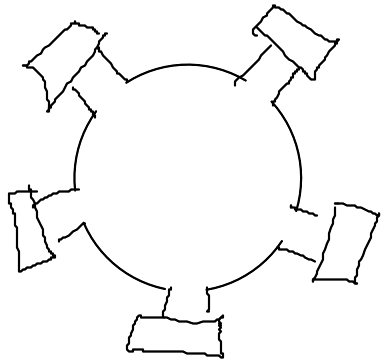
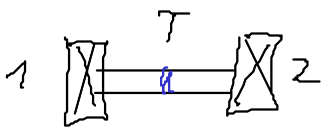
auch so



$$\frac{h}{2e} I_\alpha = [N_\alpha - R_\alpha]\mu_\alpha - \sum_{\beta \neq \alpha} T_{\alpha\beta}\mu_\beta$$

## Beispiele

1) 2 Kontakt - System




$$I_1 = -I_2 = I$$

$$T_{12} = T_{21} = T \;;\quad R_1 = R_2 = R = N - T$$

Zahl der Kanäle, (beliebig)

(wählt man N zu groß, wird mehr reflektiert)

$$\frac{h}{2e} I = [N - R]\mu_1 - T\mu_2$$

$$-\frac{h}{2e} I = [N - R]\mu_2 - T\mu_1$$

Differenz

$$2\frac{h}{2e} I = [N - R](\mu_1 - \mu_2) + T(\mu_1 - \mu_2)$$

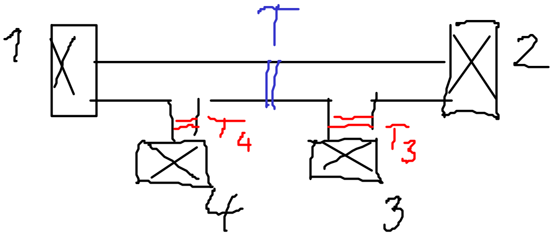
$$= (N - R + T)(\mu_1 - \mu_2) = 2T(\mu_1 - \mu_2)$$

$$\Rightarrow \quad I = \frac{2e}{h}(\mu_1 - \mu_2) = G_{2p}\frac{\mu_1 - \mu_2}{e}$$

Summe

$$0 = (N - R)(\mu_1 + \mu_2) - T(\mu_1 + \mu_2) = 0 \checkmark$$

## 2) 4-Kontakt-System mit 1 Kanal




$T_3$ und $T_4$ sollen eine schwache Kopplung der Spannungs-Messkontakte bringen

$\Rightarrow \quad T_3, T_4 \ll 1$

zur Vereinfachung $T_3 = T_4 = \mathcal{T} \ll 1$

(1)
$$\frac{h}{2e} I = (1-R_1)\mu_1 - T_{12}\mu_2 - T_{13}\mu_3 - T_{14}\mu_4$$

(2)
$$-\frac{h}{2e} I = (1-R_2)\mu_2 - T_{21}\mu_1 - T_{23}\mu_3 - T_{24}\mu_4$$

(3)
$$0 = (1-R_3)\mu_3 - T_{31}\mu_1 - T_{32}\mu_2 - \cancel{T_{34}\mu_4}$$

(4)
$$0 = (1-R_4)\mu_4 - T_{41}\mu_1 - T_{42}\mu_2 - \cancel{T_{43}\mu_3}$$

Abschätzung:

$$T_{12} = T_{21} = T + O(\mathcal{T})$$
$$T_{13} = T\mathcal{T} = T_{31} = T_{24} = T_{42}$$
$$T_{14} = \mathcal{T} + R\mathcal{T} = T_{41} = T_{23} = T_{32}$$
$$R = 1 - T$$

vernachlässige
$$T_{34} = \mathcal{T}^2 T = T_{43} \ll 1$$

(3) − (4)

$$0 = (1-R_3)(\mu_3-\mu_4) - T_{31}(\mu_1-\mu_2) - T_{32}(\mu_2-\mu_1)$$
$$0 = (T_{13}+T_{23}+\cancel{T_{43}})(\mu_3-\mu_4) - (T_{32}-T_{31})(\mu_2-\mu_1)$$
$$\Rightarrow \mu_3-\mu_4 = \frac{T_{32}-T_{31}}{T_{32}+T_{31}}(\mu_2-\mu_1) = \frac{\mathcal{T}(1+R)-\mathcal{T}T}{\mathcal{T}(1+R)+\mathcal{T}T}(\mu_2-\mu_1)$$
$$= \frac{2R}{2}(\mu_2-\mu_1)$$

$$\Longrightarrow \quad \mu_3 - \mu_4 = R(\mu_2 - \mu_1)$$

(1) − (2)

$$\frac{2h}{2e} I = (1-R)(\mu_1 - \mu_2) - T_{12}(\mu_2 - \mu_1) + \mathcal{O}(J)$$

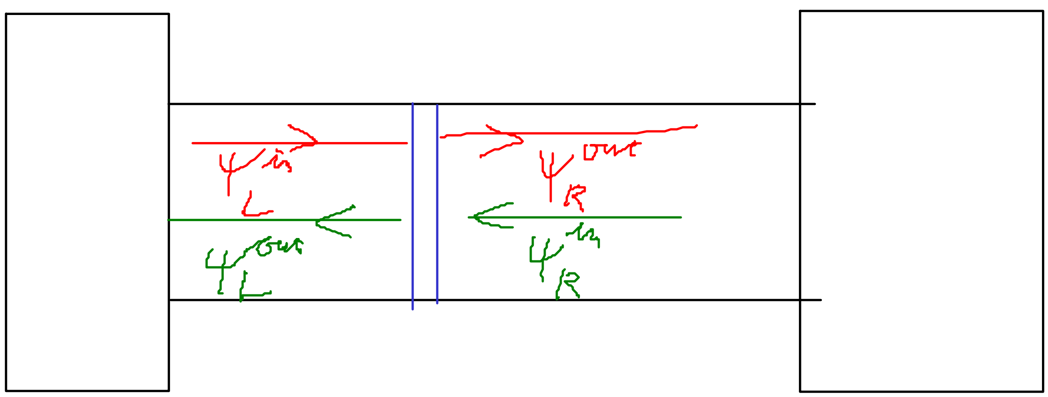
$$= (1 - R + T)(\mu_1 - \mu_2)$$

$$= 2T(\mu_1 - \mu_2)$$

$$\Rightarrow \quad G_{2P} = \frac{eI}{\mu_1 - \mu_2} = \frac{2e^2}{h} T$$

und $G_{4P} = \frac{eI}{\mu_4 - \mu_3} = \frac{eI}{R(\mu_1 - \mu_2)} = \frac{2e^2}{h} \frac{T}{R}$

## 2.8 S-Matrix für Streuung




$$\psi_L^{in}(x,y) = \sum_{n=1}^{N_L} a_n^L \chi_n^L(y) e^{ik_n x}$$

$$\psi_L^{out}(x,y) = \sum_{n=1}^{N_L} b_n^L \chi_n^L(y) e^{-ik_n x}$$

$$\psi_R^{in}(x,y) = \sum_{m=1}^{N_R} a_m^R \chi_m^R(y) e^{ik_m x}$$

$$\psi_R^{out}(x,y) = \sum_{m=1}^{N_R} b_m^R \chi_m^R(y) e^{-ik_m x}$$

S - Matrix

$$
\begin{pmatrix} b_1^L \\ b_2^L \\ \vdots \\ b_N^L \\ b_1^R \\ b_2^R \\ \vdots \\ b_N^R \end{pmatrix}
=
\left(\begin{array}{cccc|cc}
r_{11} & r_{12} & & r_{1N_L} & t'_{11} & t'_{1N_R} \\
\vdots & & & & & \\
r_{N_L 1} & \cdots & & r_{N_L N_L} & t'_{N_L 1} & t'_{N_L N_R} \\
\hline
t_{11} & & & t_{1 N_L} & r'_{11} & r'_{N_R 1} \\
 & & & & & \\
t_{N_R 1} & & & t_{N_R N_L} & r_{N_R 1} & r_{N_R N_R}
\end{array}\right)
\begin{pmatrix} a_1^L \\ a_2^L \\ \vdots \\ a_N^L \\ a_1^R \\ \vdots \\ a_N^R \end{pmatrix}
$$

$a_1^L \Rightarrow b_1^L$ Kanal 1 reflektiert nach Kanal 1 $\Rightarrow r_{11}$

$a_1^L \Rightarrow b_1^R$ Kanal 1 transmittiert auf Kanal 1 $\Rightarrow t_{11}$

$S$ ist quadrat. $(N_L + N_R)^2$ Matrix

$b = S \cdot a$

$(N_L + N_R)$ dim. Vektoren

oder (symbolisch)

$\psi^{out} = S\, \psi^{in}$

Die S-Matrix kann auf mehrere Kontakte verallgemeinert werden $S \Longleftrightarrow t_{\alpha\beta, mn}\,;\ r_{\alpha\alpha, mn}$

## Eigenschaften

a) Beh. $S$ ist unitär $\qquad SS^+ = S^+S = 1$

$$\Rightarrow \langle \psi^{in} | \psi^{in} \rangle =$$

$$\int dx\, dy \sum_{n=1}^{N_L+N_R} \sum_{n'=1}^{N_L+N_R} a_n^* a_{n'}\, \chi_n^*(y)\, \chi_{n'}(y)\, e^{\pm i k_n x} e^{\pm i k_{n'} x}$$

$$= L \sum_n |a_n|^2$$

Orthogonalität nutzen

$$\langle \psi^{out} | \psi^{out} \rangle = L \sum_n |b_n|^2 \qquad (\text{analog})$$

$$= L \sum_{nn'n''} S^*_{nn'} a^*_{n'} S_{nn''} a_{n''}$$

$$= L \sum_{nn'n''} (S^+ S)_{n'n''} a^*_{n'} a_{n''} = L_n \sum_n |a_n|^2$$

$$= \langle \psi^{in} | \psi^{in} \rangle$$

<u>dies Beweist nicht</u> dass $S$ unitär sein muss

auch $\langle \psi^{out} | \psi^{out} \rangle \overset{?}{=} \langle \psi^{in} | \psi^{in} \rangle$

warum ? kein phys. Gesetz

<u>jetzt: Beweis dass</u> $S$ unitär folgt aus Stromerhaltung

$$I_L^{in} = \mathrm{Re}\, \frac{e}{m} \int dy\, \psi_L^{in\,*} (-i\hbar \nabla_x) \psi_L^{in}$$

$$= \frac{e\hbar}{m} \sum_{n=1}^{N_L} |a_n^L|^2 k_n \qquad (?)$$

$$I_L = I_L^{in} - I_L^{out} = I_R = I_R^{out} - I_R^{in} \qquad (\text{rechts} = \text{links})$$

$$\Rightarrow \quad I_L^{in} + I_R^{in} = I_L^{out} + I_R^{out} \qquad (\text{in} = \text{out})$$

$$\Rightarrow \sum_{n=1}^{N_L+N_R} |a_n|^2 k_n = \sum_{n=1}^{N_L+N_R} |b_n|^2 k_n$$

$$\frac{m}{e\hbar} I^{out} = \sum_{m=1} |b_m|^2 k_m = \sum_m \sum_{nn'} S^*_{mn} a_n S_{mn'} a_{n'} k_m$$

$$= \sum_{nn'} \sum_m S^+_{nm} S_{mn'} k_m a^*_n a_{n'}$$

<u>führe neue Matrix ein</u>

$$\tilde{S}_{mn} = S_{mn} \sqrt{\frac{k_m}{k_n}} \iff S_{mn} = \sqrt{\frac{k_n}{k_m}} \tilde{S}_{mn}$$

$$\Rightarrow \tilde{S}^+_{mn} = \tilde{S}^*_{nm} = S^*_{nm} \sqrt{\frac{k_n}{k_m}} = S^+_{mn} \sqrt{\frac{k_n}{k_m}}$$

$$S^{+}_{mn} = \tilde{S}^{+}_{mn} \sqrt{\frac{k_m}{k_n}}$$

$$= \sum_{nn'} \sum_{m} \tilde{S}^{+}_{nm} \sqrt{\frac{k_n}{k_m}} \; \tilde{S}_{mn'} \sqrt{\frac{k_{n'}}{k_m}} \; k_m \; a_n^* a_{n'}$$

$$= \sum_{nn'} \left( \sum_{m} \tilde{S}^{+}_{nm} \tilde{S}_{mn'} \right) \sqrt{k_n k_{n'}} \; a_n^* a_{n'}$$

wenn $\tilde{S}^{+} \tilde{S} = \mathbb{1}$ also $\tilde{S}$ unitär

$$= \sum_{n} |a_n|^2 k_n \quad \checkmark \quad \text{Stromerhaltung}$$

folgt aus $\tilde{S}$ unitär auch $S$ unitär ?

$$\sum_{m} S^{+}_{nm} S_{mn'} = \sum_{m} \tilde{S}^{+}_{nm} \sqrt{\frac{k_n}{k_m}} \; \tilde{S}_{mn'} \sqrt{\frac{k_{n'}}{k_m}}$$

$$\sqrt{\frac{k_m}{k_n}} \sqrt{\frac{k_{n'}}{k_m}} \Rightarrow \mathbb{1}$$

---

b) Für Zeitumkehrinvariante Probleme

$\Rightarrow S = S^{T}$ $\qquad \Rightarrow t_{mn} = t_{nm}$

$\qquad t_{\alpha\beta\, mn} = t_{\beta\alpha\, nm}$

$\Rightarrow G_{\alpha\beta} = G_{\beta\alpha}$

mit Magnetfeld <u>nicht</u> Zeitumkehrinvariant

$$S(\vec{B}) = S^{T}(-\vec{B}) \quad \rightarrow \quad t_{\alpha\beta\, mn}(\vec{B}) = t_{\beta\alpha\, nm}(-\vec{B})$$

$$\rightarrow G_{\alpha\beta}(\vec{B}) = G_{\beta\alpha}(-\vec{B})$$