

# Phasenübergang

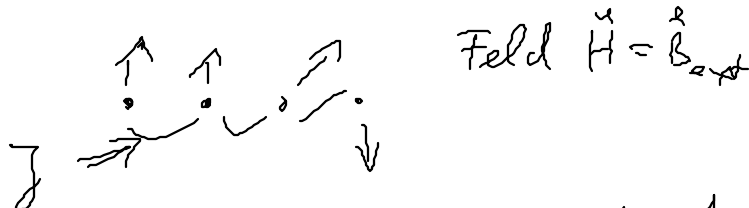
$\vec{M}$  intensiv

$\vec{H}$  extensiv ( $\vec{M} \Rightarrow V \cdot \vec{M}$ )

$$\vec{B} = \vec{H} + 4\pi \vec{M} \quad \delta W = - \frac{\vec{H} \cdot d\vec{B}}{4\pi}$$

$$\vec{H} \leftarrow \text{const} \quad d\vec{B} = 4\pi d\vec{M}$$

## Heisenberg Modell



- Für Gase  $\langle \vec{H} \rangle = 0$
- für Ising  $\vec{m}_i = g \mu_B \vec{S}_i$  ; Hamiltonoperator  $\hat{H} = -H \sum_i m_i = -H \sum_i S_i^x$

$$\vec{S}_i = \frac{\hbar}{2} \vec{\sigma}_i$$

• Zustandssumme  $Z = \sum_{\text{Zustände}} e^{-\beta E_n} = \sum_{\{\sigma_i\}} e^{-\beta \sum_i \sigma_i} = \prod_i \left( e^{-\beta \hbar \sigma_i} + e^{\beta \hbar \sigma_i} \right) = \left( 2 \cosh(\beta \hbar) \right)^N$

$\sigma_i = 1 \quad \sigma_i = -1$

z.B. Zustand  $|\uparrow_1\rangle |\downarrow_2\rangle |\downarrow_3\rangle |\uparrow_4\rangle$

allgemein  $|\sigma_1\rangle |\sigma_2\rangle \dots |\sigma_N\rangle \quad \sigma_i = \pm 1$

$$H = -\hbar \sum_i \sigma_i \Rightarrow E_n = -\hbar \sum_i \sigma_i$$

•  $\vec{M} = - \frac{\partial G}{\partial \vec{H}} = - \frac{\partial}{\partial \hbar} \left( - \frac{1}{\beta} \ln Z \right) \frac{\partial \hbar}{\partial \vec{H}} = \frac{1}{\beta} \left( \frac{\partial}{\partial \hbar} \ln \left( 2 \cosh(\beta \hbar) \right)^N \right) \frac{g \mu_B \hbar}{2}$

$$= \tanh(\beta \hbar) \frac{N}{\beta} \frac{g \mu_B \hbar}{2}$$

$$\vec{M} = \sum_i \frac{g \mu_B \hbar}{2} \langle \sigma_i \rangle = N \frac{g \mu_B \hbar}{2} \langle \sigma_z \rangle$$

$$\vec{H} = H_z \hat{z}$$

$$\langle \sigma_z \rangle = \tanh(\beta \hbar)$$

## Ferromagnetismus des Heisenberg-Modell

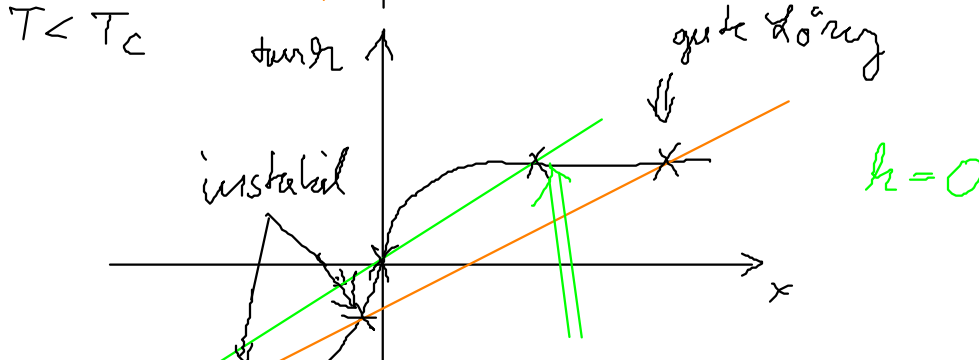
$\vec{M}_F$ : Molekularfeld  $\hat{=}$  mean field

$n, N$ : nächste Nachbarn

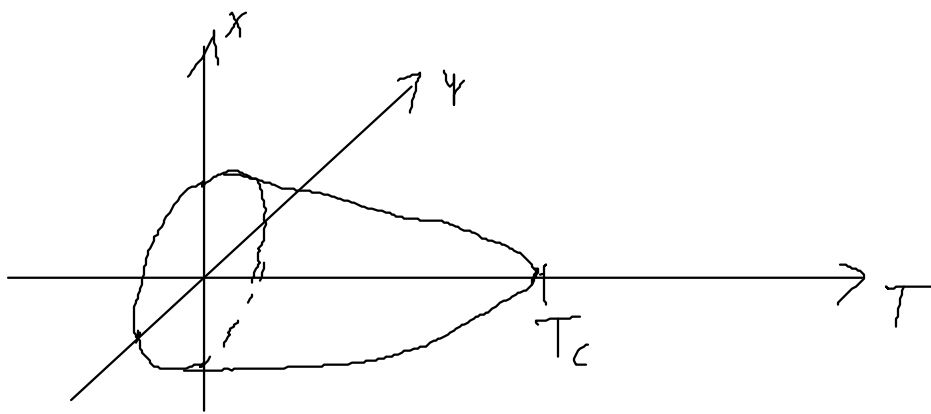
$z$ : Koordinationszahl  $\hat{=}$  Zahl der n. N

- Feld in  $z$ -Richtung  $\Rightarrow$  alle  $\sigma_i \Rightarrow \sigma_z$  und  $\vec{H} \parallel \vec{H}_{MF}$

$\tanh(\beta h + x) = \frac{T}{T_c} x$ 
 $x = \beta h_{MF}$ 
 $\beta = \frac{1}{k_B T}$



wobei die beiden minima thermodyn. Potential



- Spontane Symmetrie brechung:  
System wählt spontan eine Symmetrie

**MF als Variationsverfahren thermodyn. Potential  $G$**

$G_{\text{var}} = \text{Tr}(A W_{MF}) + k_B T \text{Tr}(W_{MF} \ln(W_{MF}))$

$W_{MF} = \frac{1}{Z_{MF}} e^{-\beta H_{MF}}$   
 $\ln W_{MF} = -\ln Z_{MF} - \beta H_{MF}$