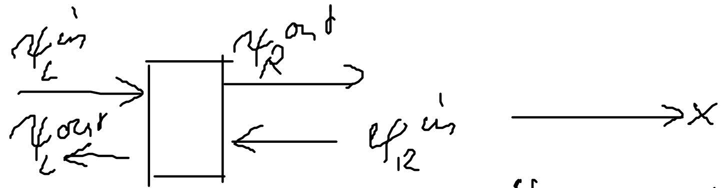
S-Matrix

$\tilde{S}$ ist unitär ⇐ Stromerhaltung

$\bar{S}$ ist nicht unitär

korrigierte Version

• normierte Zustände:



$$\psi_L^{in}(x,y) = \frac{1}{\sqrt{2\pi\hbar}} \sum_{n=1}^{N_L} a_n^L \frac{\chi_n^L(y)}{\sqrt{v_n}} e^{ik_n x} \qquad \psi_L^{out} = \frac{1}{\sqrt{2\pi\hbar}} \sum_{n=1}^{N_L} b_n^L \frac{\chi_n^L(y)}{\sqrt{v_n}} e^{-ik_n x}$$

$$\psi_R^{in}(x,y) = \frac{1}{\sqrt{2\pi\hbar}} \sum_{m=1}^{N_R} a_m^R \frac{\chi_m^R(y)}{\sqrt{v_m}} e^{-ik_m x} \qquad \psi_R^{out} = \frac{1}{\sqrt{2\pi\hbar}} \sum_{m=1}^{N_R} b_m^R \frac{\chi_m^R(y)}{\sqrt{v_m}} e^{ik_m x}$$

⇒ jeder Kanal trägt zum Leitwert $\frac{2e^2}{h}$ bei

$N^{1\,dim}$ · Gruppengeschw. $= \frac{1}{\frac{\partial E}{\partial k}} \frac{\partial E}{\partial k} = 1$

• Strom $I_L^{in} = \mathrm{Re}\left[\int dy \, \frac{e}{m} \left(\psi_L^{in\,*} (-i\hbar\nabla)\, \psi_L^{in}\right)\right]$

$$= \frac{1}{2\pi\hbar} \frac{e\hbar}{m} \sum_n \frac{|a_n^L|^2}{v_n} k_n = \frac{e}{h} \sum_n |a_n|^2$$

• S-Matrix verknüpft

$$\begin{pmatrix} b_1^L \\ \vdots \\ b_{N_L}^L \\ b_1^R \\ \vdots \\ b_{N_R}^R \end{pmatrix} = \underbrace{\left(\begin{array}{c|c} r_{11} & t'_{11} \\ \hline t & r' \end{array}\right)}_{S} \begin{pmatrix} a_1^L \\ \vdots \\ a_{N_L}^L \\ a_1^R \\ \vdots \\ a_{N_R}^R \end{pmatrix}$$

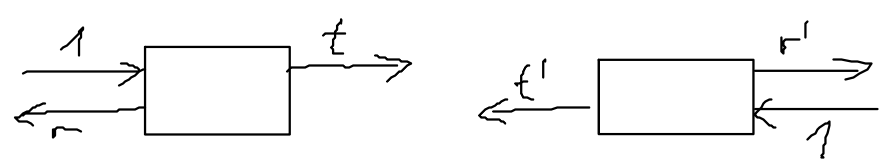
$$\Rightarrow \sum_n |a_n|^2 \overset{?}{=} \sum_m |b_m|^2 = \sum_m \sum_n S_{mn}^* a_n^* \sum_{n'} S_{mn'} a_{n'}$$

$$= \sum_{n n'} \underbrace{\left(\sum_m S_{nm}^+ S_{mn'}\right)}_{\delta_{nn'}} a_n^* a_{n'} = \sum |a_n|^2$$

• Wenn S unitär ist ⇒ Stromerhaltung

S-Matrix für 1 Kanal

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$



• r, t beziehen sich auf von links einlaufende Welle

$$\begin{pmatrix} b^L \\ b^R \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} r \\ t \end{pmatrix}$$

• r', t' beziehen sich auf von rechts einlaufende Welle

$$\begin{pmatrix} b^L \\ b^R \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} t' \\ r' \end{pmatrix}$$

• Zusammenhang zwischen r, r', t und t'

komplex: $t = |t| e^{i\tau}$, $t' = |t'| e^{i\tau'}$

$r = |r| e^{i\rho}$, $r' = |r'| e^{i\rho'}$

• Unitarität $S^\dagger S = \mathbb{1}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \overset{!}{=} \begin{pmatrix} r^* & t^* \\ t'^* & r'^* \end{pmatrix} \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} = \begin{pmatrix} |r|^2 + |t|^2 & r^* t' + t^* r' \\ t'^* r + r'^* t & |t'|^2 + |r'|^2 \end{pmatrix}$$

$$\Rightarrow \underbrace{|r|^2}_{R} + \underbrace{|t|^2}_{T} = \underbrace{|r'|^2}_{R'} + \underbrace{|t'|^2}_{T'} = 1$$

$$r^* t' + r' t^* = 0 \Rightarrow |r||t'| = |r'||t|$$

$$\rho + \tau = \rho' - \tau' + \pi \quad \text{mod } 2\pi$$

$$S S^\dagger = \mathbb{1} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} r^* & t^* \\ t'^* & r'^* \end{pmatrix} = \begin{pmatrix} |r|^2 + |t'|^2 & r t^* + t' r'^* \\ t r^* + r' t'^* & |t|^2 + |r'|^2 \end{pmatrix}$$

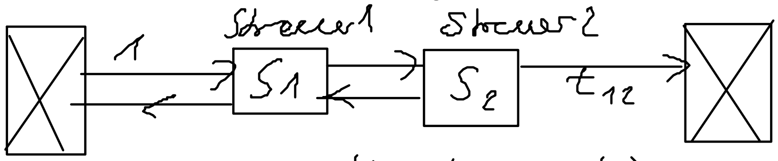
$$\Rightarrow |r|^2 + |t'|^2 = 1 \quad \Rightarrow |t|^2 = |t'|^2, \quad |r|^2 = |r'|^2$$

• **Freie Parameter** $|t| = \sqrt{1 - |r|^2}$, τ, ρ, ρ' mit $\tau' = \rho + \rho' - \tau + \pi$ mod 2π

$$S = \begin{pmatrix} |r| e^{i\rho} & -|t| e^{i((\rho+\rho') - \tau)} \\ |t| e^{i\tau} & |r| e^{i\rho'} \end{pmatrix} = e^{i\frac{\rho+\rho'}{2}} \begin{pmatrix} |r| e^{i\frac{\rho-\rho'}{2}} & -t^* e^{i\frac{\rho+\rho'}{2}} \\ t e^{-i\frac{\rho+\rho'}{2}} & |r| e^{-i\frac{\rho-\rho'}{2}} \end{pmatrix}$$

$$= e^{i\varphi} \begin{pmatrix} \tilde{r} & -\tilde{t}^* \\ \tilde{t} & \tilde{r}^* \end{pmatrix} \quad \text{mit} \quad |\tilde{r}|^2 + |\tilde{t}|^2 = 1$$

2.9 Serienschaltung von Streuern



$$\begin{pmatrix} r_1 & t_1' \\ t_1 & r_1' \end{pmatrix} \begin{pmatrix} r_2 & t_2' \\ t_2 & r_2' \end{pmatrix}$$

$R_1 = |r_1|^2$

$|T_1| = |t_1|^2$

1) klassische Analyse

Widerstand $G_{12,4\ell}^{-1} = G_1^{-1} + G_2^{-1}$

$$G_1^{-1} = \frac{h}{2e^2}\begin{cases} \frac{1}{T_1} & \text{2 Punkt-Widerstand} \\ \frac{R_1}{T_1} & \text{4 Punkt-Widerstand} \end{cases}$$

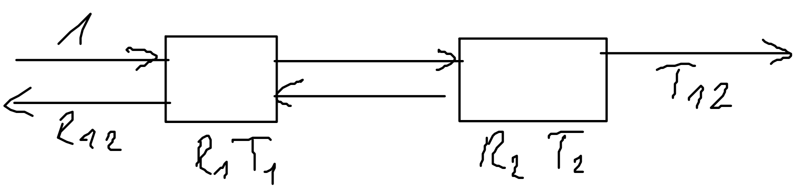
• Welcher Widerstand spielt hier eine Rolle?

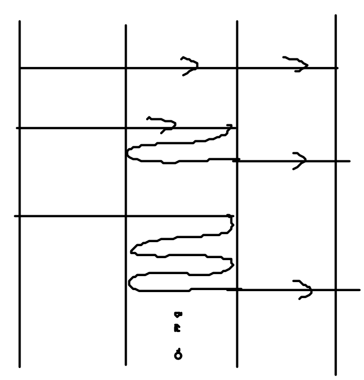
$$G_{2p}^{-1} = G_{4p}^{-1} + \underbrace{\frac{h}{2e^2}}_{\text{Kontaktwiderstand}}$$

• Kontaktwiderstand ist nur am Kontakt zu addieren

⇒ In Serienschaltung addieren sich die $G_{4\ell}^{-1}$

$$\Rightarrow G_{12,4\ell}^{-1} = \frac{h}{2e^2}\left(\frac{R_1}{T_1} + \frac{R_2}{T_2}\right) = \frac{h}{2e^2}\frac{(T_2 R_1 + T_1 R_2)}{T_1 T_2} = \frac{h}{2e^2}\frac{R_1 + R_2 - 2R_1R_2}{T_1 T_2}$$





$$T_{12} = T_1 T_2 \left(1 + R_1 R_2 + (R_1 R_2)^2 + \dots\right) = \frac{T_1 T_2}{1 - R_1 R_2}$$

$$R_{12} = R_1 + \frac{T_1 R_2 T_1}{1 - R_1 R_2} = \frac{R_1 + R_2 - 2R_1R_2}{1 - R_1 R_2}$$

$$\Rightarrow G_{12,4\ell}^{-1} = \frac{h}{2e^2}\frac{R_{12}}{T_{12}} = \frac{h}{2e^2}\frac{R_1 + R_2 - 2R_1R_2}{T_1 T_2}$$ wie oben

2) quantenmechanische Analyse für kohärenten Transport

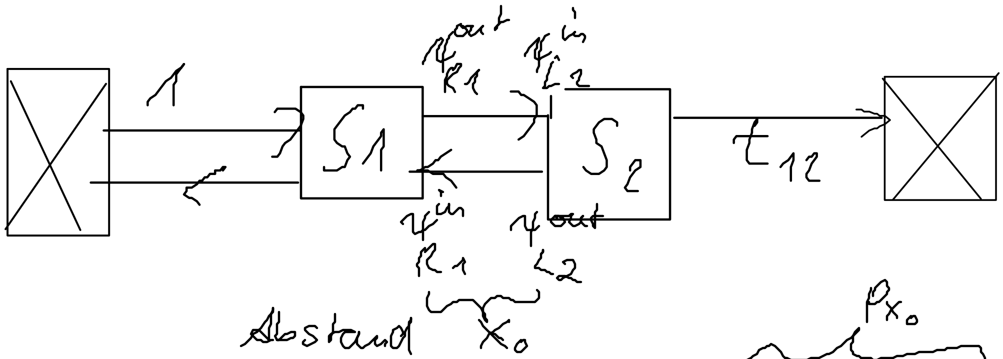
• S_i verknüpft ψ^{in} mit $\psi^{out} = S\psi^{in}$

• Brauche aber M, die ψ^L mit $\psi^R = M\psi^L$ verknüpft

$$\begin{pmatrix}\psi_L^{out}\\ \psi_R^{out}\end{pmatrix} = \begin{pmatrix} r & t' \\ t & r'\end{pmatrix}\begin{pmatrix}\psi_L^{in}\\ \psi_R^{in}\end{pmatrix} \xleftrightarrow{\text{Übung}} \begin{pmatrix}\psi_R^{out}\\ \psi_R^{in}\end{pmatrix} = \underbrace{\begin{pmatrix} -\frac{\Delta}{t'} & \frac{r'}{t'} \\ -\frac{r}{t'} & \frac{1}{t'}\end{pmatrix}}_{M}\begin{pmatrix}\psi_L^{in}\\ \psi_L^{out}\end{pmatrix}$$

mit $\Delta = rr' - tt'$

- Brauche außerdem P_{x_0}

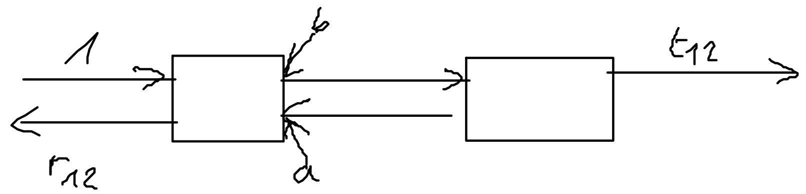


$$\psi_{L2}^{in} = e^{ikx_0}\,\psi_{R1}^{out}$$
$$\psi_{R1}^{in} = e^{ikx_0}\,\psi_{L2}^{out}$$

$$\begin{pmatrix}\psi_{L2}^{in}\\ \psi_{L2}^{out}\end{pmatrix} = \overbrace{\begin{pmatrix} e^{ikx_0} & 0 \\ 0 & e^{-ikx_0}\end{pmatrix}}^{P_{x_0}}\begin{pmatrix}\psi_{R1}^{out}\\ \psi_{R1}^{in}\end{pmatrix}$$

$$\begin{pmatrix}\psi_{R2}^{out}\\ \psi_{R2}^{in}\end{pmatrix} = M_2\, P_{x_0}\, M_1 \begin{pmatrix}\psi_{L1}^{in}\\ \psi_{L1}^{out}\end{pmatrix}$$ (beschreibt Problem bereits)

- Bedingung: $\psi_{R2}^{in} = 0$ Zusammenhang von ψ_{L1}^{in} und ψ_{L1}^{out}
- einfacheres Vorgehen (nicht Matrixmultiplikation)

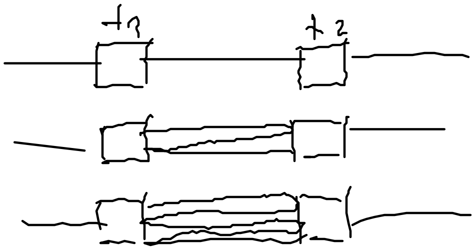


$t_{12}, r_{12} = ?$
$a, b = ?$

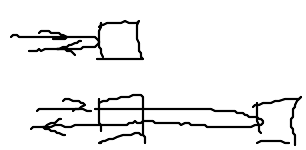
$$r_{12} = r_1 + a t_1' \qquad b = t_1 + a r_1'$$
$$t_{12} = b e^{ikx_0} t_2 \qquad a = b e^{ikx_0} r_2 e^{ikx_0}$$ Übung

$$\Rightarrow t_{12} = \frac{t_1 t_2 e^{ikx_0}}{1 - r_1' r_2 e^{2ikx_0}} \qquad r_{12} = r_1 + \frac{t_1 t_1' r_2 e^{2ikx_0}}{1 - r_1' r_2 e^{2ikx_0}}$$

$$t_{12} = t_1 t_2 e^{ikx_0}\left(1 + r_1' r_2 e^{2ikx_0} + \dots\right)$$



$$r_{12} = r_1 + t_1 t_1' r_2 e^{2ikx_0} + \dots$$



$$\Rightarrow \quad T_{12} = |t_{12}|^2 = \frac{T_1 T_2}{1 + R_1 R_2 - 2\sqrt{R_1 R_2}\cos\Theta} \qquad \Theta = 2kx_0 + \varphi_1' + \varphi_2$$

$$R_{12} = |r_{12}|^2 = 1 - T_{12} = \frac{R_1 + R_2 - 2\sqrt{R_1 R_2}\cos\Theta}{1 + R_1 R_2 - 2\sqrt{R_1 R_2}\cos\Theta}$$

im Allgemeinen

$$\Rightarrow \quad G^{-1}_{12,\,4P} = \frac{h}{2e^2}\,\frac{R_{12}}{T_{12}} = \frac{h}{2e^2}\,\frac{R_1 + R_2 - 2\sqrt{R_1 R_2}\cos\Theta}{T_1 T_2} \neq G^{-1}_{12,\,kl}$$