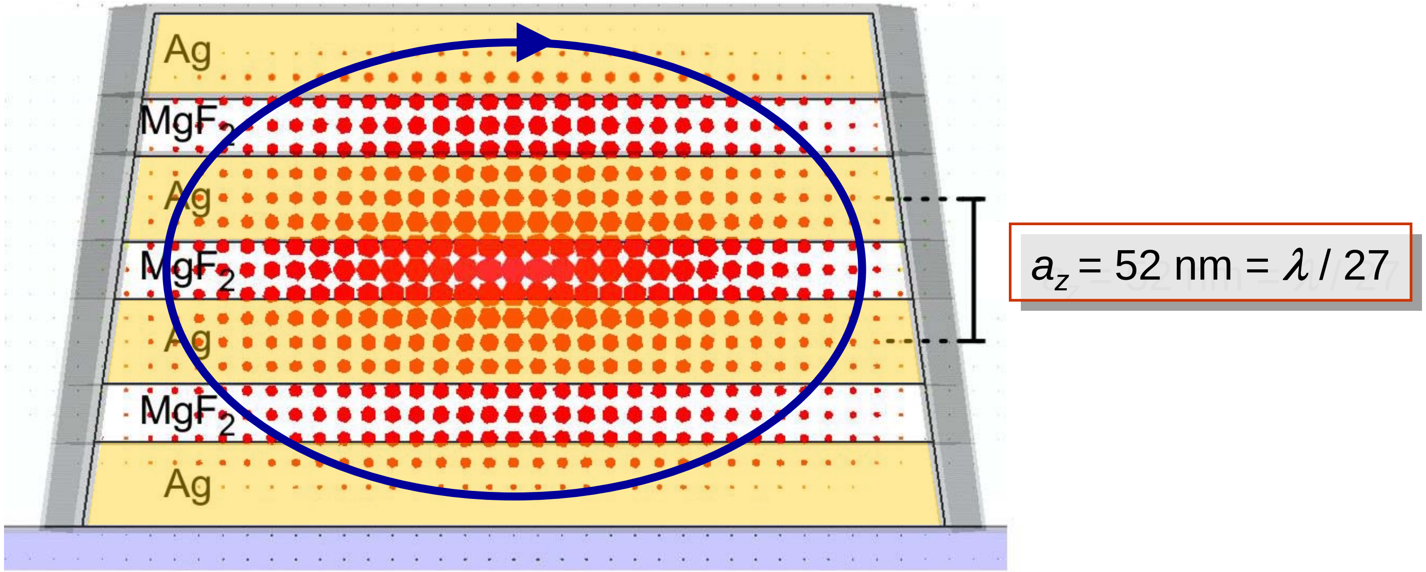
So far, we have only demonstrated metamaterial monolayers.

However, it is well known that monolayers can have properties distinct from the bulk.

So what about several functional layers?

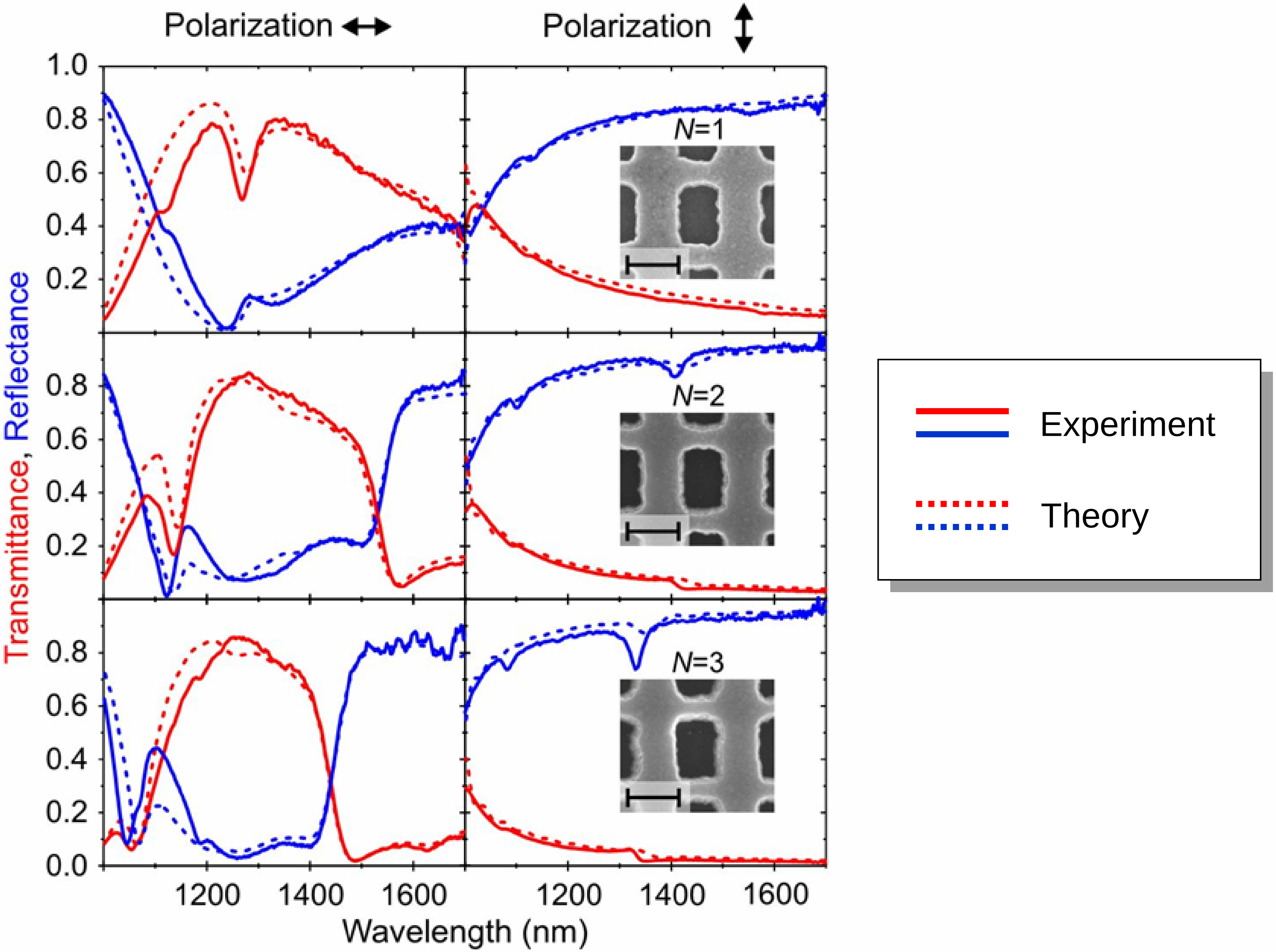
Theoretical proposal: S. Zhang et al., Opt. Express 14, 6778 (2006)

N=3 functional layer sample



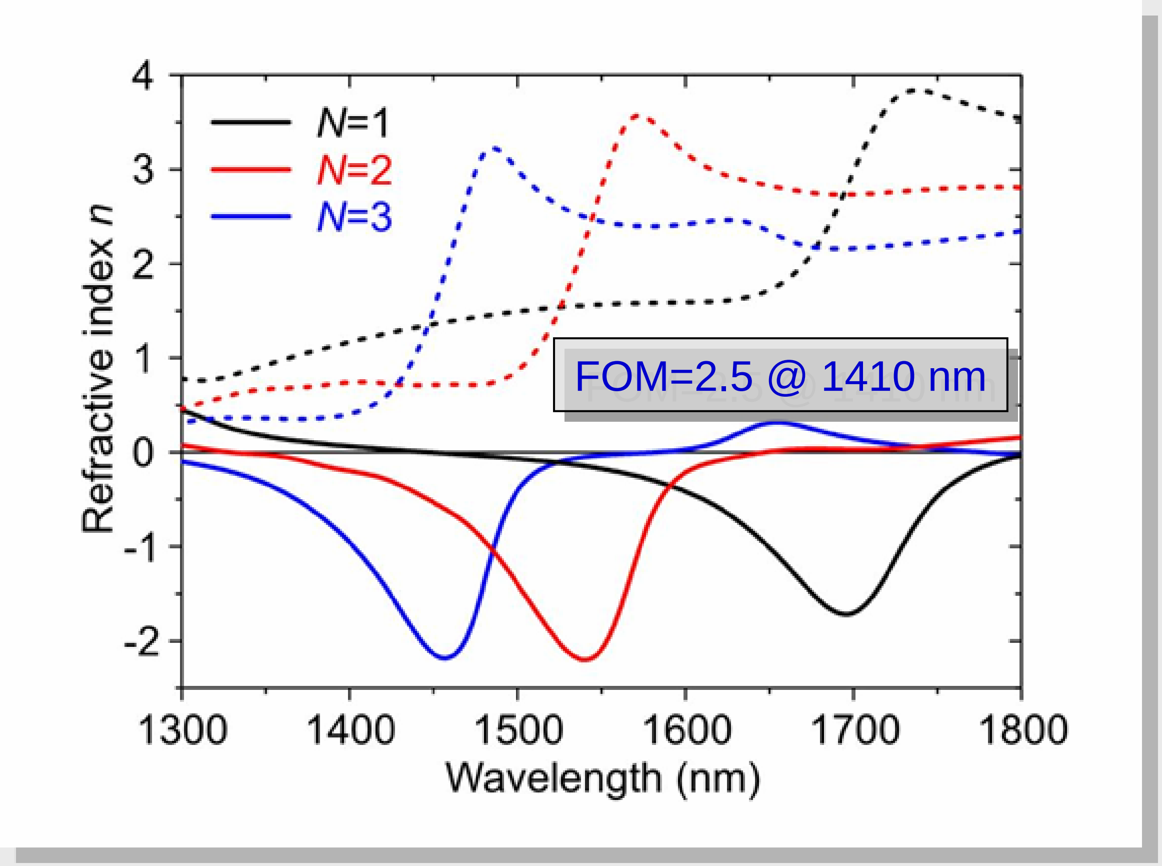
$a_z = 52\text{ nm} = \lambda / 27$

magnetic field and current @ 1410-nm wavelength where $\mu<0$



500-nm scale bar

Towards 3D metamaterials



G. Dolling et al., Opt. Lett., in press (2007)

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    - 1.2. Paradigms of nanophotonics
        - 1.2.1. Evanescent waves & the limit of electrostatics
        - 1.2.2. Mie-scattering of nanoparticles
        - 1.2.3. Nanoscale metal particles: Particle plasmons
        - 1.2.4. A nano-aperture in an ideal metal film: Bethe-Bouwkamp theory

2. Photonic band gap (PBG) materials
    - 2.1. One-dimensional Photonic Crystals (i.e., dielectric mirrors)
    - 2.2. Two-dimensional Photonic Crystals
        - 2.2.1. Negative refraction and the superprism effect
        - 2.2.2. "Semiconductors for light": Waveguides and defect cavities
    - 2.3. Three-dimensional Photonic Crystals
        - 2.3.1. Quantum optics: Modified Planck's law
        - 2.3.2. Fabrication: Overview and state-of-the-art
        - 2.3.3. Fabrication: Current efforts within the CFN

3. Metamaterials
    - 3.1. Left-handed or Veselago materials (LHM)
    - 3.2. "Perfect lenses" made from LHM
    - 3.3. Towards metamaterials @ optical frequencies (CFN activities)

4. **Plasmonics**
    - 4.1. Field-enhancement & surface-enhanced Raman scattering (SERS)
    - 4.2. Surface-plasmon amplification by stimulated emission of radiation (SPASER)
    - 4.3. Transmission through sub-wavelength hole arrays

Field-enhancement effects in metal nano-optics ...

... are analogous to the physics of lightning-rods

... can enhance local fields by factors $>10^3$,
leading to intensity enhancements of $>10^6$ and
enhancements of $>(10^6)^N$ for a N-th order nonlinear
optical process

... for example, allow to detect Raman scattering
from single molecules near to a metal nanostructure
(started with rough metal films, hence the name)

J.J. Lasema ed., "Modern Techniques in Raman spectroscopy", John Wiley & Sons (1996)

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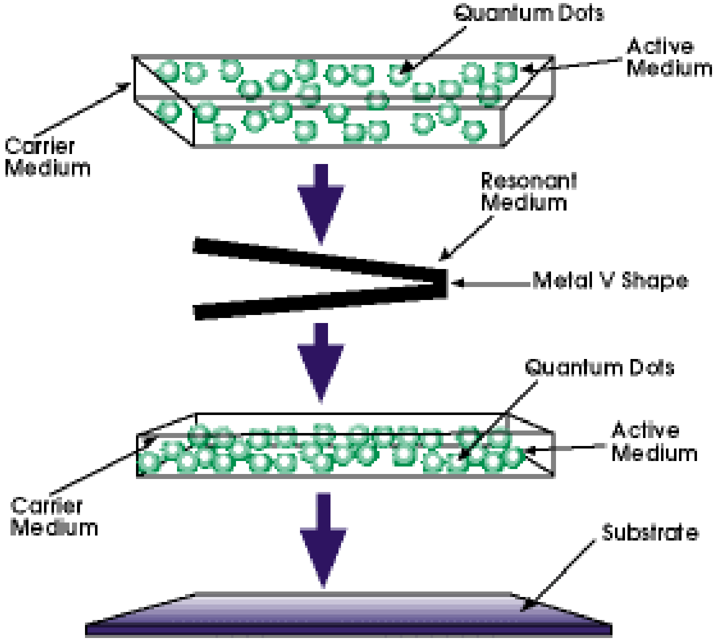
LASER nm-analoge: SPASER

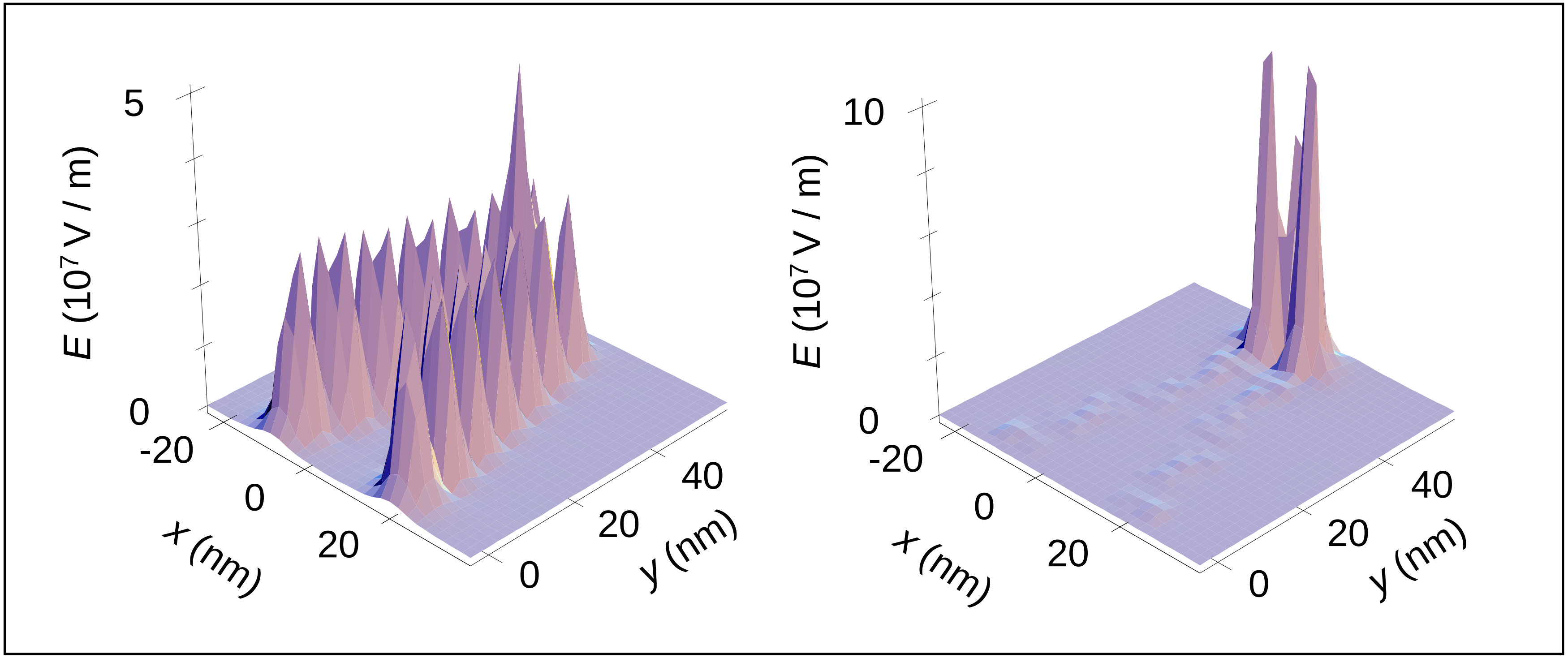


- **use nano metal object to get field enhancement (replaces LASER cavity)**
- **"V"-shape appears ideal**
- **quantum dots serve as gain medium**

D.J. Bergmann and M.I. Stockman, Phys. Rev. Lett. 90, 027402 (2003)

Eigenmodes of silver “V“

dark mode @ 1.63 eV **luminous mode @ 1.56 eV**



D.J. Bergmann and M.I. Stockman, Phys. Rev. Lett. 90, 027402 (2003)

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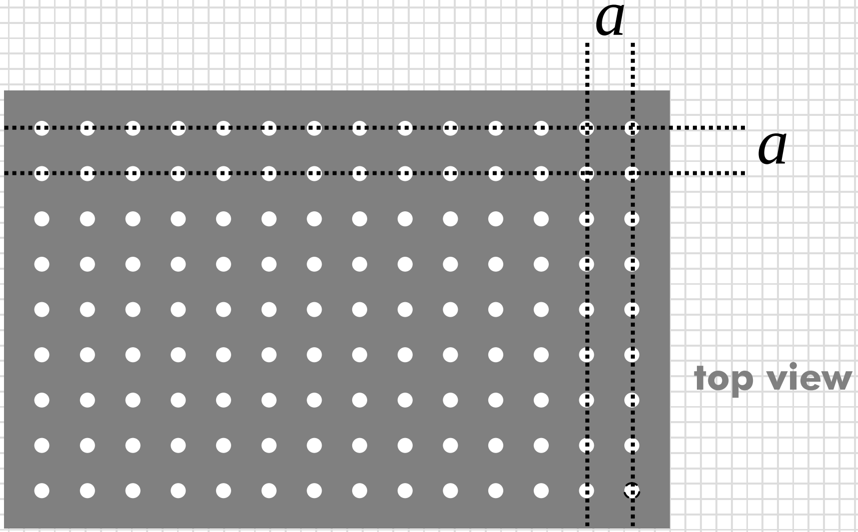
Consider a real metal film, perforated with a periodic array of sub-wavelength holes.

?

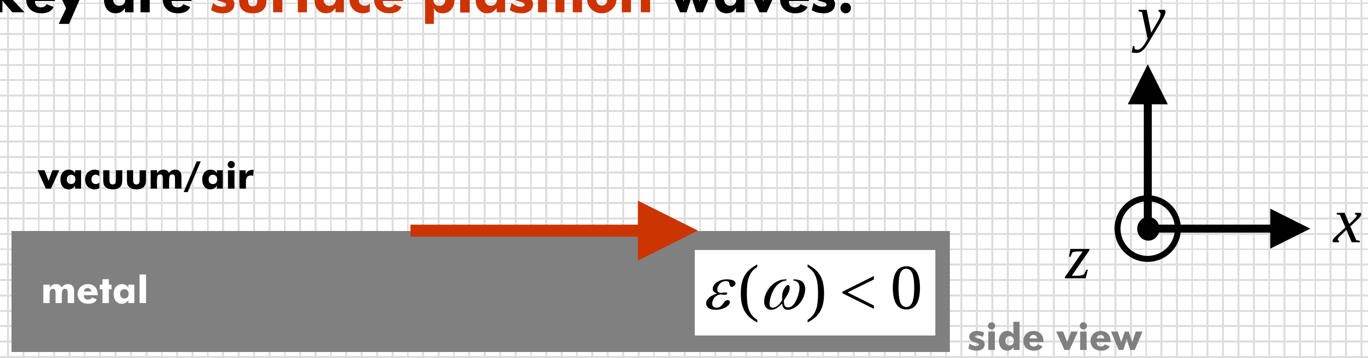


Naively, one would expect a transmission smaller than anticipated from geometrical optics (see section 1.2.4.).

In sharp contrast to this, experiments have shown much larger transmission for particular wavelengths.

T.W. Ebbesen et al., Nature 391, 667 (1998)

The key are surface plasmon waves:



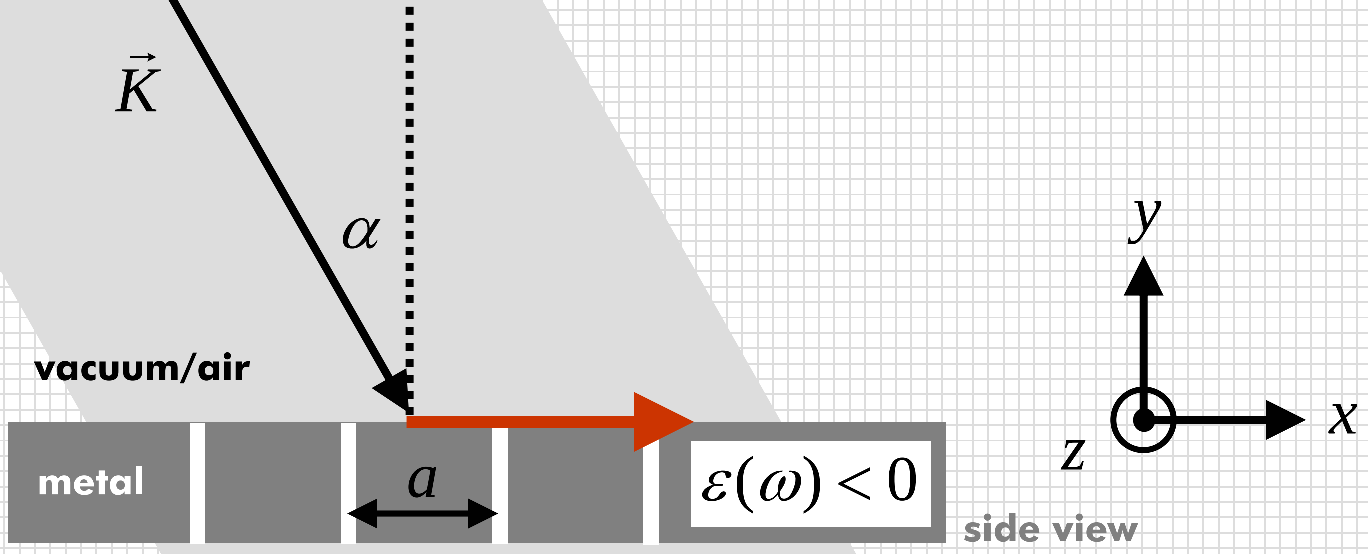
The ansatz (air) for the electric field

$$E_y = E_y^0 \exp(\mathrm{i}(K_x^{\mathrm{sp}} x + K_y^{\mathrm{sp}} y - \omega t)) + \mathrm{c.c.}$$

together with the boundary conditions leads to

$$K_x^{\mathrm{sp}} = \sqrt{\frac{\varepsilon(\omega)}{1+\varepsilon(\omega)}}\,\frac{\omega}{c_0} > \frac{\omega}{c_0} \Rightarrow (K_y^{\mathrm{sp}})^2 = \left(\frac{\omega}{c_0}\right)^2 - (K_x^{\mathrm{sp}})^2 < 0$$

i.e., to evanescent waves.

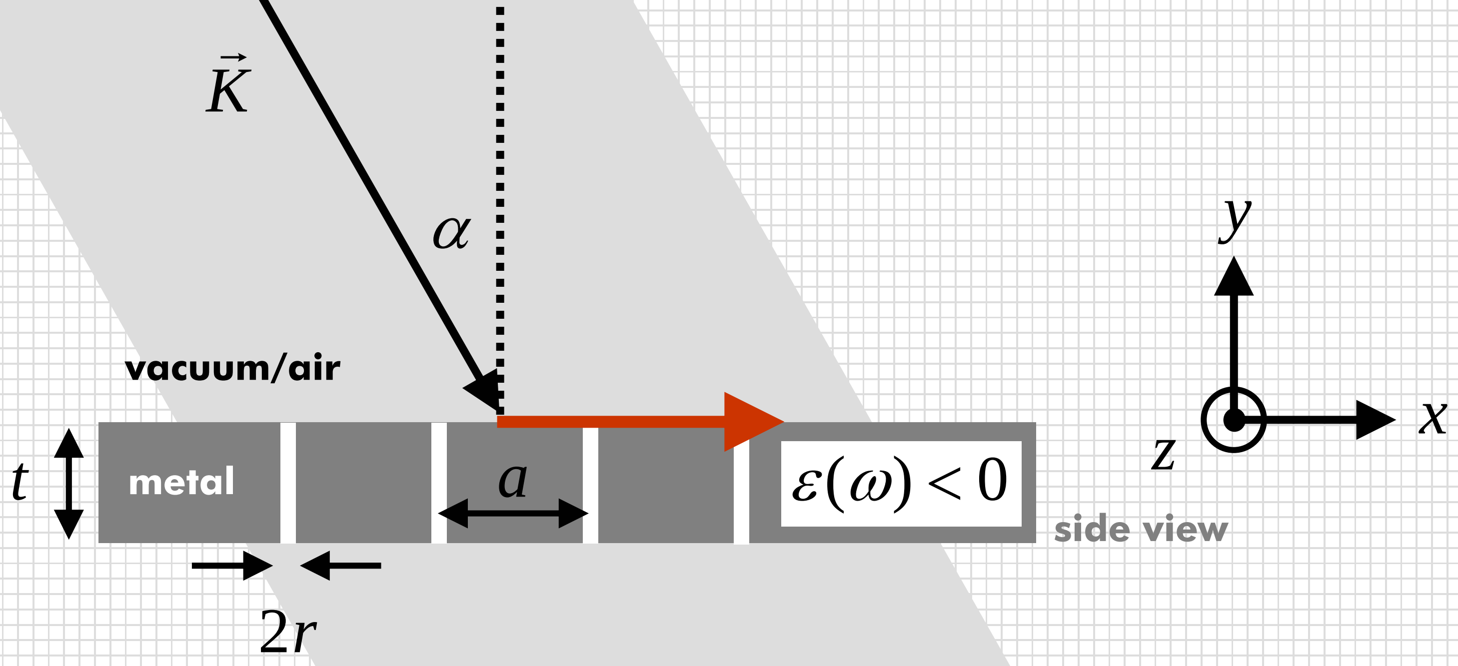


Usually, one cannot couple from vacuum because of wavevector and frequency conservation.

For the arrays, reciprocal lattice vectors can be added

$$K_x = \frac{\omega}{c_0}\sin(\alpha) = \sqrt{\frac{\varepsilon(\omega)}{1+\varepsilon(\omega)}}\,\frac{\omega}{c_0} \pm \boxed{N_x \frac{2\pi}{a}}$$

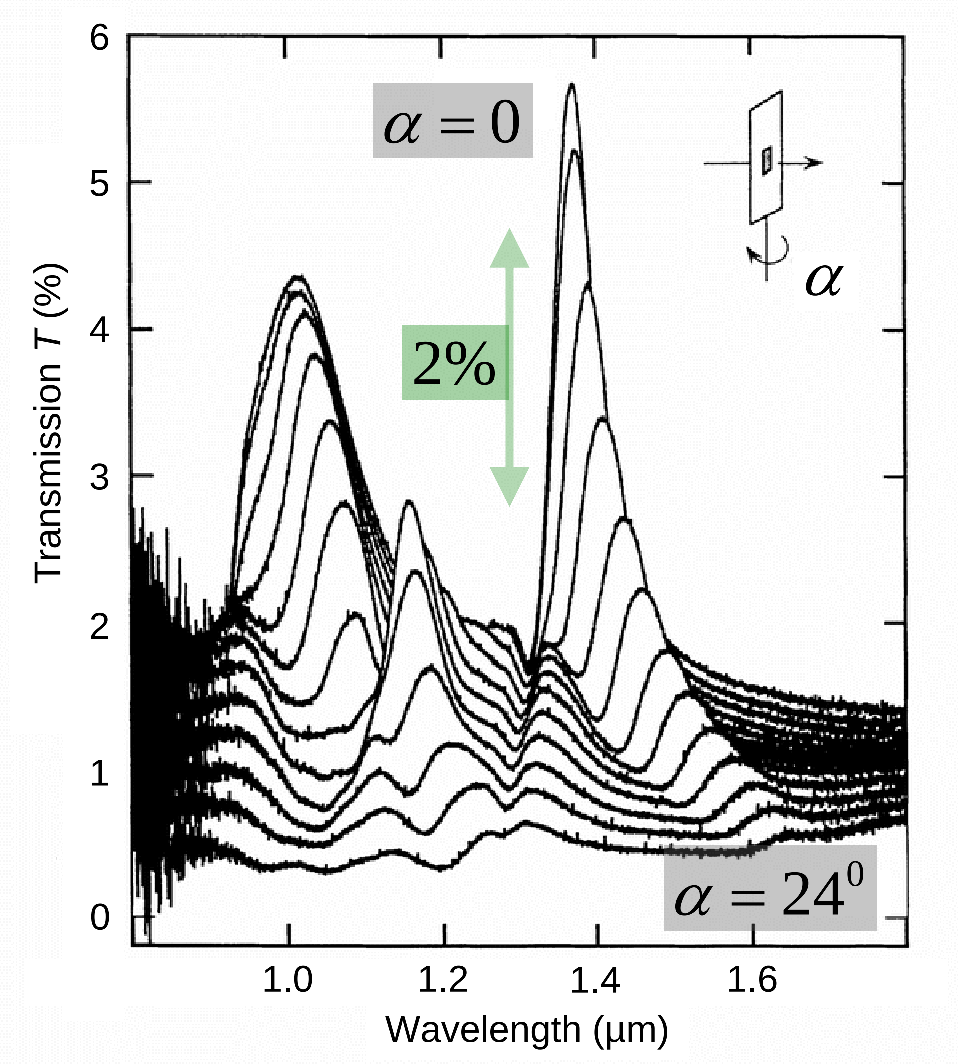
hence, the conservation laws can be fulfilled.



The coupling to the surface plasmons via reciprocal lattice vectors leads to enhanced transmission.

This qualitative explanation has been confirmed by detailed numerical calculations.

Measured spectra on perforated silver:

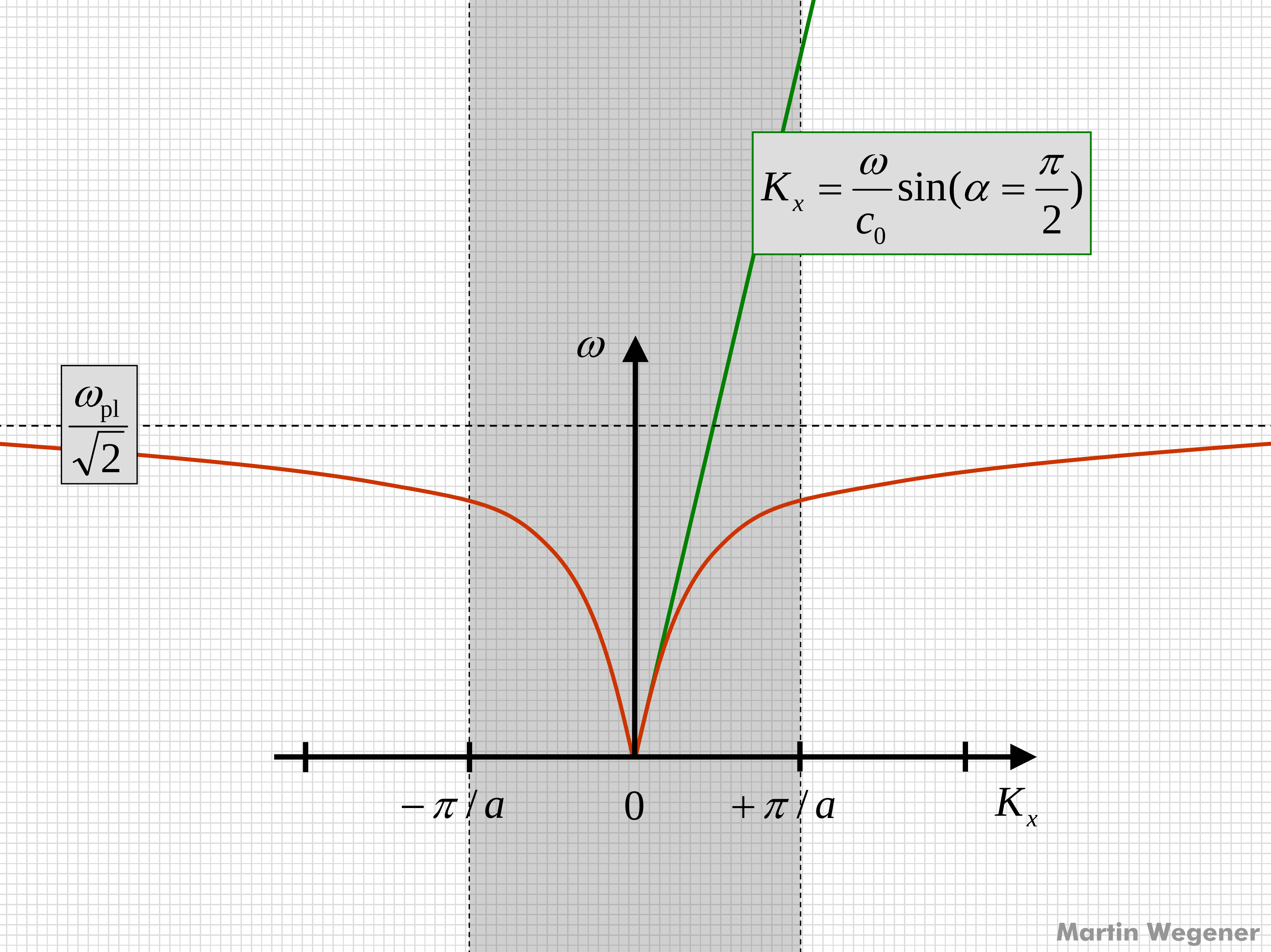


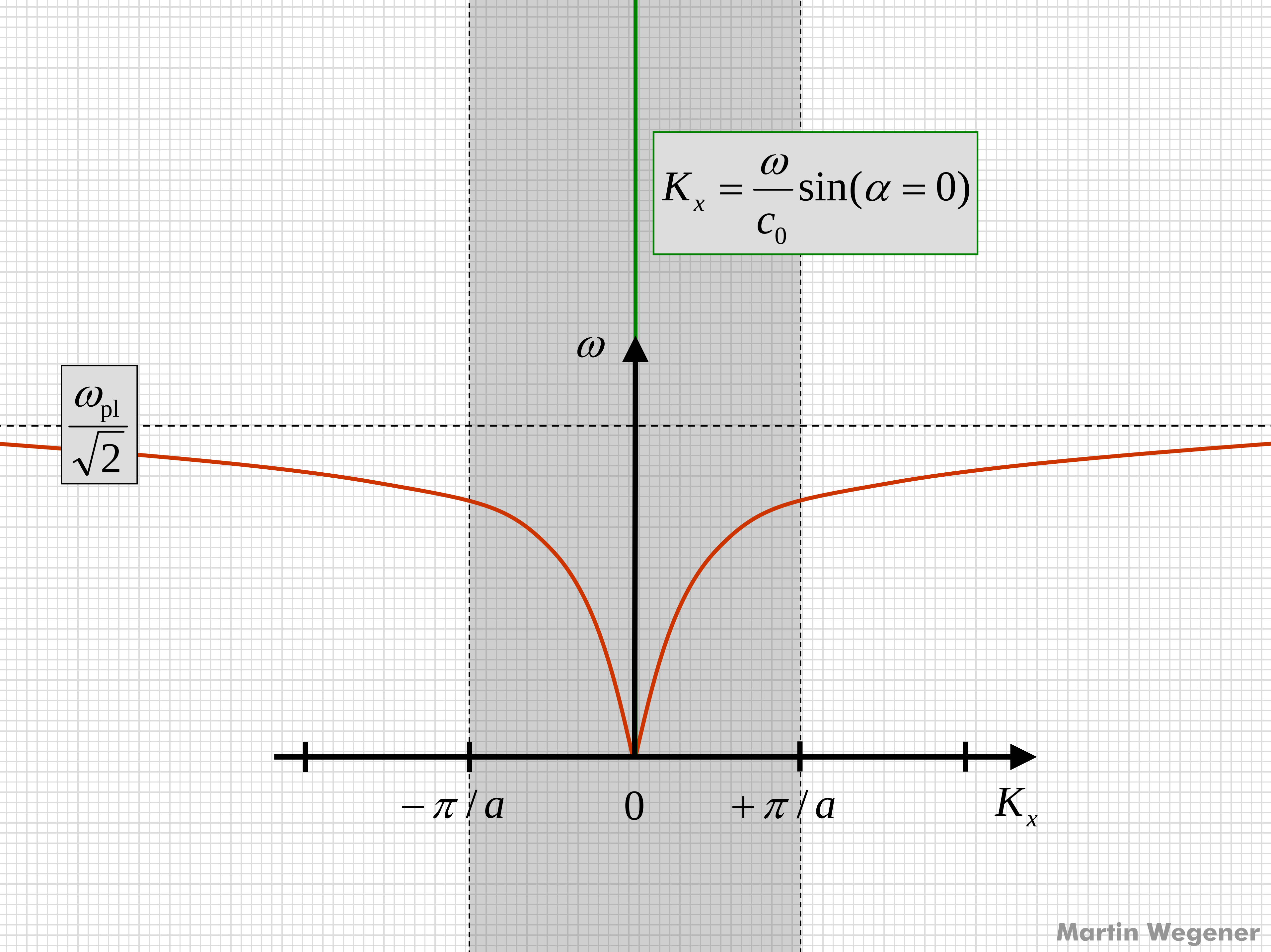
$$a = 900\,\mathrm{nm}$$
$$2r = 150\,\mathrm{nm}$$
$$t = 200\,\mathrm{nm}$$

geometric optics:

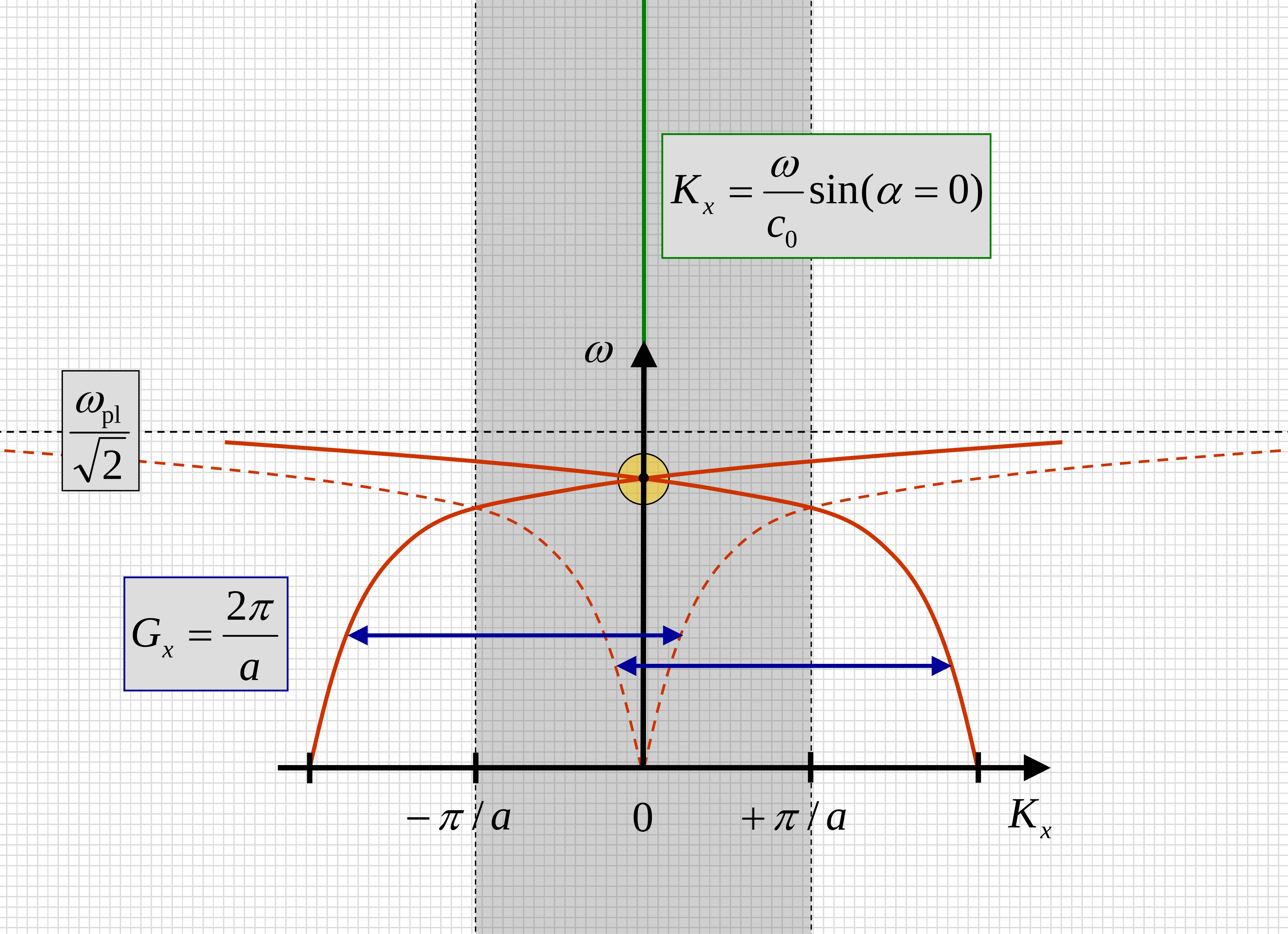
$$T = \frac{\pi\, r^2}{a^2} \approx 2\%$$

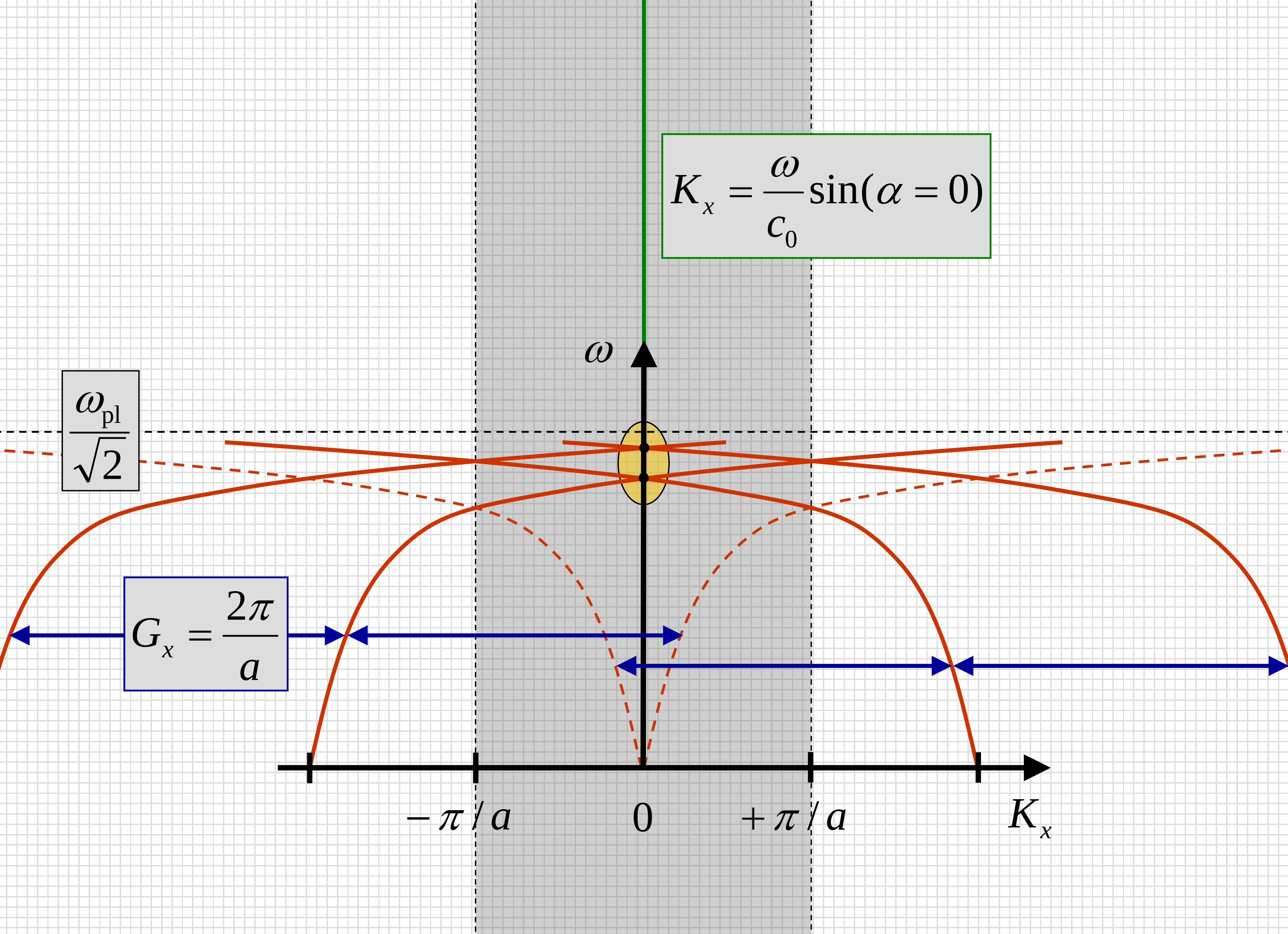
T.W. Ebbesen et al., Nature 391, 667 (1998)


$K_x = \frac{\omega}{c_0} \sin(\alpha = \frac{\pi}{2})$
ω
$\frac{\omega_{\text{pl}}}{\sqrt{2}}$
$-\pi / a$
0
$+\pi / a$
K_x
Martin Wegener



Martin Wegener



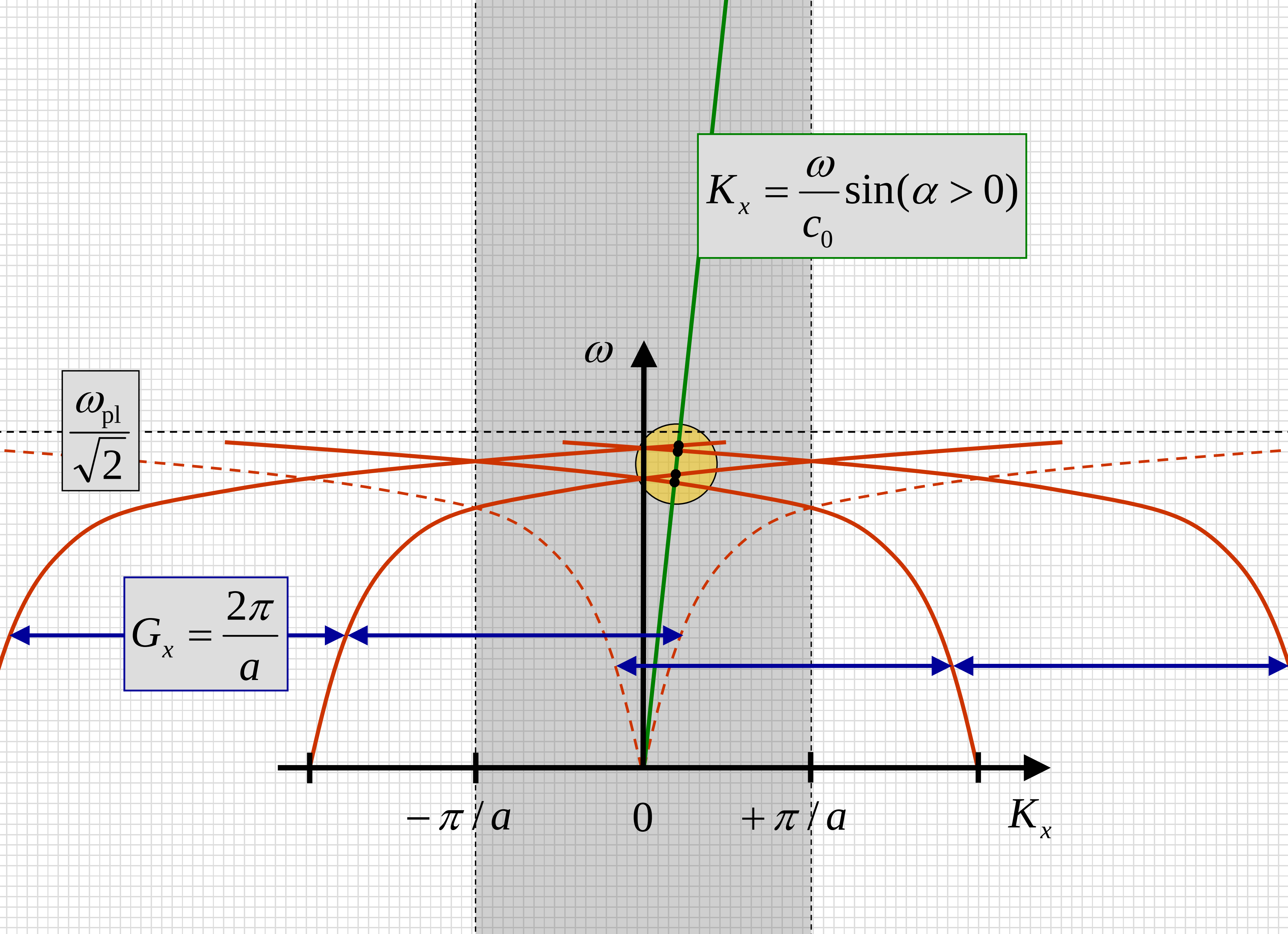


$$K_x = \frac{\omega}{c_0} \sin(\alpha = 0)$$

$$\frac{\omega_{\mathrm{pl}}}{\sqrt{2}}$$

$$G_x = \frac{2\pi}{a}$$

Martin Wegener

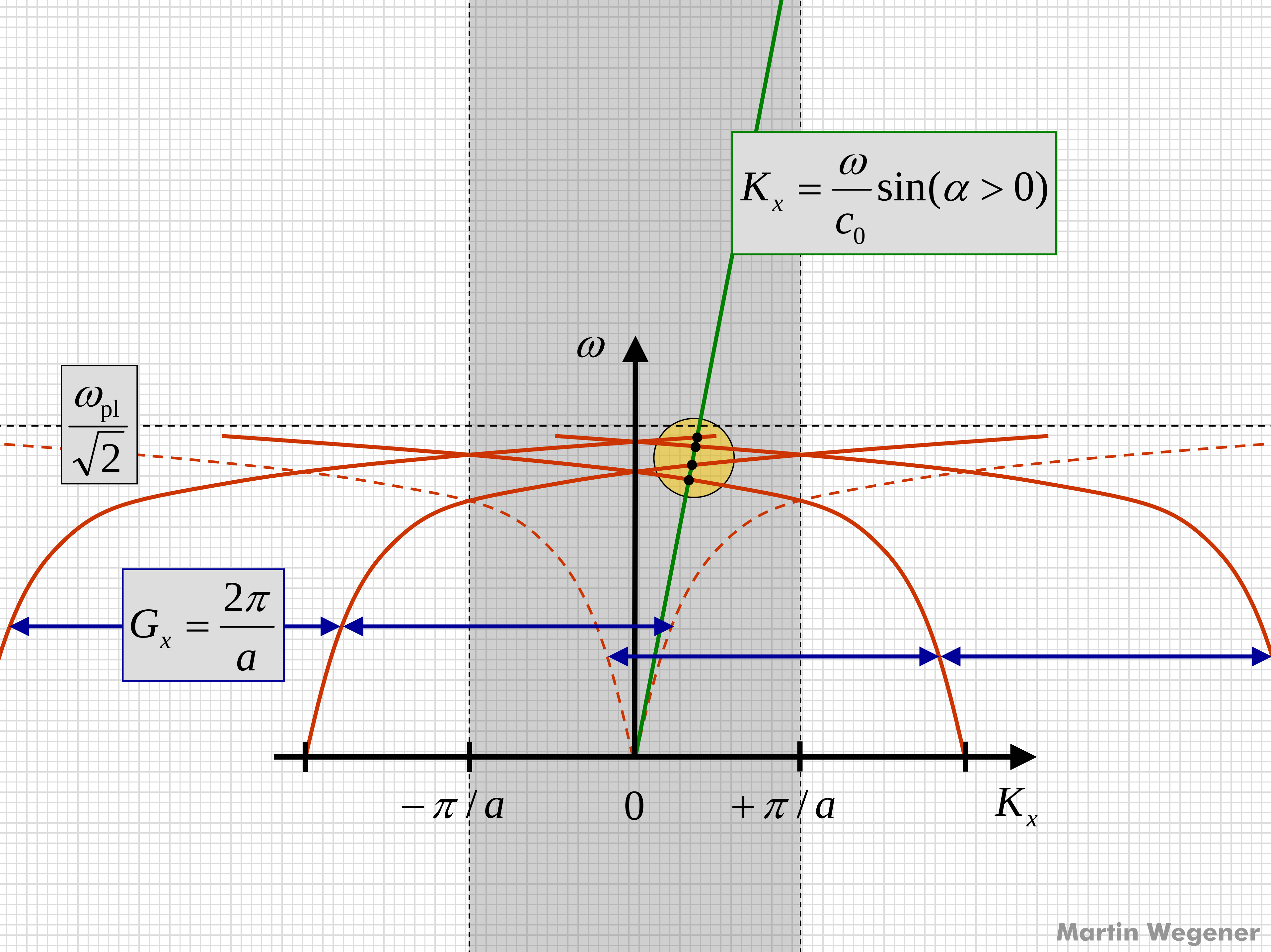


$$K_x = \frac{\omega}{c_0}\sin(\alpha > 0)$$

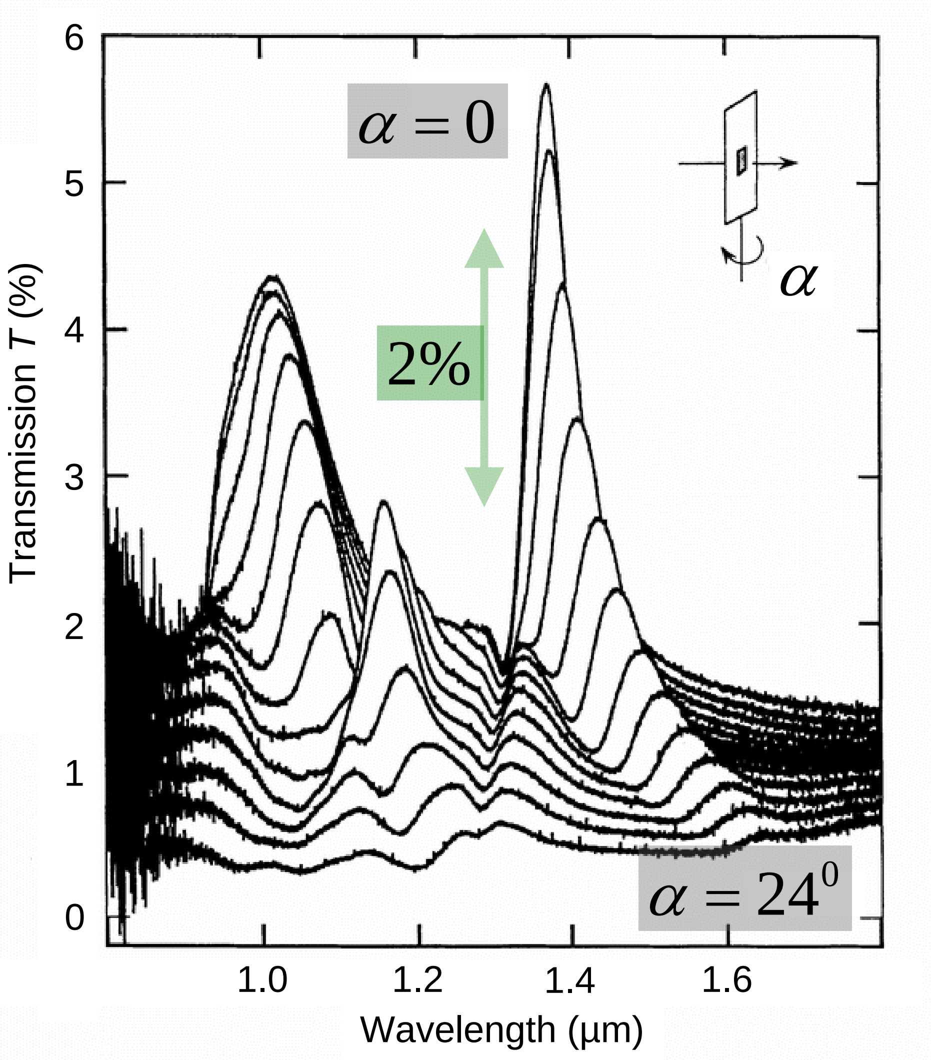
$$\frac{\omega_{\mathrm{pl}}}{\sqrt{2}}$$

$$G_x = \frac{2\pi}{a}$$

Martin Wegener


$K_x = \frac{\omega}{c_0}\sin(\alpha > 0)$
ω
$\frac{\omega_{\text{pl}}}{\sqrt{2}}$
$G_x = \frac{2\pi}{a}$
$-\pi / a$
0
$+\pi / a$
K_x

Measured spectra on perforated silver:



$a = 900$ nm

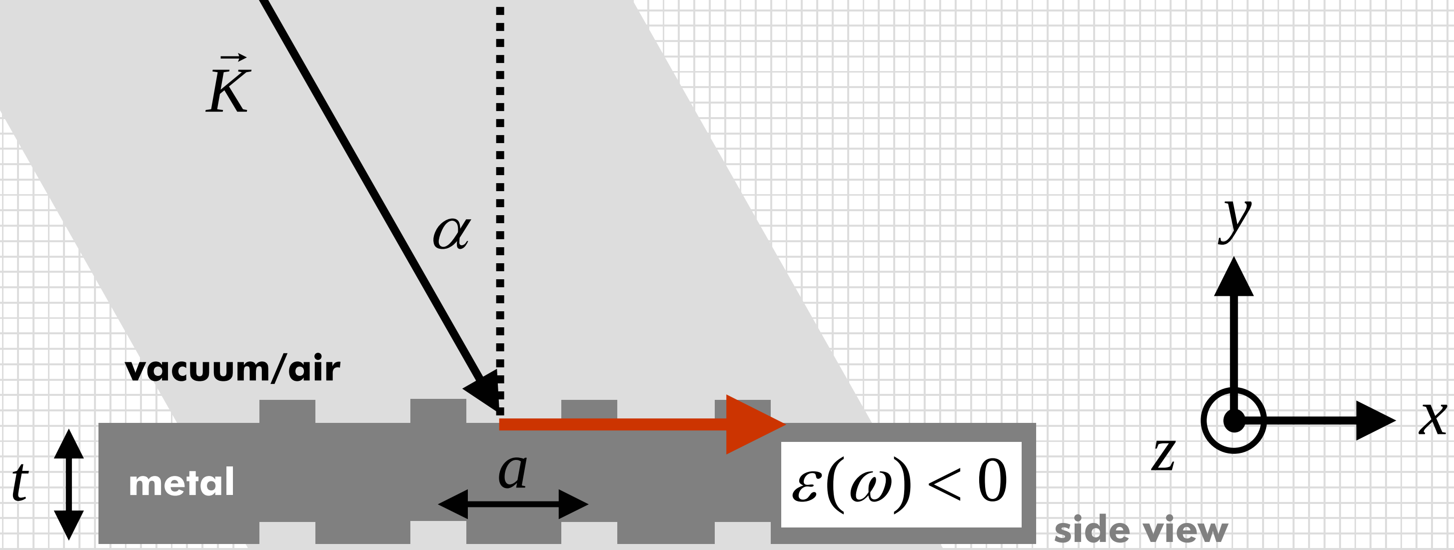
$2r = 150$ nm

$t = 200$ nm

geometric optics:

$$T = \frac{\pi r^2}{a^2} \approx 2\%$$

T.W. Ebbesen et al., Nature 391, 667 (1998)



But! The holes are not essential for obtaining enhanced transmission through the metal film.

Any periodic modulation allows for coupling to the surface plasmon polaritons via reciprocal lattice vectors.

U. Schröter and D. Heitmann, Phys. Rev. B 60, 4992 (1999)
I.R. Hooper and J.R. Sambles, Phys. Rev. B 67, 235404 (2003)

1. **Basics**
    - 1.1. Analogy between photonics and electronics
    - 1.2. Paradigms of nanophotonics
        - 1.2.1. Evanescent waves & the limit of electrostatics
        - 1.2.2. Mie-scattering of nanoparticles
        - 1.2.3. Nanoscale metal particles: Particle plasmons
        - 1.2.4. A nano-aperture in an ideal metal film: Bethe-Bouwkamp theory

2. **Photonic band gap (PBG) materials**
    - 2.1. One-dimensional Photonic Crystals (i.e., dielectric mirrors)
    - 2.2. Two-dimensional Photonic Crystals
        - 2.2.1. Negative refraction and the superprism effect
        - 2.2.2. "Semiconductors for light": Waveguides and defect cavities
    - 2.3. Three-dimensional Photonic Crystals
        - 2.3.1. Quantum optics: Modified Planck's law
        - 2.3.2. Fabrication: Overview and state-of-the-art
        - 2.3.3. Fabrication: Current efforts within the CFN

3. **Metamaterials**
    - 3.1. Left-handed or Veselago materials (LHM)
    - 3.2. "Perfect lenses" made from LHM
    - 3.3. Towards metamaterials @ optical frequencies (CFN activities)

4. **Plasmonics**
    - 4.1. Field-enhancement & surface-enhanced Raman scattering (SERS)
    - 4.2. Surface-plasmon amplification by stimulated emission of radiation (SPASER)
    - 4.3. Transmission through sub-wavelength hole arrays

Martin Wegener

The End