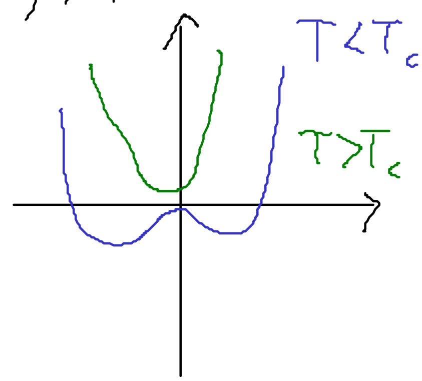
Landau-Theorie der Phasenübergänge

$$G_{var}(T,\vec{H},\vec{m}) \longrightarrow \frac{\partial G}{\partial m} = 0 \Rightarrow \vec{m}_0(T,\vec{H})$$

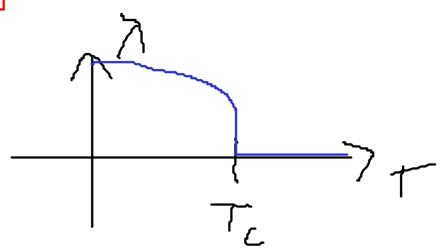
$$\Rightarrow G(T,\vec{H},\overset{\text{Minimum}}{\vec{m}_0(T,\vec{H})})$$

$T \sim T_c$ $\qquad G_{var} = a(T)\,\vec{m}^2 + b(T)\,\vec{m}^4$



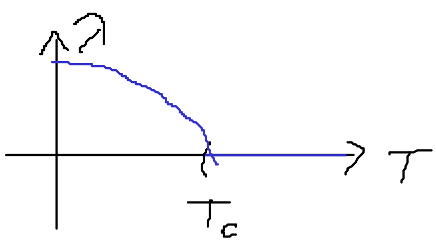
Def. Phasenübergang

1. Art $\qquad$ Ordnungsparameter λ



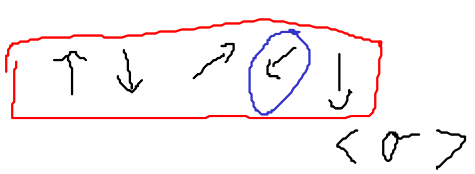
Sprung bei T_c

2. Art



(1. Ableitung hat Sprung)

(n-te Art: $(n-1)$te Ableitung des Ordnungsparameters springt)

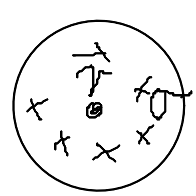


$\langle \sigma \rangle$

Nachteil: starke Schwankungen

$$m(\vec{r}) = \sum_{i \in \text{Bereich um } \vec{r}} \vec{\sigma}_i$$

Coarse graining



Schwankung klein

$$Z = \sum_n e^{-\beta E_n}$$

$\vec m(\vec r)$ fest

$n(\vec m(\vec r))$ Zahl von Bereichen mit $\vec m(\vec r)$

$$= \text{"}\sum_{m(\vec r)}\text{"} \sum_{n(\vec m(\vec r))} e^{-\beta E_n}$$

$$= \sum_{m(\vec r)} Tr_{\vec m(\vec r)}\left(e^{-\beta H}\right) = \int Dm \, Tr_m\left(e^{-\beta H}\right)$$

Pfadintegral

$$Tr_m\left(e^{-\beta H}\right) = e^{-\beta F(m)} =: Z_m$$

$$F(m) := -\frac{1}{\beta} \ln\left(Tr_m e^{-\beta H}\right) = -\frac{1}{\beta} \ln(Z_m)$$

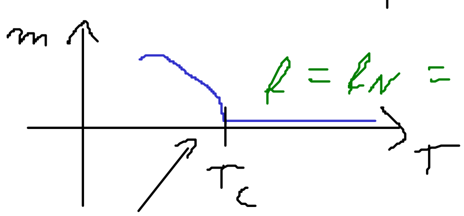
$$= \int Dm \, e^{-\beta F(m)} = \text{"}\sum_m\text{"} e^{-\beta F(m)}$$

$F(m)$ Landau freie-Energie-Funktional

$F(m)$ i.A. nicht berechenbar, daher Postulat

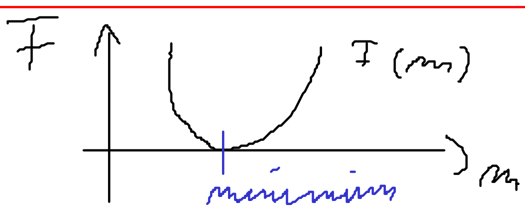
$$F\left(\{\vec m(\vec r)\}, T, \vec H\right) = \int d^3\vec r \, f\left(m(\vec r), T, \vec H\right)$$

$$f(\vec r) = f_N + f_0 \left(\frac{a(T)}{2} \vec m(\vec r)^2 + \frac{b(T)}{4} \vec m(\vec r)^4 + \frac{1}{2} \rho_0^2 \left|\vec\nabla_{\vec r} m\right|^2 \right) - hm$$



Entwicklung um klein m

Molekularfeldnäherung



$$\int dx\, e^{-\chi(x)} \approx e^{-\chi(x_{min})}$$

$$Z_{MF} = e^{-\beta F(m_{min})} \qquad m_{min} = ?$$

$$\delta F = \int dV \Big[f_0 \big\{ a(T)\, m(\vec r)\, \delta m(\vec r) + b(T)\, m^3(\vec r)\, \delta m(\vec r) - \rho_0^2 \nabla^2 m(\vec r)\, \delta m(\vec r) \big\} - h\, \delta m \Big]$$

$$\delta(\nabla m \nabla m) = \nabla \delta m\, \nabla m + \nabla m\, \nabla \delta m = 2 \nabla \delta m\, \nabla m$$

$$\delta F = \int dV\, (\ldots)\, \delta m$$

$$\Rightarrow f_0 (a m + b m^3 - \rho_0^2 \nabla^2 m) - h = 0 \Rightarrow m_{min}$$

homogene Lösungen $\quad \nabla^2 m = 0$

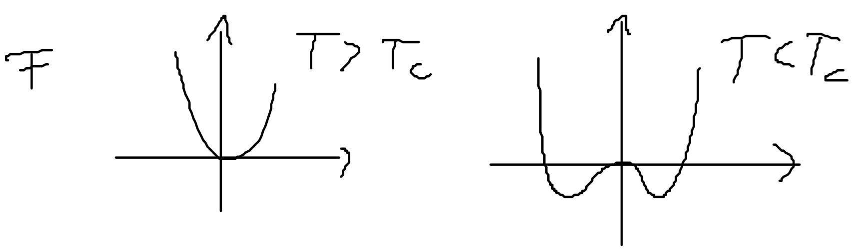
$$a m + b m^3 = \frac{h}{f_0}$$

weitere Bed. $\quad a = \varepsilon = \dfrac{T - T_c}{T_c} \qquad (T \sim T_c)$

b const $\qquad b > 0$

$h = 0 \qquad T > T_c \quad m = 0$

$\qquad\qquad T < T_c \quad m = \pm\sqrt{-\dfrac{a}{b}} = \pm\sqrt{\dfrac{|\varepsilon|}{b}}$



Wärmekapazität

$$Z_{MF} = e^{-\beta F(m_{min})} = e^{-\beta G(T, H)}$$

$$\Leftrightarrow \quad G = F(m_{min})$$

$$c_H = -T \frac{\partial^2 G}{\partial T^2}$$

$$F = V \cdot f_0 \left(\frac{a}{2} m^2 + \frac{b}{4} m^4 \right) - V h m + V f_N$$

(kein Gradient da homogen)

$\boxed{h = 0}$

$T > T_c$ $\quad F = V f_N$

$T < T_c$

$$F = V f_0 \left(\frac{a}{2} \left(-\frac{a}{b}\right) + \frac{b}{4}\left(\frac{a^2}{b^2}\right)\right) + V f_N = V f_N - V f_0 \left(\frac{1}{4} \frac{a^2}{b} \right)$$

$$= V f_N - V f_0 \frac{(T - T_c)^2}{4 T_c^2 b}$$

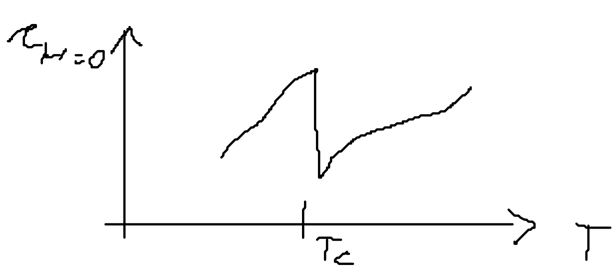
$$c_{H=0} = c_N + T \frac{V f_0}{2 T_c^2 b} \qquad T < T_c$$

Normale Wärmekapazität

$$c_{H=0} = c_N \qquad T > T_c$$

$$c_N = -T \frac{\partial^2}{\partial T^2} (V f_N(T))$$

Wärmekapazität hat einen Sprung bei $T = T_c$



Nicht homogene Situation

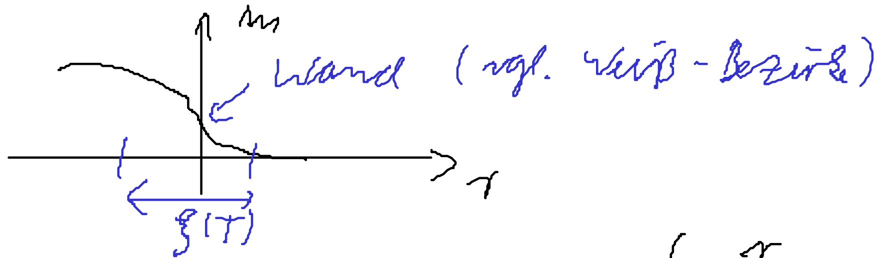
$a < 0\,, \quad T < T_c$

$$a m + b m^3 - \xi_0^2 \nabla^2 m = 0$$

z.B.

↑↑ ? ? ? ↓↓

Randbedingungen fest



Lösung $\quad m = m_0 \tanh\left(\frac{r}{2\xi(T)}\right)$

$$\xi(T) = \frac{\xi_0}{\sqrt{2|a|}}$$ Korrelationslänge

bei $T \to T_c \qquad a = \frac{T - T_c}{T_c} \Rightarrow \xi(T) \longrightarrow \infty$