Hohlraum-Strahlung (Photonen, Phononen)

$$\sum_n f(\varepsilon) = 2V\int \frac{d^3p}{(2\pi\hbar)^3} f(\varepsilon) = 2V\int \frac{4\pi p^2\,dp}{(2\pi\hbar)^3} f(\varepsilon)$$

$\varepsilon = cp \qquad dp = \frac{d\varepsilon}{c} \qquad \Rightarrow g(\varepsilon) = \frac{4\pi\varepsilon^2}{(2\pi\hbar c)^3}$ **für Photonen**

Spin $(2s+1) = 2$?

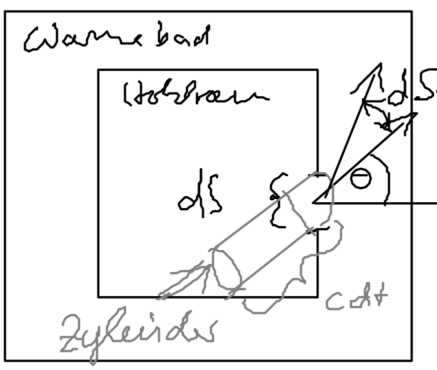
$Z(T,V)$ kanonisch oder $Z(T,V,\mu=0)$ großkanonisch

$$\boxed{x = \beta\hbar\omega}$$

$$\Omega = 2V k_B T \int_0^\infty d\omega \frac{4\pi\omega^2}{(2\pi c)^3} \ln(1-e^{-x}) = \frac{2V k_B T}{(\beta\hbar)^3}\int_0^\infty dx \frac{4\pi x^2}{(2\pi c)^3}\ln(1-e^{-x})$$

$$= \frac{2V(k_B T)^4}{(2\pi\hbar c)^3} 4\pi \int_0^\infty dx\, x^2 \ln(1-e^{-x})$$

$$U = F + TS = F + T\left(4\frac{F}{T}\right) = -3F$$



$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$ Wärmekapazität

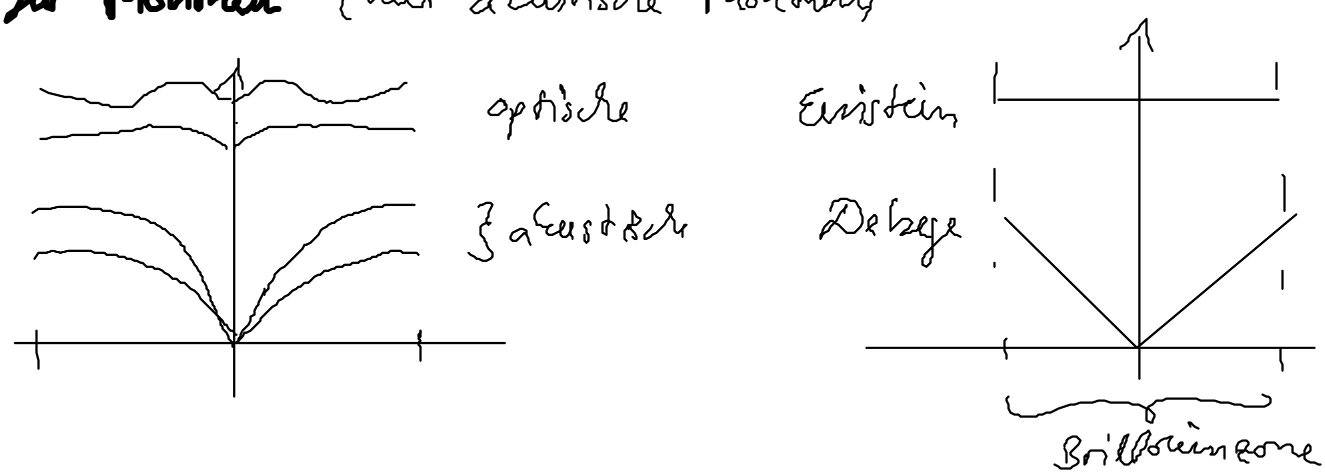
$$U\,d\omega = \frac{1}{V}\frac{1}{e^{\beta\hbar\omega}-1}\frac{4\pi k^2 dk}{(2\pi)^3}(2V)\hbar\omega$$

$$\hbar\omega \ll k_B T: \quad u(\omega,T) = \frac{\hbar}{\pi^2 c^3}\frac{\omega^3}{1+\beta\hbar\omega-1}$$

$dV_{Zylinder} = c\,dt\,(dA\cos\Theta) \qquad dE = dV\, u(\omega,T)\, d\omega\, d\Omega$

Abgestrahlte Leistung $\qquad dI(\theta) = \frac{dN}{dt} E$

für Phononen (nur akustische Phononen)



ideale Systeme

- keine exakte Rechnung möglich ⇒ Näherungen verwenden

z.B $V_{Coulomb} = \frac{e^2}{|\vec{r}_1 - \vec{r}_2|}$ (cgs-System)

$H = H_{kin}(\{p_i\}) + H_{pot}(\{r_i\}) + H_{int}(\{r_i\})$

(H_{int}: Wechselwirkung)

- klas. Betrachtung
- quasi reale Systeme

Virialentwicklung

$Z = Tr(e^{-\beta H})$

- Virialkoeffizienten B und C

$$f(x) = \ln(1-x) \quad f'(x) = -\frac{1}{1-x} \quad f'' = -\frac{1}{(1-x)^2} \quad f''' = -\frac{2}{(1-x)^3}$$

$$f^{(n)} = -\frac{(n-1)!}{(1-x)^n}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} x^n f^{(n)}(x=0) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$-\Omega = k_B T Z_1 e^{\beta\mu} + k_B T \left(Z_2 - \frac{Z_1^2}{2}\right) e^{2\beta\mu}$$

$$N = Z_1 e^{\beta\mu} + (2 Z_2 - Z_1^2) e^{2\beta\mu}$$

$$e^{2\beta\mu} = \left(\frac{N}{Z_1}\right)^2$$

$$-\Omega = k_B T \left(Z_1 e^{\beta\mu} + \left(Z_2 - \frac{Z_1^2}{2}\right) e^{2\beta\mu}\right) = k_B T \left(N - \left(Z_2 - \frac{Z_1^2}{2}\right) e^{2\beta\mu}\right)$$

$$\vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2} \qquad \vec{r} = \vec{r}_1 - \vec{r}_2$$

$$\int d\vec{r}_1 \, d\vec{r}_2 \, f(\vec{r}_1 - \vec{r}_2) = \int d^2R \, f(\vec{r}_1 - \vec{r}_2)$$

$$B = -V \left(\frac{\frac{1}{2} V \int dr \, e^{-\beta V(r)}}{\lambda_T^6 \left(\frac{V}{\lambda_T^3}\right)^2} - \frac{1}{2} \right) = -\frac{1}{2} V \left(\frac{\int e^{-\beta V(r)}}{V} - 1 \right)$$

$$= -\frac{1}{2} \int d^3r \left(e^{-\beta V(r)} - 1 \right)$$

$b = \frac{1}{2} \frac{4\pi}{3} (2r_0)^3$ $\qquad$ $B = b - a/\beta$

↑ 2 Teilchen (ununterscheidbar)

$pV = k_B T N\left(1 + bn - \frac{an}{k_B T}\right)$

$pV + Nan = k_B T N(1 + bn)$

$V(p + an^2) = k_B T N(1 + bn)$

$$\boxed{V(p + an^2)(1 - bn) = k_B T N}$$

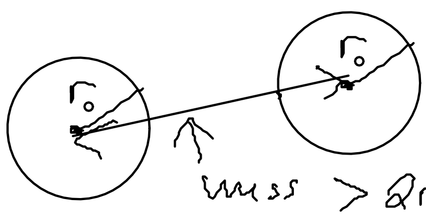
$$\boxed{\frac{N}{V} = n}$$

$$\boxed{\frac{1}{1+x} \approx 1 - x}$$ x klein

für $nb \ll 1$

$n r_T^3 \ll 1$

Van der Waals-Gleichung



muss $> 2r_0$ sein, sonst ausgeschlossenes Volumen (Teilchen ineinander, nicht mehr frei)

$V - V_0 = V - Nb$

$(V - V_0)^n = (V - Nb)^n = V^n \left(1 - \frac{Nb}{V}\right)^n$

$$\boxed{N! = \left(\frac{N}{e}\right)^N}$$